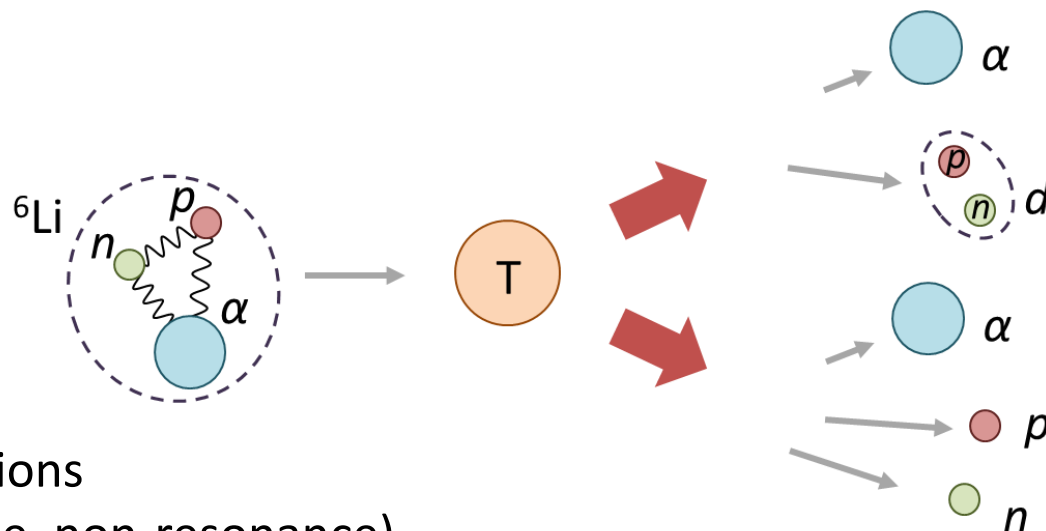


Theoretical study on multichannel breakup reactions

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¹NIT, Gifu College, ²RIKEN, ³RCNP, Osaka Univ., ⁴Osaka City Univ. & NITEP, ⁵Kyushu Univ.



Key words

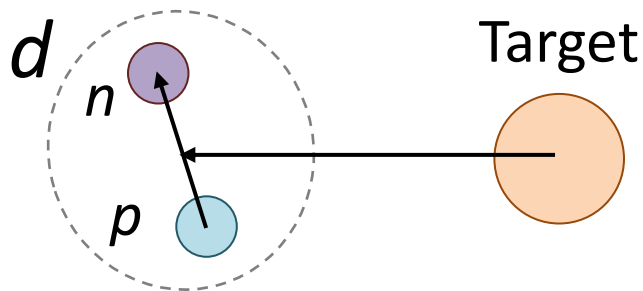
- Multichannel breakup reactions
- Continuum states (resonance, non-resonance)
- Discretized continuum states

2021/06/24

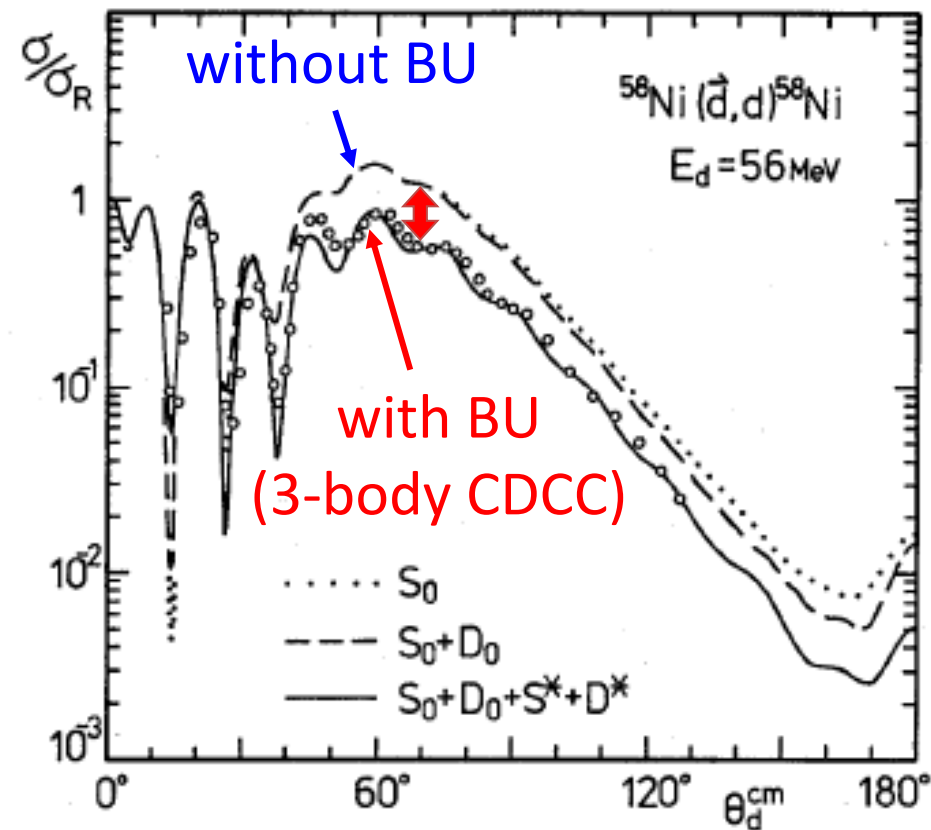
Reaction Seminar 2021 (Zoom)

CDCC: Continuum-Discretized Coupled-Channels

- ✓ CDCC is a fully quantum mechanical method for treating projectile-BU effects.
- ✓ CDCC was born as a theory for d -scattering \Rightarrow 3-body CDCC



Three-body CDCC has been **widely applied** and **successful** in describing many kinds of three-body reactions.



BU effects in elastic scattering

M. Yahiro, Y. Iseri, H. Kameyama, M. Kamimura, and M. Kawai, Prog. Theor. Phys. Suppl. No. 89 (1986), 32.

Development of CDCC

Three-body CDCC (1980s-)

Four-body CDCC (2004-)

T. Matsumoto, E. Hiyama, K. Ogata, Y. Iseri, M. Kamimura, S. Chiba, and M. Yahiro, *Phys. Rev. C* **70**, 061601(R) (2004).

I. J. Thompson, F. M. Nunes, and B. V. Danilin, *Comput. Phys. Commun.* **161**, 87 (2004).

M. Rodríguez-Gallardo, J. M. Arias, J. Gómez-Camacho, A. M. Moro, I. J. Thompson, and J. A. Tostevin, *Phys. Rev. C* **80**, 051601(R) (2009).

T. Matsumoto, K. Katō, and M. Yahiro, *Phys. Rev. C* **82**, 051602(R) (2010).

P. Descouvemont, *Phys. Rev. C* **97**, 064607 (2018).

Three-body CDCC with core excitation (2014-)

R. de Diego, J. M. Arias, J. A. Lay, and A. M. Moro, *Phys. Rev. C* **89**, 064609 (2014).

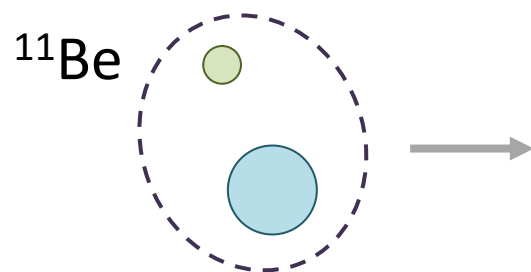
J. A. Lay, R. de Diego, R. Crespo, A. M. Moro, J.M. Arias, and R. C. Johnson, *Phys. Rev. C* **94**, 021602(R) (2016).

R. de Diego, R. Crespo, and A. M. Moro, *Phys. Rev. C* **95**, 044611 (2017).

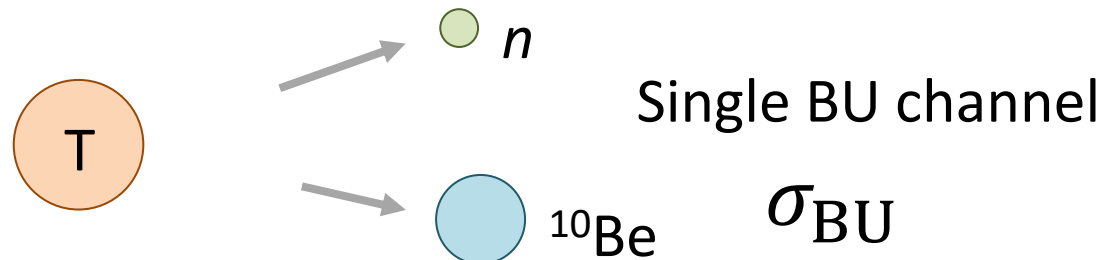
These methods address breakup reactions
including **multi-breakup channels**.

Multi-channel breakup reactions

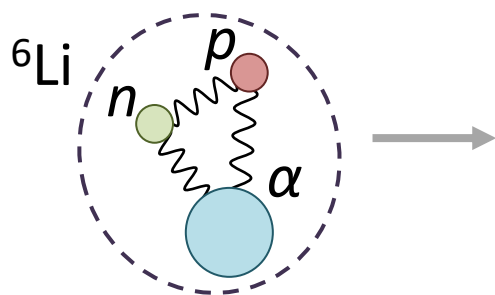
3-body BU reaction



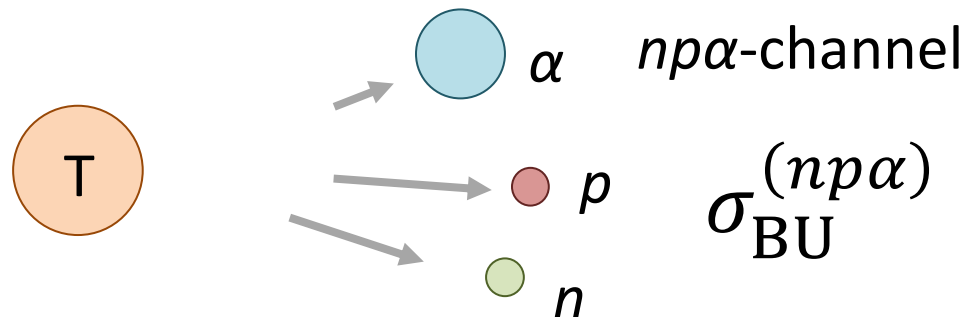
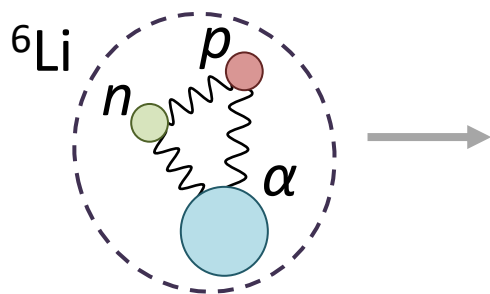
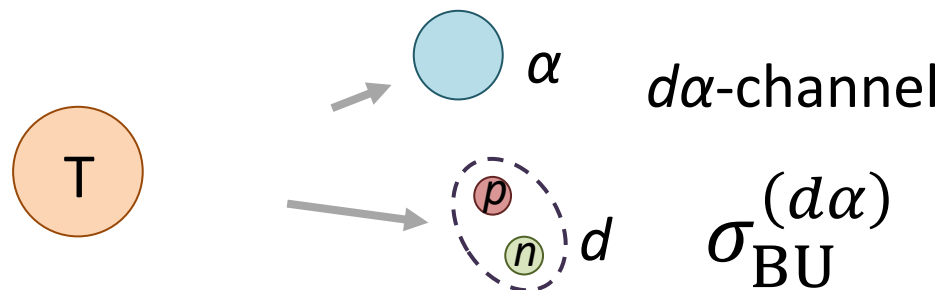
3-body CDCC: *M. Yahiro, Y. Iseri, H. Kameyama, M. Kamimura, and M. Kawai, Prog. Theor. Phys. Suppl. No. 89 (1986), 32.*



4-body BU reaction



4-body CDCC: *T. Matsumoto et al., PRC 70, 061601(R) (2004).*



Multi-channel breakup reactions

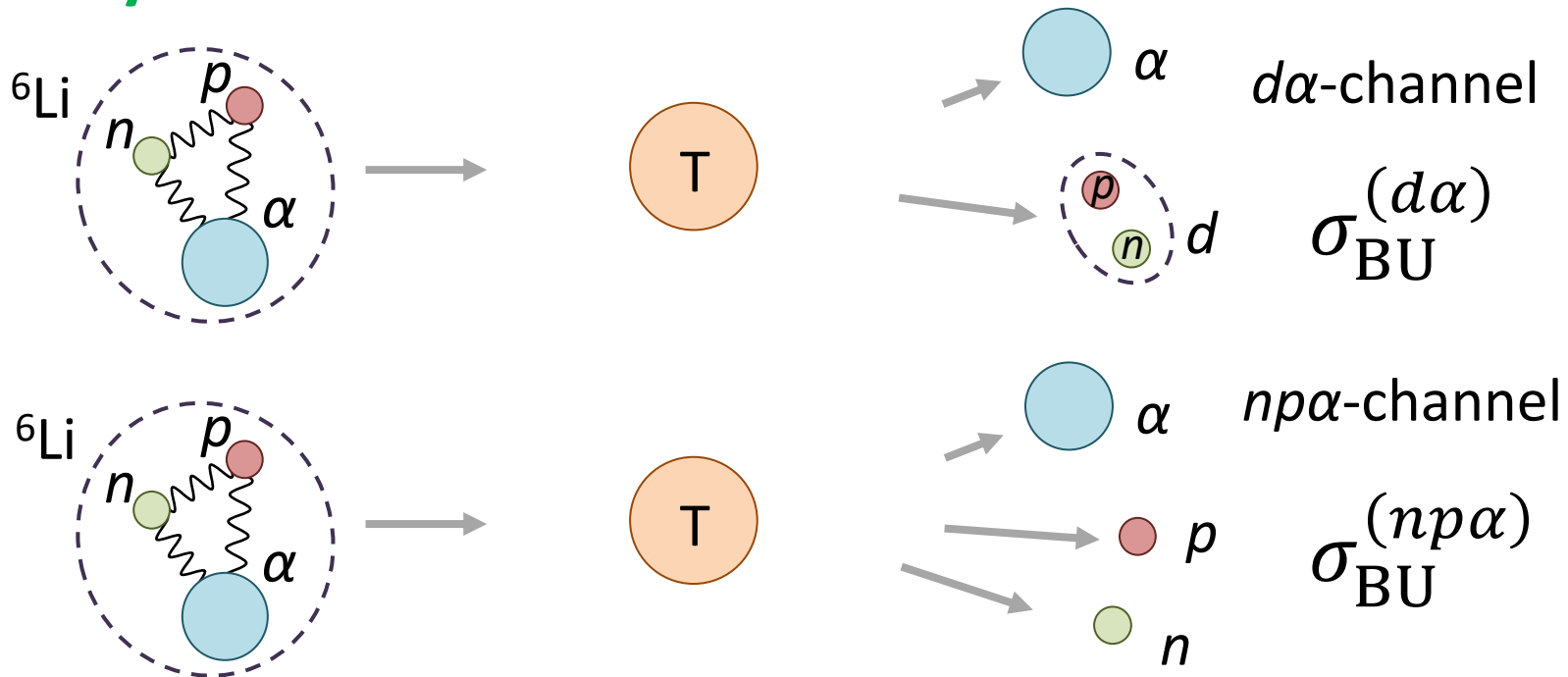
3

Dynamics of 4-body BU reaction is richer than that of 3-body BU reaction and its elucidation is important.
Each breakup cross section (**BUX**) provides **useful information**.

nd
32.

4-body BU reaction

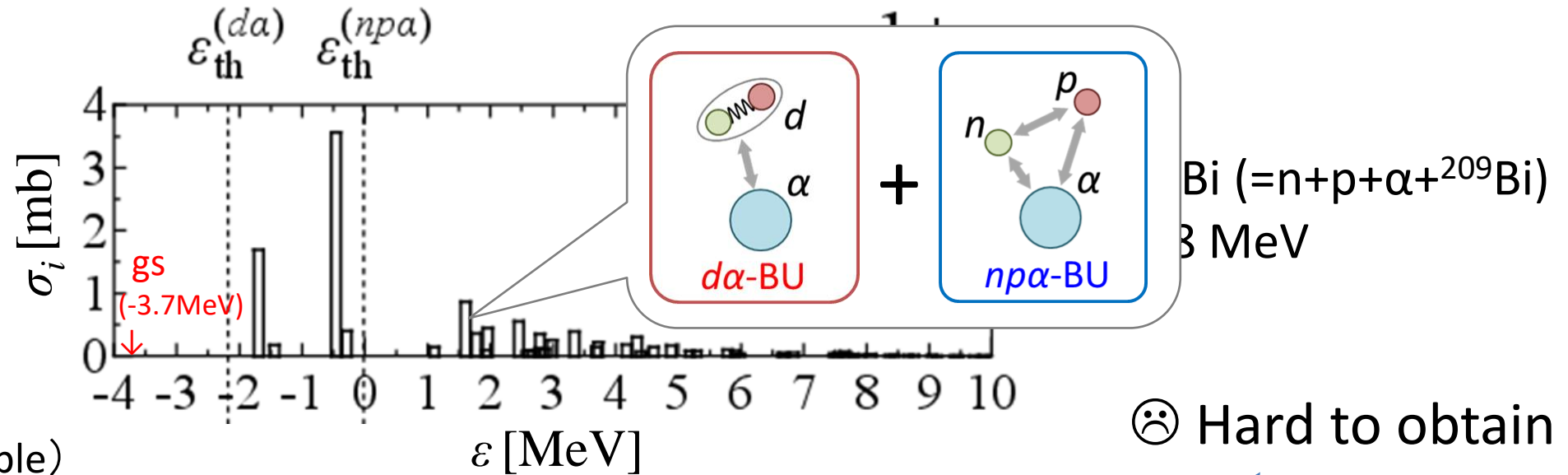
4-body CDCC: *T. Matsumoto et al., PRC 70, 061601(R) (2004).*



Problem in four-body CDCC

☹ In four-body CDCC, a BUX is obtained as a **mixture of different channels**.

e.g.) **Discretized BUX for ${}^6\text{Li}$ scattering**



(In principle)

We can decompose the discretized BUXs by taking an overlap between the **discretized state** and the **exact continuum state**.

3-body CDCC smoothing:

T. Matsumoto et al., Phys. Rev. C 68, 064607 (2003)

→ **Can we decompose BUXs easily?**

Background & Purpose

Background

- ✓ If a projectile is a 3-body system (${}^6\text{Li}=\text{n}+\text{p}+\alpha$ etc.), the continuum states are often **discretized** in reaction calculations.
- ✓ These discretized states (**Pseudostates**) are obtained as a mixture of many kinds of channels.
- ✓ The discretized BUX thus obtained is also a **mixture of many kinds of channels**.

Note
Discretization is
indispensable in CDCC

Purpose

We propose an approximate way of decomposing discretized BUXs into components of different channels.

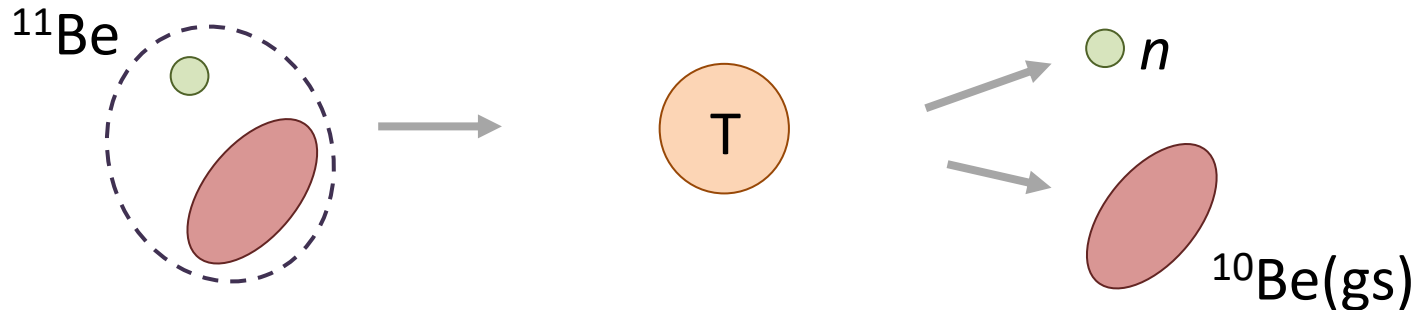
Model and Analysis

Simplifying the problem

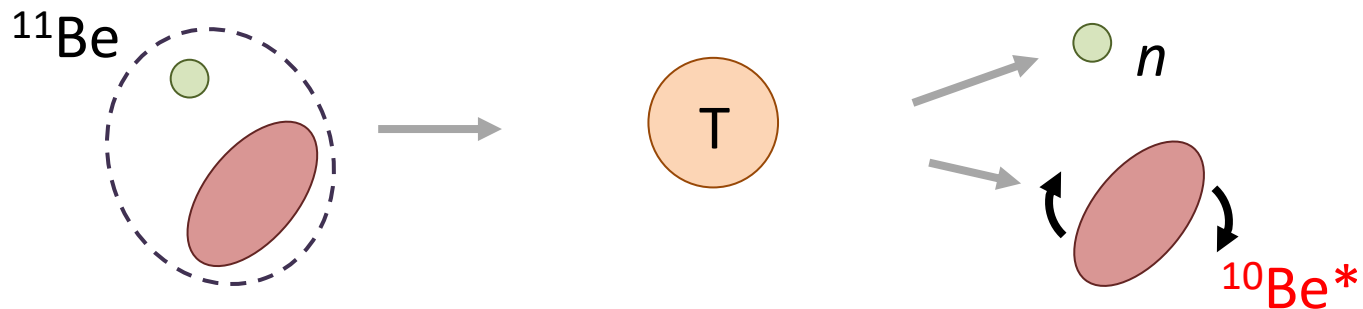
Before going to four-body scattering, we consider **three-body scattering with core excitation**.

Core-excitation
R. Crespo, A. Delgado, and A. M. Moro,
Phys. Rev. C **83**, 044622 (2011).

Core-ground channel



Core-excited channel



- Analogy to ^6Li scattering
→ **Mixture of different channels**
- Simple 2-body problem
→ We can easily obtain the **exact continuum wave functions**.

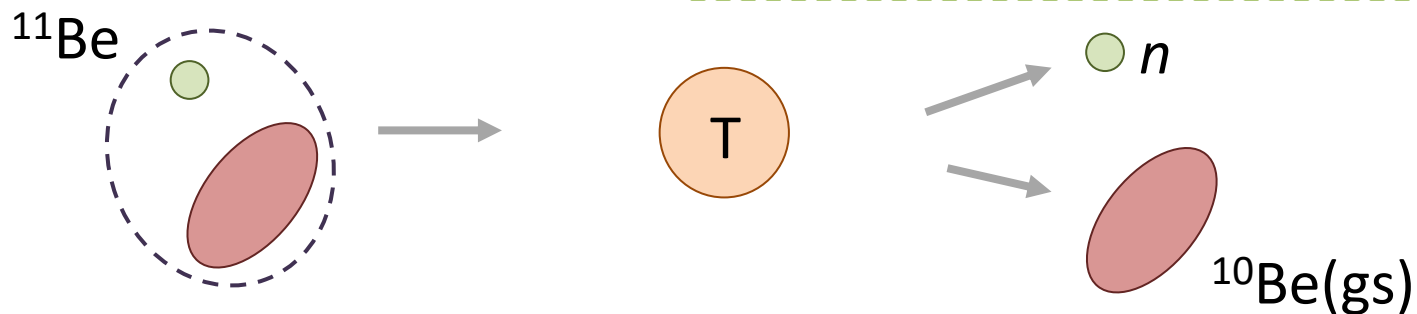


Exact vs Discretized

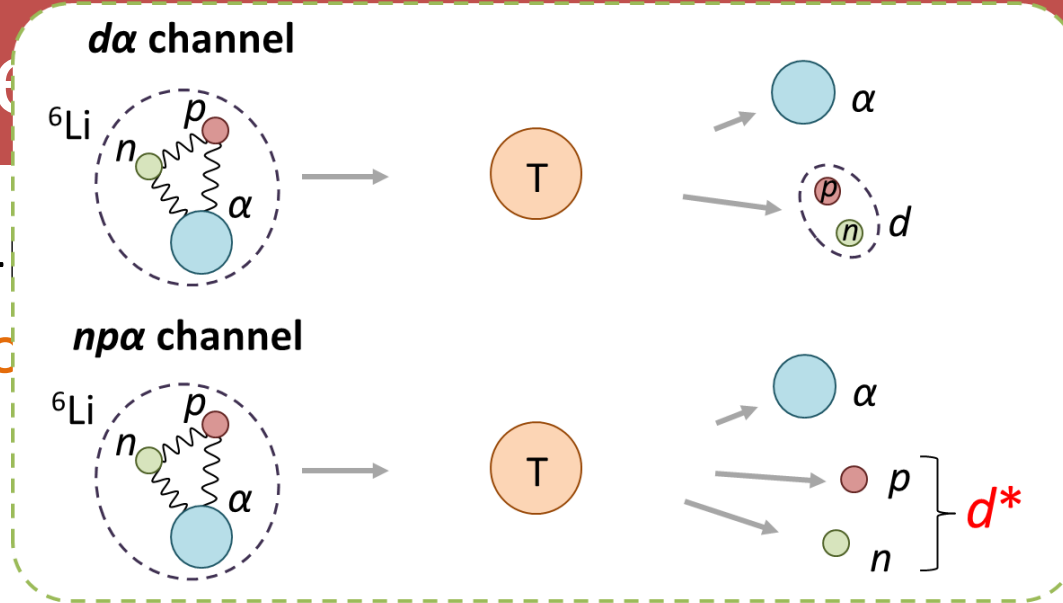
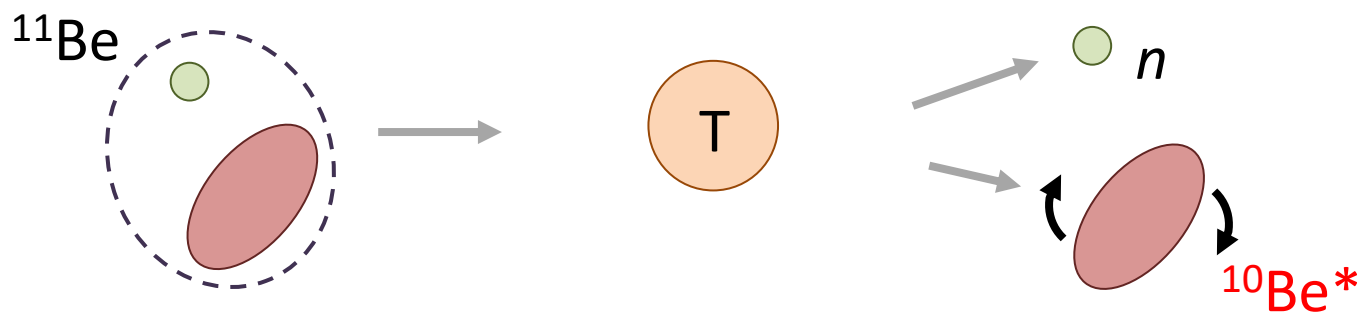
Simplifying the

Before going to four-
we consider **three-body**

Core-ground channel



Core-excited channel



Core-excitation
R. Crespo, A. Delgado, and A. M. Moro,
Phys. Rev. C **83**, 044622 (2011).

- Analogy to ${}^6\text{Li}$ scattering
→ **Mixture of different channels**
- Simple 2-body problem
→ We can easily obtain the **exact continuum wave functions.**



Exact vs Discretized

Model Hamiltonian

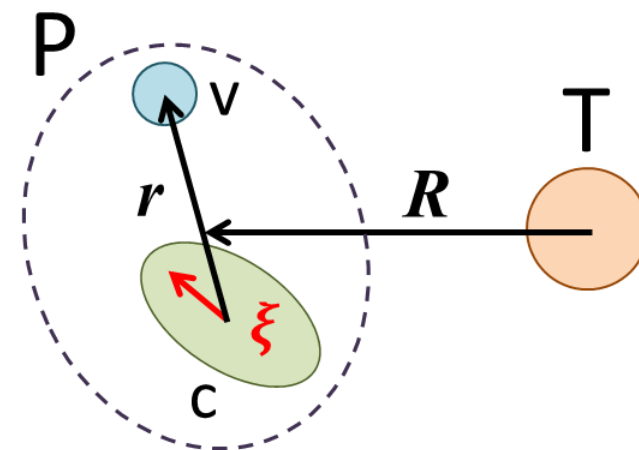
$$H_{\text{tot}} = K_R + V_{\text{VT}}(R_{\text{VT}}) + V_{\text{cT}}(\mathbf{R}_{\text{cT}}, \boldsymbol{\xi}) + h_P$$

$$h_P = K_r + V(\mathbf{r}, \boldsymbol{\xi}) + h_c(\boldsymbol{\xi})$$

DWBA with core excitation: A. M. Moro and R. Crespo, Phys. Rev. C 85, 054613 (2012).

CDCC with core excitation: R. de Diego, J. M. Arias, J. A. Lay, and A. M. Moro, Phys. Rev. C 89, 064609 (2014).

- ✓ Projectile WF is constructed with the **Particle Rotor Model**
- ✓ Reaction part is solved by the **distorted-wave Born Approximation (DWBA)**



This model enables us to calculate both the **exact (continuous)** and the **approximate (discretized)** T-matrix elements.

Model setting (Hamiltonian)

$$H_{\text{tot}} = K_R + V_{\text{VT}}(R_{\text{VT}}) + V_{\text{CT}}(\mathbf{R}_{\text{CT}}, \boldsymbol{\xi}) + h_P$$

$$h_P = K_r + V(\mathbf{r}, \boldsymbol{\xi}) + h_c(\boldsymbol{\xi})$$

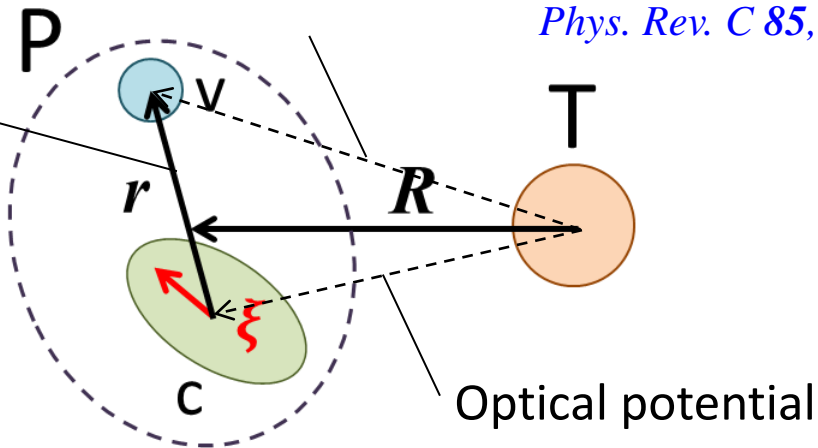
$^{11}\text{Be}+p$ at 63.7 MeV/nucl.

Gaussian: $V(r) = -45e^{-(r/1.484)^2}$

A. M. Moro and R. Crespo, Phys. Rev. C 85, 054613 (2012).

WS Potential
F.M. Nunes et al., NPA609 43 (1996).

$V_{\text{WS}} = -54.45 \text{ MeV}$
 $V_{\text{SO}} = -8.50 \text{ MeV}$
 $R = 2.483 \text{ fm}$
 $a = 0.65 \text{ fm}$



B. A. Watson et al., Phys. Rev. 182, 977 (1969).

^{10}Be core

$\beta_2 = 0.67, E(2+) = 3.368 \text{ MeV}$

Model space

$\ell = 0 - 3 \quad I = 0, 2$

Discretized and Exact continuum states of projectile

- **Discretized continuum states (with diagonalization)**

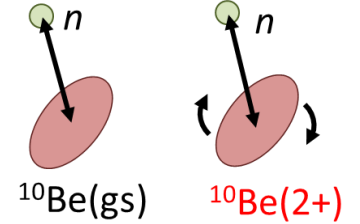
Gaussian Expansion Method (GEM)

*E. Hiyama, Y. Kino, M. Kamimura,
Prog. Part. Nucl. Phys. 51, 223 (2003).*

- ✓ Continuum states are automatically discretized

- ✓ Specified by the state number $n \rightarrow$ Several channels are mixed

$$\hat{\Psi}_{11\text{Be}}^{(n)}(\mathbf{r}, \xi) = \Phi_0(\xi)\hat{\psi}_0^{(n)}(\mathbf{r}) + \Phi_2(\xi)\hat{\psi}_2^{(n)}(\mathbf{r})$$



- **Exact continuum states (with difference method)**

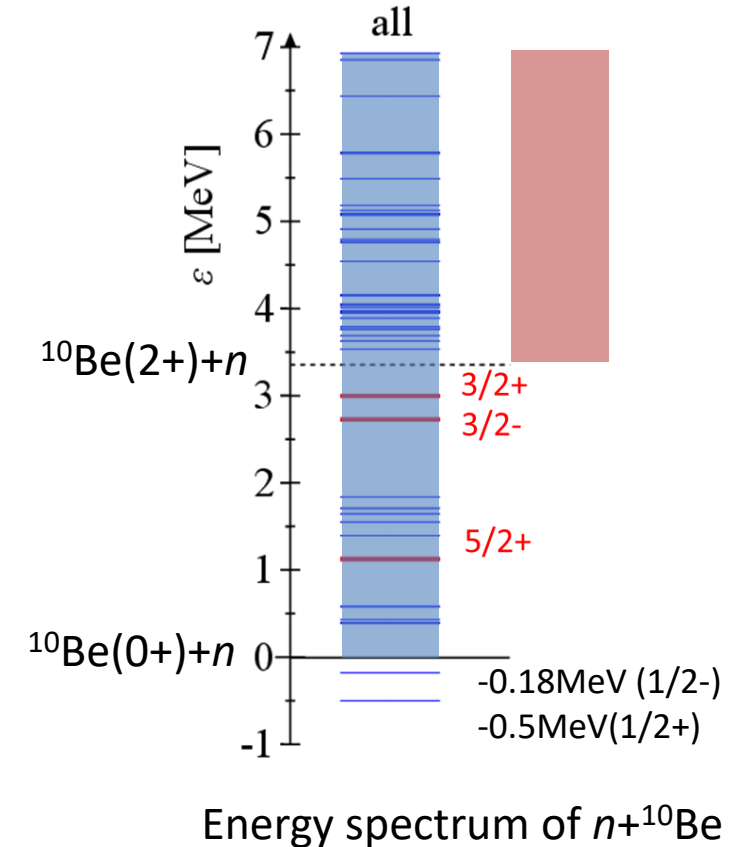
- ✓ $ch = \{\varepsilon, I\}$ is specified before solving the scattering problem

$$\Psi_{11\text{Be}}^{(ch)}(\mathbf{r}, \xi) = \Phi_0(\xi)\psi_0^{(ch)}(\mathbf{r}) + \Phi_2(\xi)\psi_2^{(ch)}(\mathbf{r})$$

**Boundary
condition**

$$\Psi_{11\text{Be}}^{(\varepsilon, I=0)}(\mathbf{r}, \xi) \rightarrow \Phi_0 e^{ik \cdot \mathbf{r}} + \Phi_0 f_{00}(\theta) \frac{e^{ikr}}{r} + \Phi_2 f_{20}(\theta) \frac{e^{ik'r}}{r}$$

$$\Psi_{11\text{Be}}^{(\varepsilon, I=2)}(\mathbf{r}, \xi) \rightarrow \Phi_2 e^{ik' \cdot \mathbf{r}} + \Phi_0 f_{02}(\theta) \frac{e^{ikr}}{r} + \Phi_2 f_{22}(\theta) \frac{e^{ik'r}}{r}$$



Discretized and Exact T -matrix in DWBA

- ✓ For the present purpose, we take the Distorted Wave Born Approximation (DWBA).

A. M. Moro and R. Crespo, Phys. Rev. C 85, 054613 (2012).

Discretized T -matrix (Approximate) ✖ with diagonalization

$$\hat{T}_{fi} = \left\langle \chi_{K'}^{(-)}(\mathbf{R}) \hat{\Psi}_f(\mathbf{r}, \xi) \left| V_{vt}(R_{vt}) + V_{ct}(\mathbf{R}_{ct}, \xi) \right| \chi_K^{(+)}(\mathbf{R}) \Psi_i(\mathbf{r}, \xi) \right\rangle$$

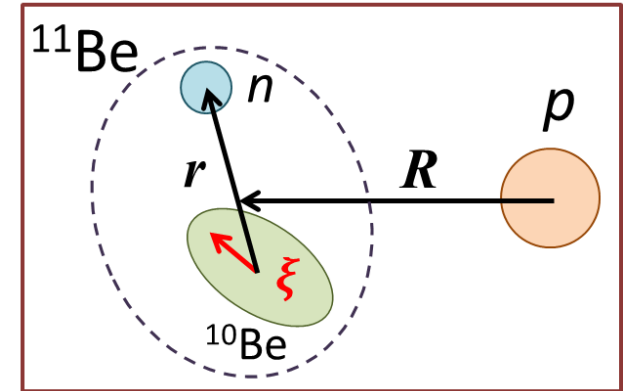
- ✓ $\hat{\sigma}_{\text{BU}}^{(f)} \propto |\hat{T}_{fi}|^2$ is obtained.

compare

Continuum T -matrix (Exact) ✖ with difference method

$$T_{fi}(\varepsilon) = \left\langle \chi_{K'}^{(-)}(\mathbf{R}) \Psi_{f[\varepsilon, I=0 \text{ or } 2]}(\mathbf{r}, \xi) \left| V_{vt}(R_{vt}) + V_{ct}(\mathbf{R}_{ct}, \xi) \right| \chi_K^{(+)}(\mathbf{R}) \Psi_i(\mathbf{r}, \xi) \right\rangle$$

- ✓ $\frac{d\sigma_{\text{BU}}^{(I=0)}}{d\varepsilon}, \frac{d\sigma_{\text{BU}}^{(I=2)}}{d\varepsilon}$ are separately obtained.

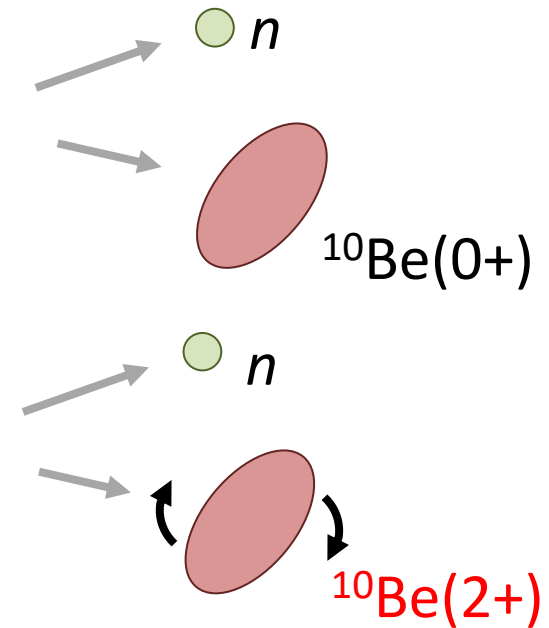
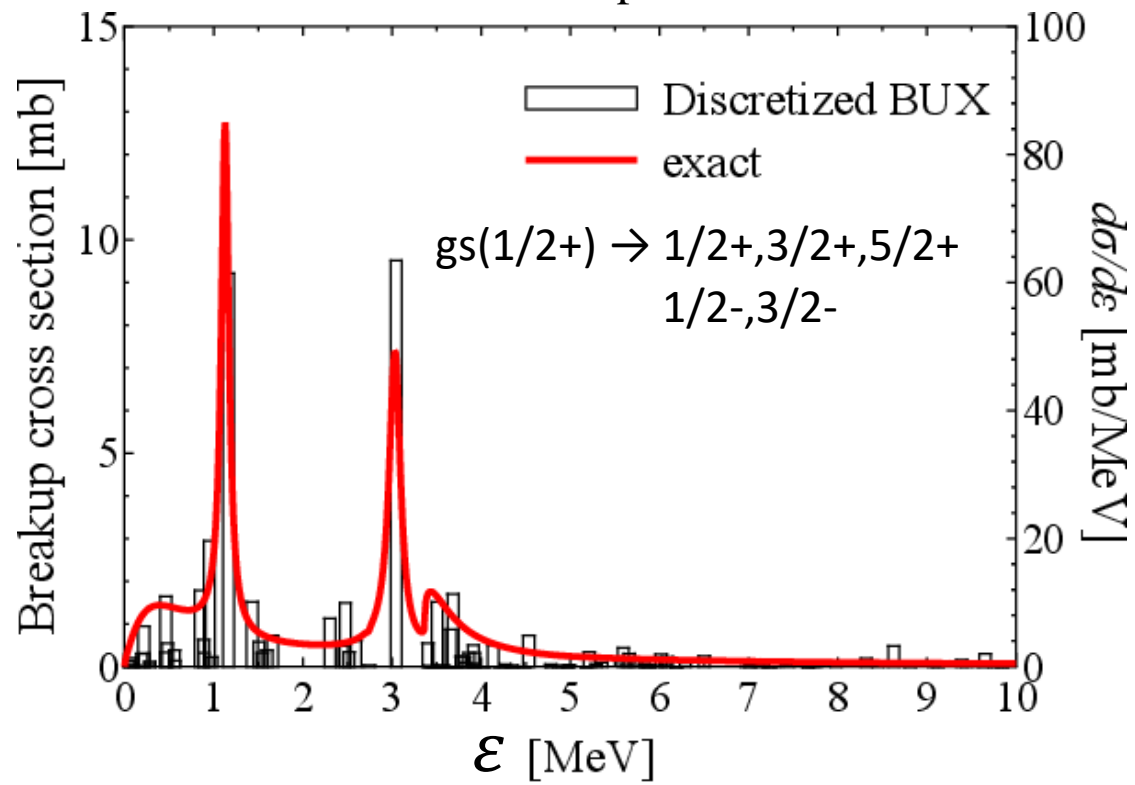


$^{11}\text{Be} + p$ at 63.7 MeV/nucleon.

Result 1: Total BUX

$$\hat{\sigma}_{\text{BU}}^{(\text{tot})} = \sum_n \hat{\sigma}_n = 54.8 \text{ mb} \quad \sigma_{\text{BU}}^{(\text{tot})} = \int \frac{d\sigma}{d\varepsilon} d\varepsilon = 54.8 \text{ mb}$$

$^{11}\text{Be} + \text{p}$ at 63.7 MeV/nucl.



Components? $\sigma_{\text{BU}}^{(0+)}$ & $\sigma_{\text{BU}}^{(2+)}$

As for $\hat{\sigma}_{\text{BU}}^{(\text{tot})}$, we can obtain the same result. ($\hat{\sigma}_{\text{BU}}^{(\text{tot})} = \sigma_{\text{BU}}^{(\text{tot})}$)

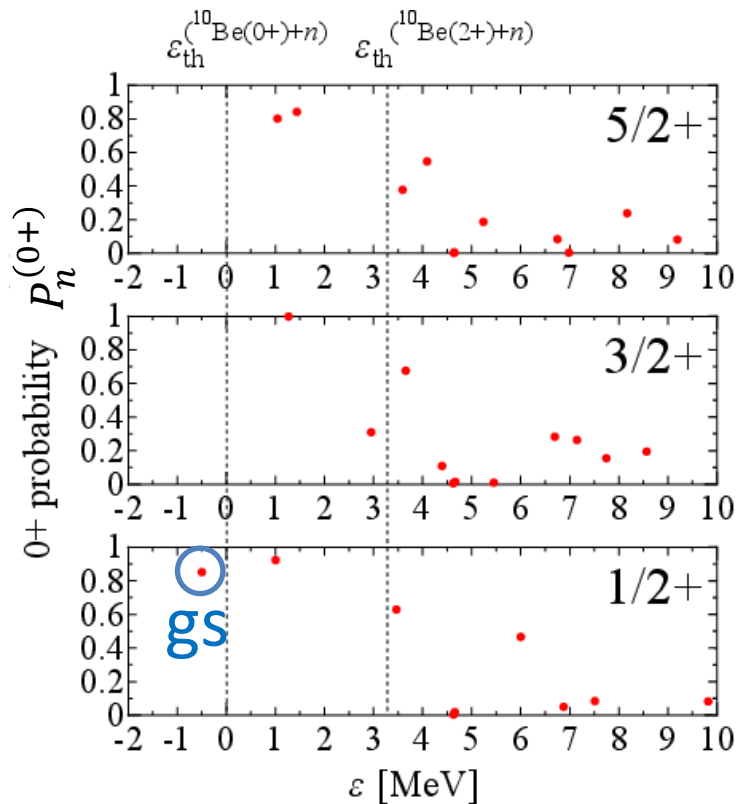
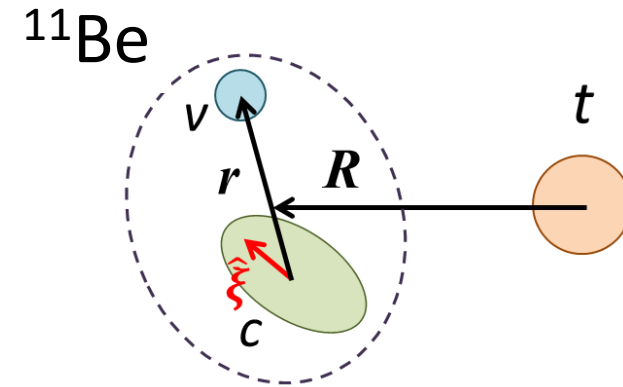
How to separate $\hat{\sigma}_{\text{BU}}^{(\text{tot})}$? \rightarrow Probability Separation “P-separation”

Projectile wf.

$$\hat{\Psi}_{^{11}\text{Be}}^{(n)}(\mathbf{r}, \xi) = \Phi_0(\xi)\hat{\psi}_0^{(n)}(\mathbf{r}) + \Phi_2(\xi)\hat{\psi}_2^{(n)}(\mathbf{r})$$

0+ probability

$$P_n^{(0+)} = \int d\mathbf{r} \left| \langle \Phi_0(\xi) | \hat{\Psi}_{^{11}\text{Be}}^{(n)}(\mathbf{r}, \xi) \rangle_{\xi} \right|^2$$



P-separation (Approx.)

$$\hat{\sigma}_{\text{BU}}^{(\text{tot})} = \sum_n \hat{\sigma}_n$$

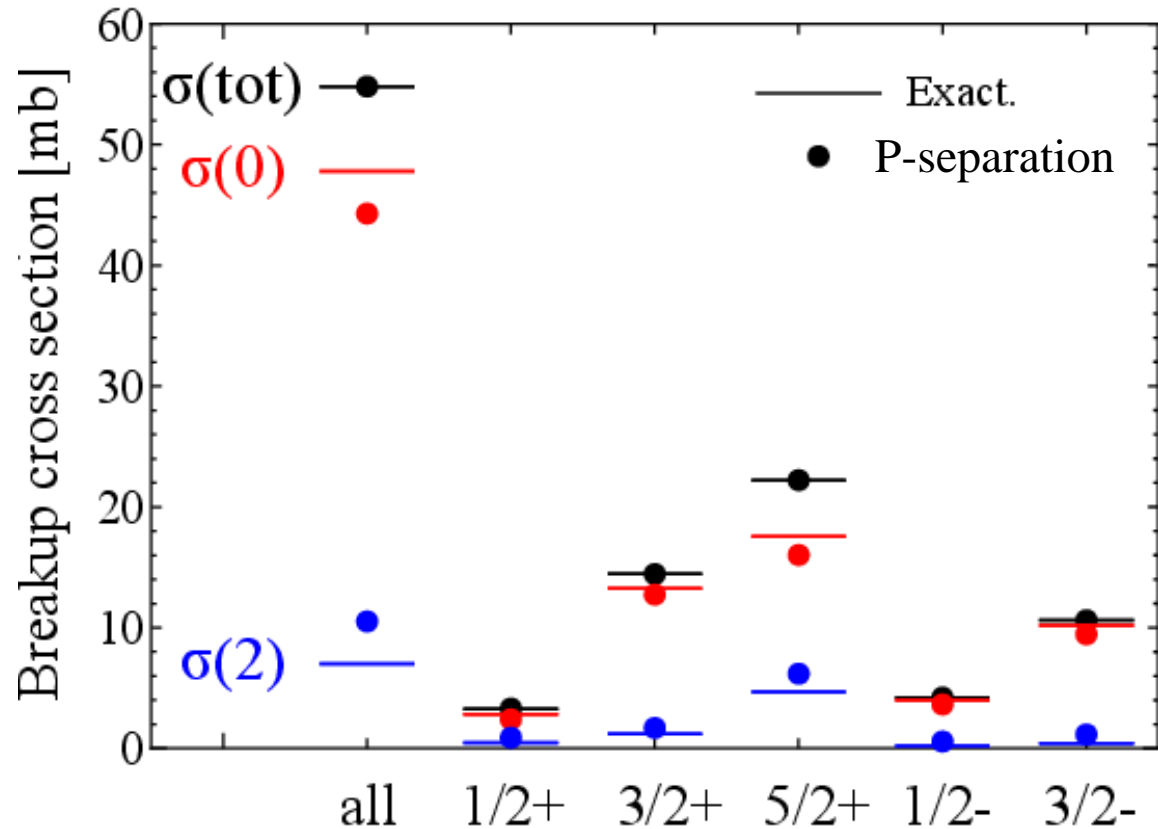
$$\hat{\sigma}_{\text{BU}}^{(0+)} \approx \sum_n P_n^{(0+)} \hat{\sigma}_n$$

$$\hat{\sigma}_{\text{BU}}^{(2+)} \approx \sum_n \left(1 - P_n^{(0+)} \right) \hat{\sigma}_n$$

$$P_n^{(0+)} = 1 \text{ for } \epsilon_n \leq \epsilon_{\text{th}}$$

^{11}Be is not broken up into $n+^{10}\text{Be}(2+)$ below ϵ_{th} .

Result 2: Decomposition of discretized BUXs



DWBA analysis

$^{11}\text{Be}+p$ at 63.7 MeV/nucl.

- $^{10}\text{Be}(0^+)+n+p$
- $^{10}\text{Be}(2^+)+n+p$

--- P-separation (Approx.) ---

$$\hat{\sigma}_{\text{BU}}^{(\text{tot})} = \sum_n \hat{\sigma}_n \quad \left(P_n^{(0^+)} = 1 \text{ for } \varepsilon_n \leq \varepsilon_{\text{th}} \right)$$

$$\hat{\sigma}_{\text{BU}}^{(0^+)} \approx \sum_n P_n^{(0^+)} \hat{\sigma}_n \quad \hat{\sigma}_{\text{BU}}^{(2^+)} \approx \sum_n P_n^{(2^+)} \hat{\sigma}_n$$



Almost identical

--- Exact solution ---

$$\sigma_{\text{BU}}^{(\text{tot})} = \int \frac{d\sigma}{d\varepsilon} d\varepsilon$$

$$\sigma_{\text{BU}}^{(0^+)} = \int \frac{d\sigma^{(0^+)}}{d\varepsilon} d\varepsilon \quad \sigma_{\text{BU}}^{(2^+)} = \int \frac{d\sigma^{(2^+)}}{d\varepsilon} d\varepsilon$$

☺ Discretized BUXs ($\hat{\sigma}_{\text{BU}}^{(\text{tot})}$) are decomposed into each component ($\hat{\sigma}_{\text{BU}}^{(0^+)}$, $\hat{\sigma}_{\text{BU}}^{(2^+)}$) very well.

Validity of P-separation (Systematic analysis)

We perform a systematic analysis to validate the P-separation.

- ✓ The **different configurations** are prepared by changing V and/or ϵ_2 .
- ✓ The potential is common for the ground and continuum states.

Table: Potential sets and the ground state properties

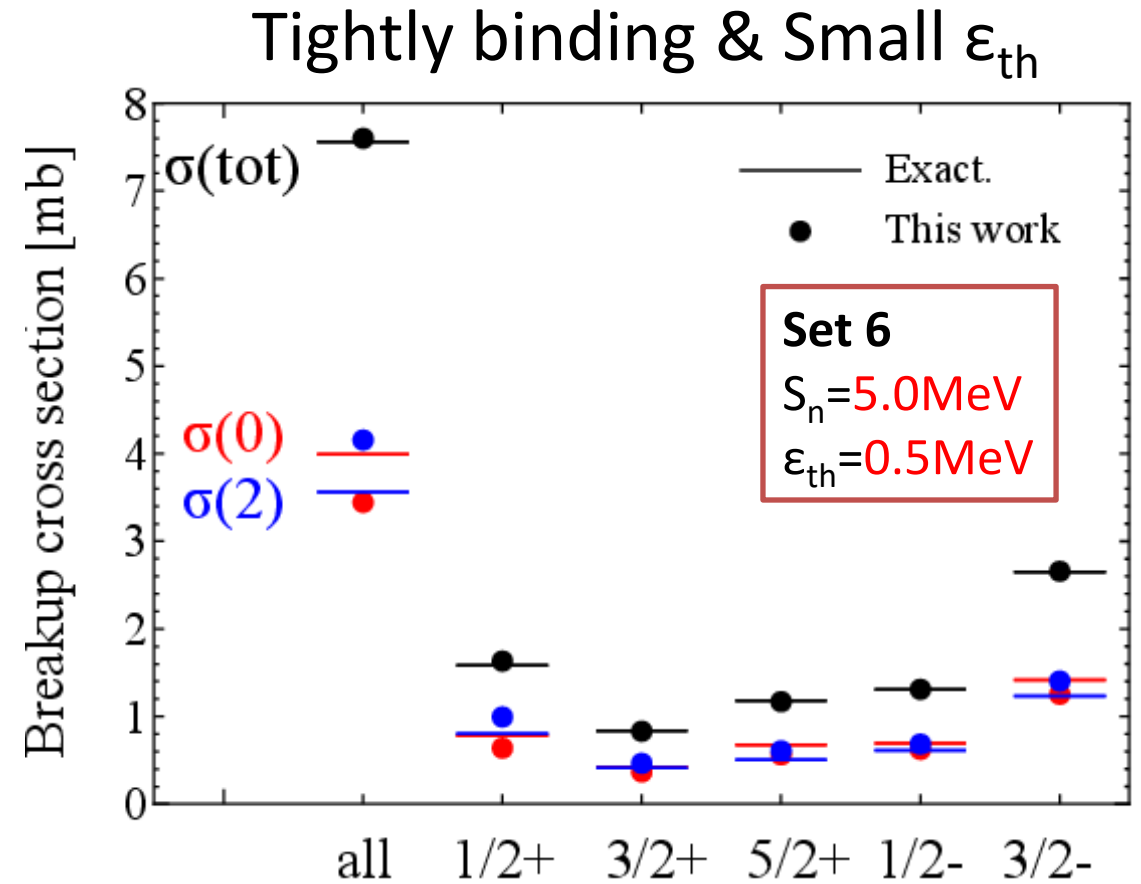
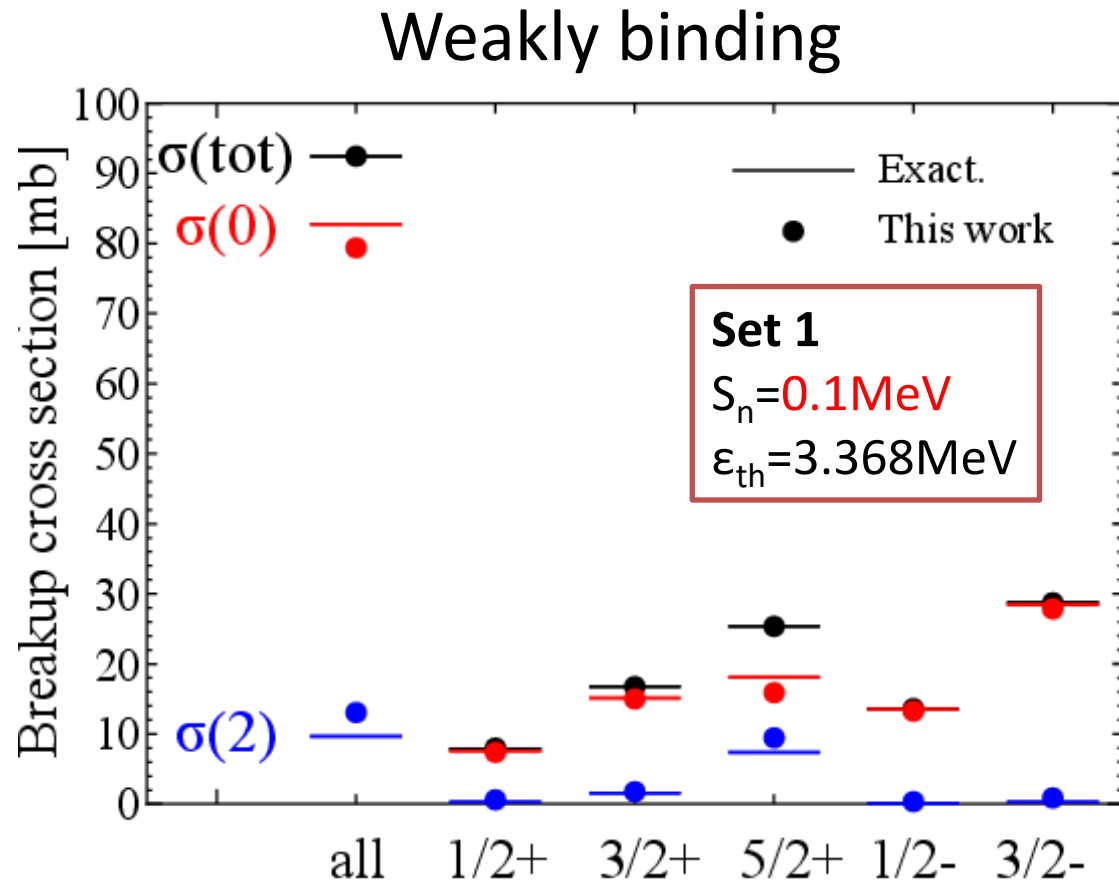
set	S_n	V_0	V_{so}	ϵ_2	$P_{gs}(0)$	$P_{gs}(2)$
1	0.1	-51.924	-8.5	3.368	0.943	0.057
2	0.5	-54.45	-8.5	3.368	0.855	0.145
3	0.5	-52.988	-1.0	0.5	0.792	0.208
4	1.0	-56.475	-8.5	3.368	0.788	0.212
5	5.0	-67.059	-8.5	3.368	0.577	0.423
6	5.0	-65.670	-1.0	0.5	0.545	0.455

Weakly-bound gs

Original (done)

Tightly-bound gs

Result 3: Validity of the P-separation



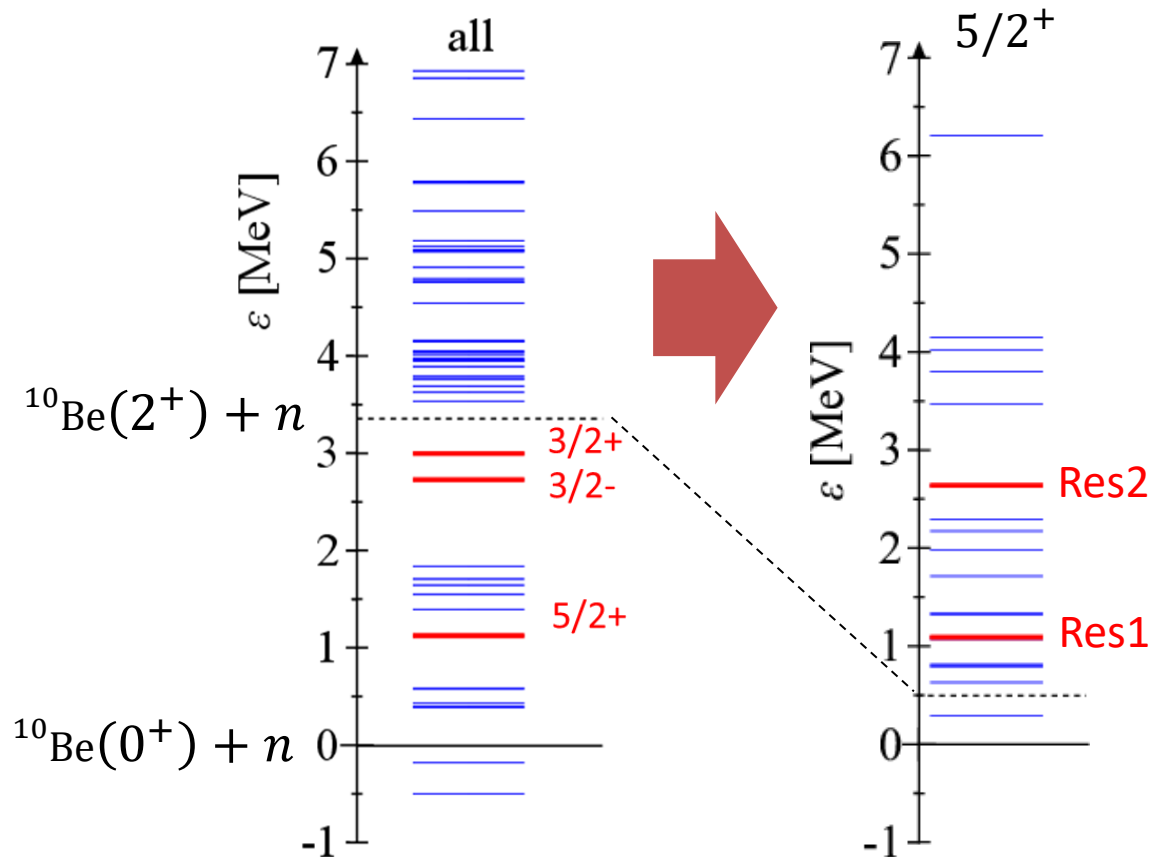
The P-separation works well regardless of the configurations.

Q. What will happen if the resonance(s) exist(s) above ε_{th} ?

So far

All the resonances appear below ε_{th} .

- The separation is **trivial**.



From now on

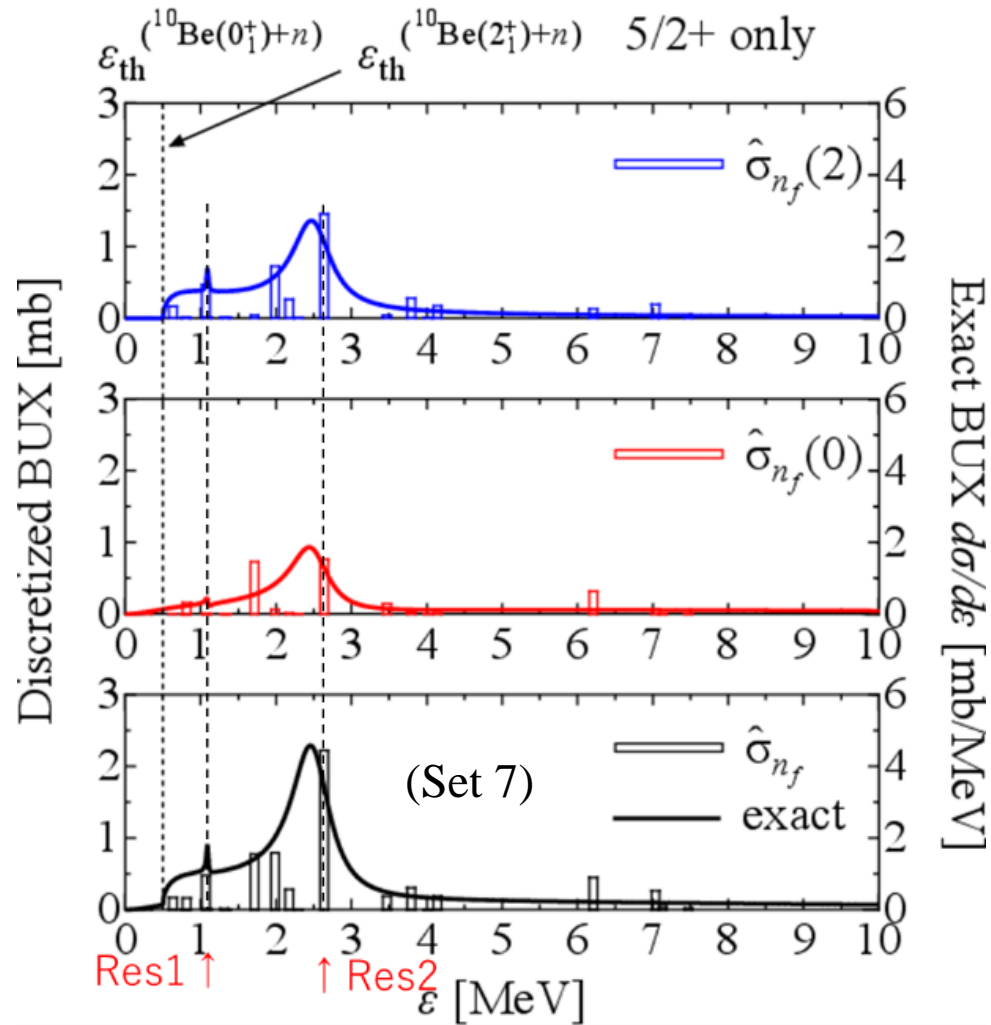
Is the P-separation still valid even if the resonance(s) exist(s) above ε_{th} ?

To construct the resonances above ε_{th} in the $5/2^+$ state, we found that the deeper potential is necessary.

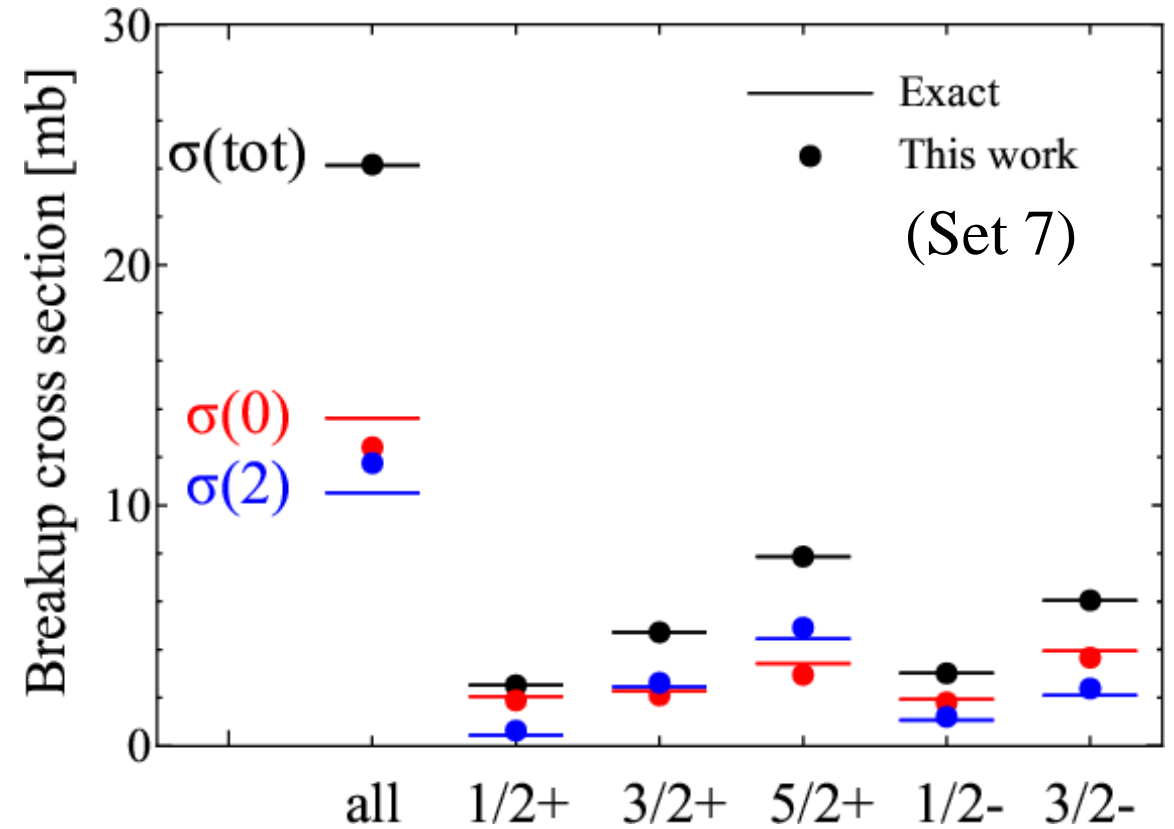
Depth : $V_0 = -54.45 \rightarrow -85.791$ MeV

Threshold: $\varepsilon_{\text{th}} = 3.368 \rightarrow 0.5$ MeV

P-separation with resonances above ε_{th}



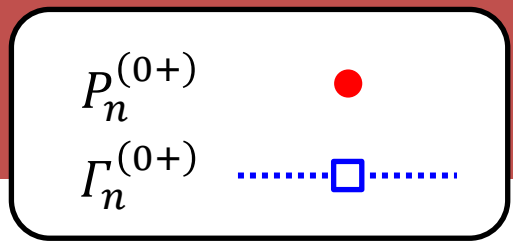
- ✓ Two resonances make the characteristic peaks, respectively.



☺ The P-separation works well regardless of the resonance position.

Why P-separation works well?

$P_n^{(0+)}$ vs $\Gamma_n^{(0+)}$



$P_n^{(0+)}$: Proportion of the core-ground component

$$P_n^{(0+)} \equiv \int d\mathbf{r} \left| \left\langle \Phi_0(\xi) \left| \widehat{\Psi}_{11\text{Be}}^{(n)}(\mathbf{r}, \xi) \right. \right\rangle_{\xi} \right|^2$$

↑ Core-ground state ↑ n -th discretized state

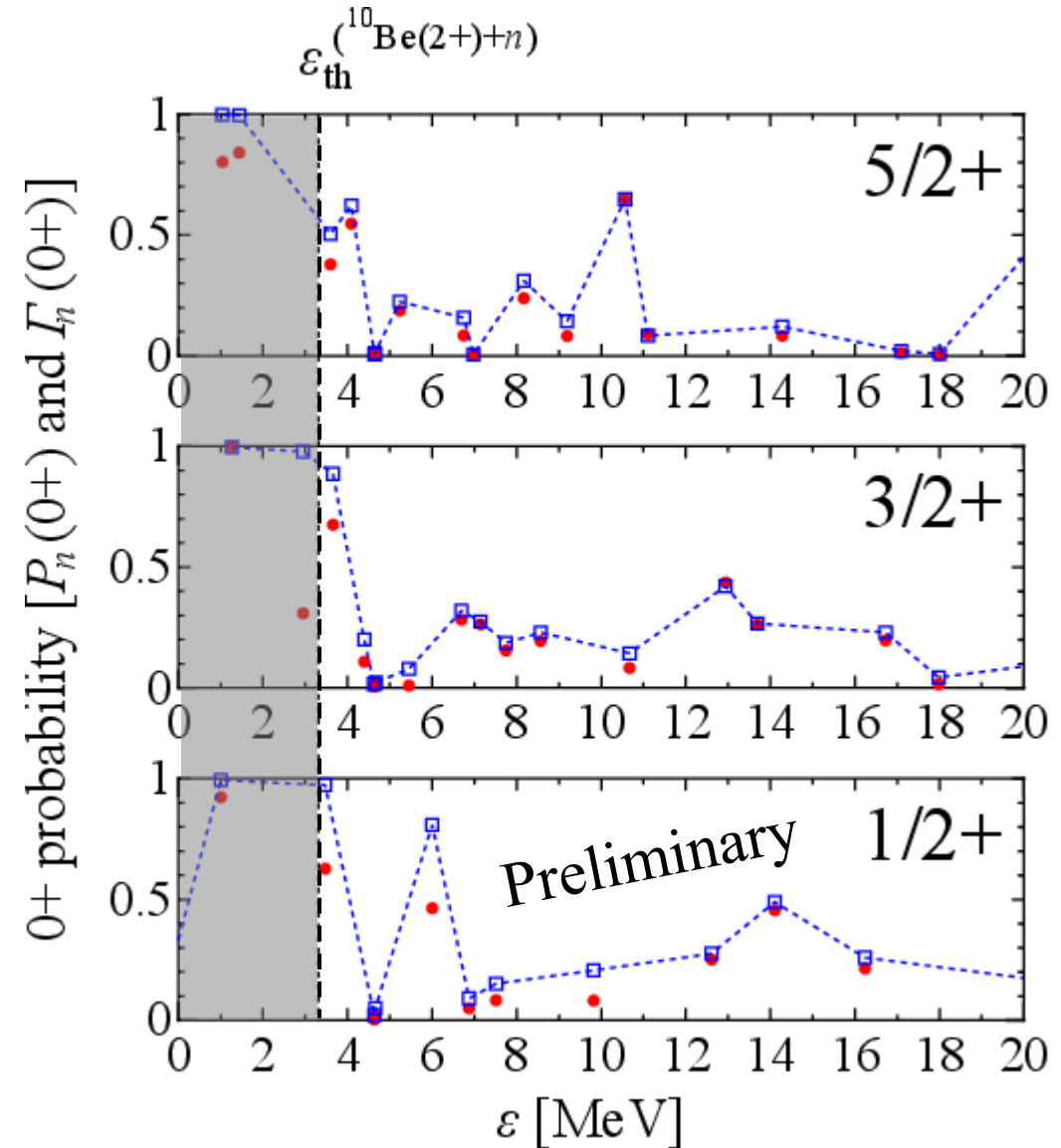
$$P_n^{(0+)} + P_n^{(2+)} = 1$$

$\Gamma_n^{(0+)}$: Proportion of the core-ground channel

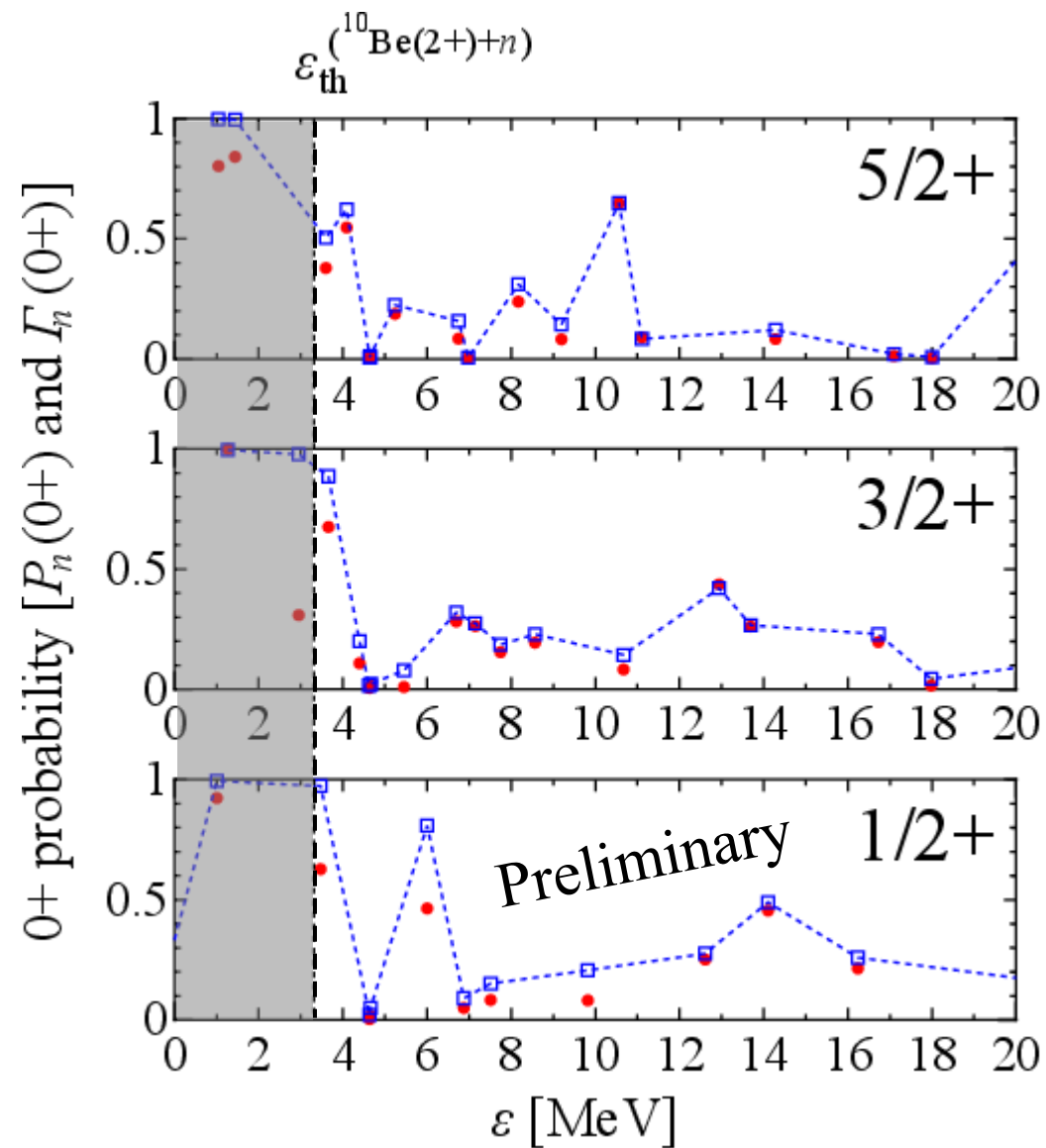
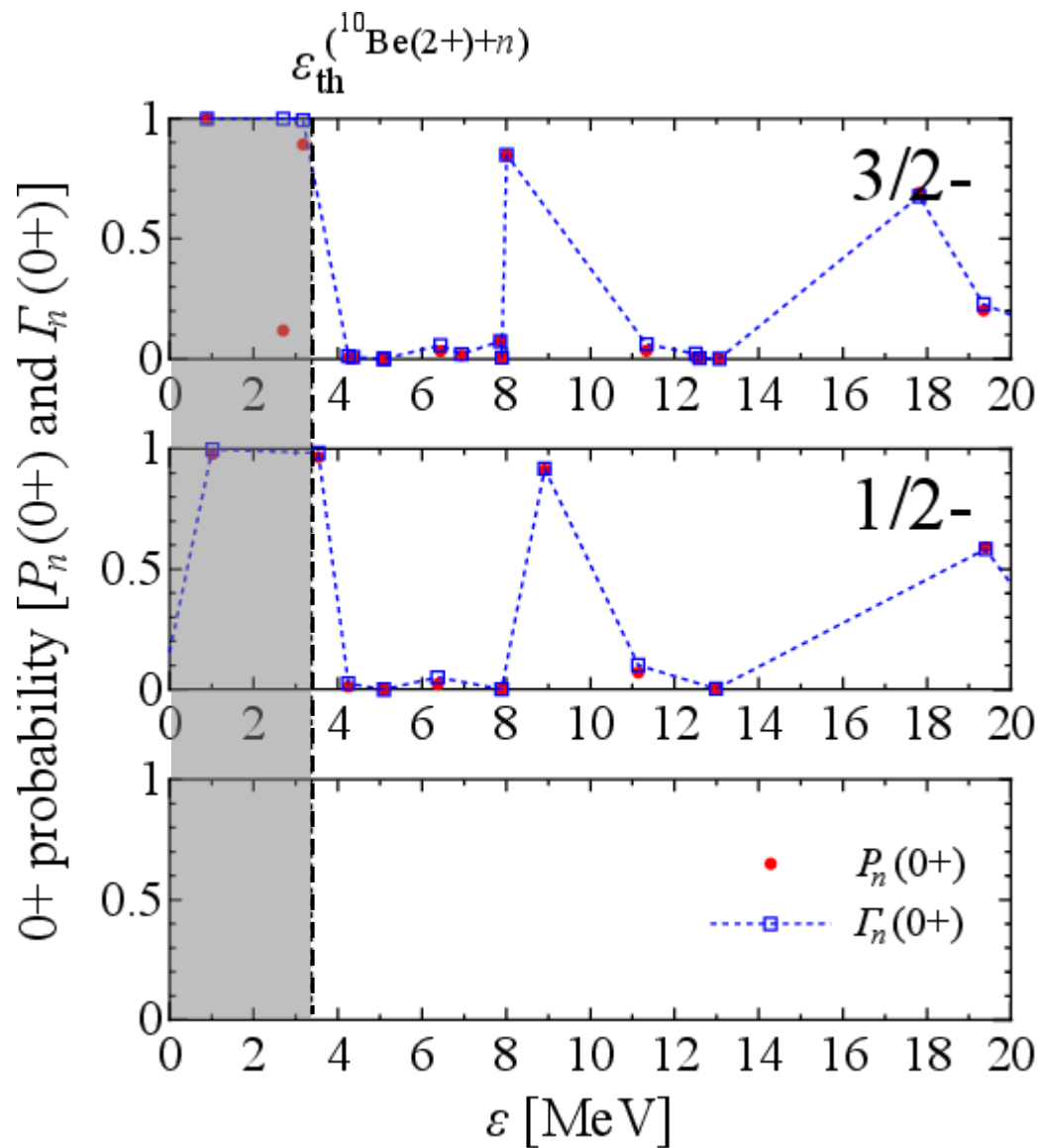
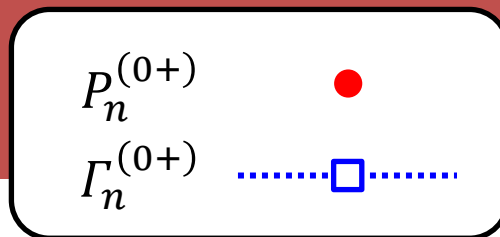
$$\Gamma_n^{(0+)} \equiv \int d\varepsilon \left| \left\langle \Psi_{\varepsilon}^{(lj, I=0)}(\mathbf{r}, \xi) \left| \widehat{\Psi}_{11\text{Be}}^{(n)}(\mathbf{r}, \xi) \right. \right\rangle \right|^2$$

↑ Exact continuum state ↑ n -th discretized state

$$\Gamma_n^{(0+)} + \Gamma_n^{(2+)} = 1$$

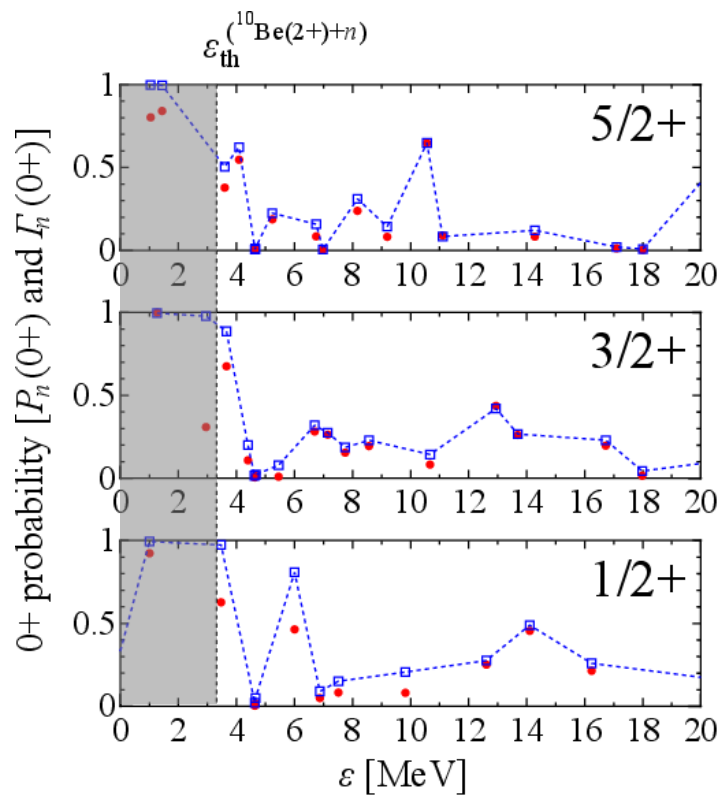


Analysis: $P_n^{(0+)}$ vs $\Gamma_n^{(0+)}$

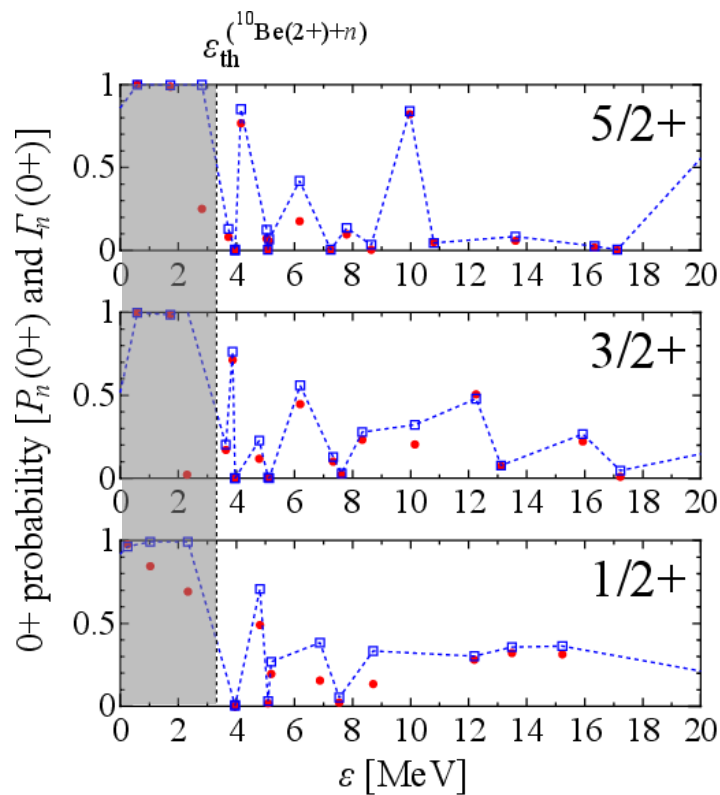


Different cases: $\Gamma_n^{(0+)}$ vs $P_n^{(0+)}$

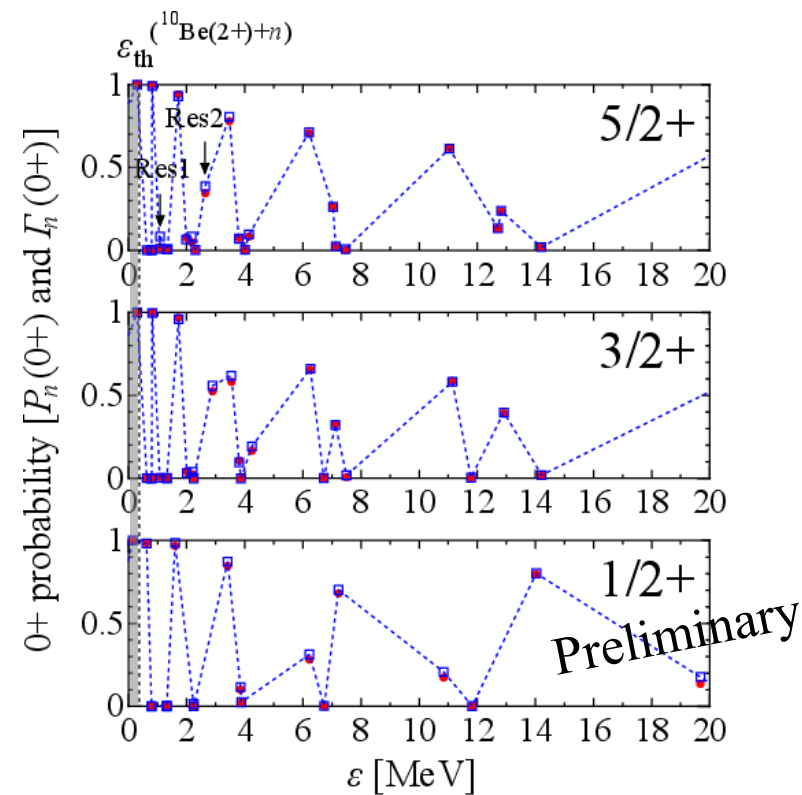
Set2 (original)



Set4 (deeper V)



Set7 (deeper V & smaller ε_{th})



$P_n^{(0+)} \approx \Gamma_n^{(0+)} \quad (\varepsilon_n > \varepsilon_{\text{th}})$
 is realized regardless of

- Potential parameters
- Deformation parameter β_2
- Basis parameters

Preliminary

Short summary

We have proposed an approximate treatment (**P-separation**) for decomposing discretized BUXs.

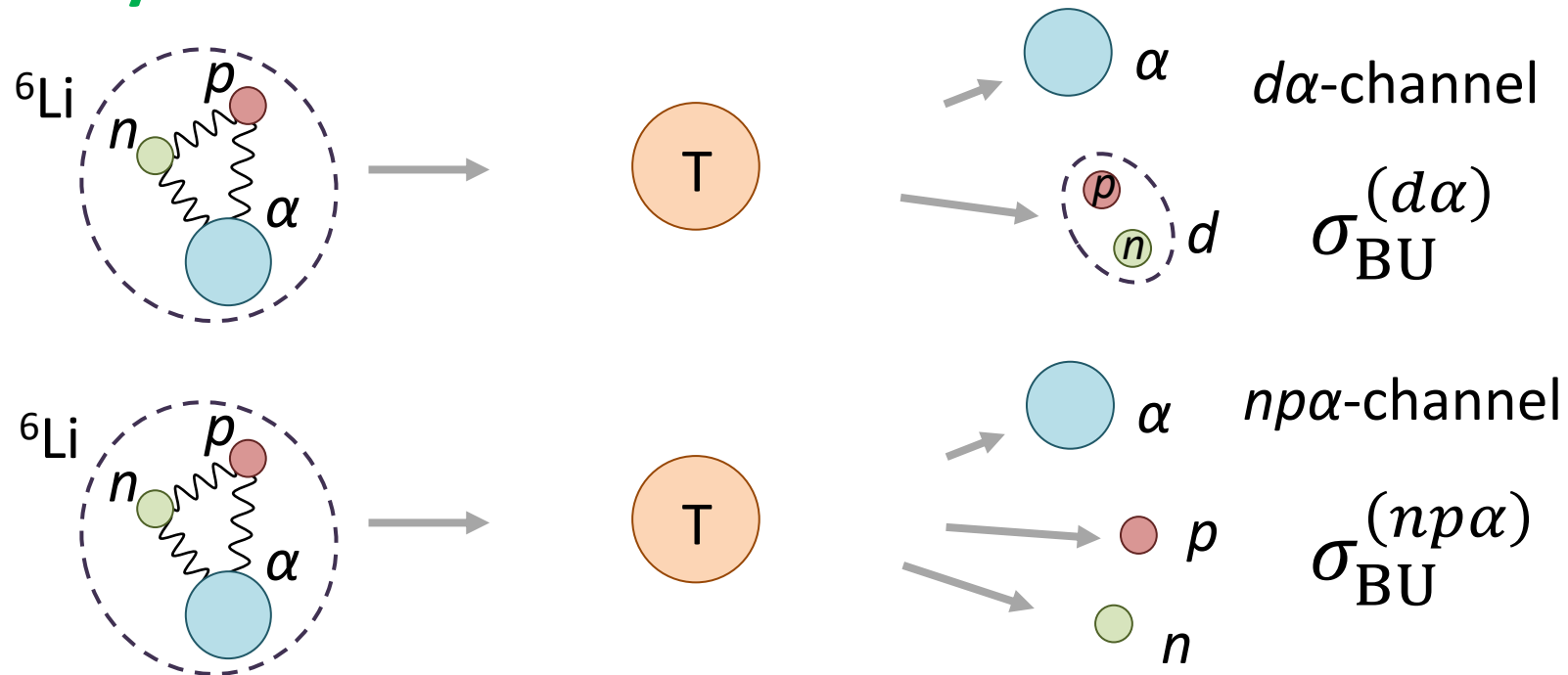
S. Watanabe, K. Ogata, and T. Matsumoto, PRC 103, L031601 (2021).

- ✓ We applied the P-separation to ^{11}Be scattering with core excitation.
 - $^{11}\text{Be}+T \rightarrow ^{10}\text{Be}(\text{gs})+n+T$ and $^{11}\text{Be}+T \rightarrow ^{10}\text{Be}(2+)+n+T$
- ✓ The **P-separation reproduces the exact BUXs well** regardless of the configurations and/or the resonance positions of ^{11}Be .
- ✓ We also found that $P_n^{(0+)} \approx \Gamma_n^{(0+)}$ is realized.

Application to four-body scattering

4-body BU reaction

4-body CDCC: *T. Matsumoto et al., PRC 70, 061601(R) (2004).*



Application to four-body CDCC

We investigate $d\alpha$ - and $n\alpha$ -BUX of ${}^6\text{Li}$ scattering ($n+p+\alpha+T$).

Low energy: ${}^6\text{Li}+{}^{208}\text{Pb}$ at 39 MeV

High energy: ${}^6\text{Li}+{}^{208}\text{Pb}$ at 210 MeV

Model

- ✓ 1+, 2+, and 3+ states are included
- ✓ Coulomb BU is neglected

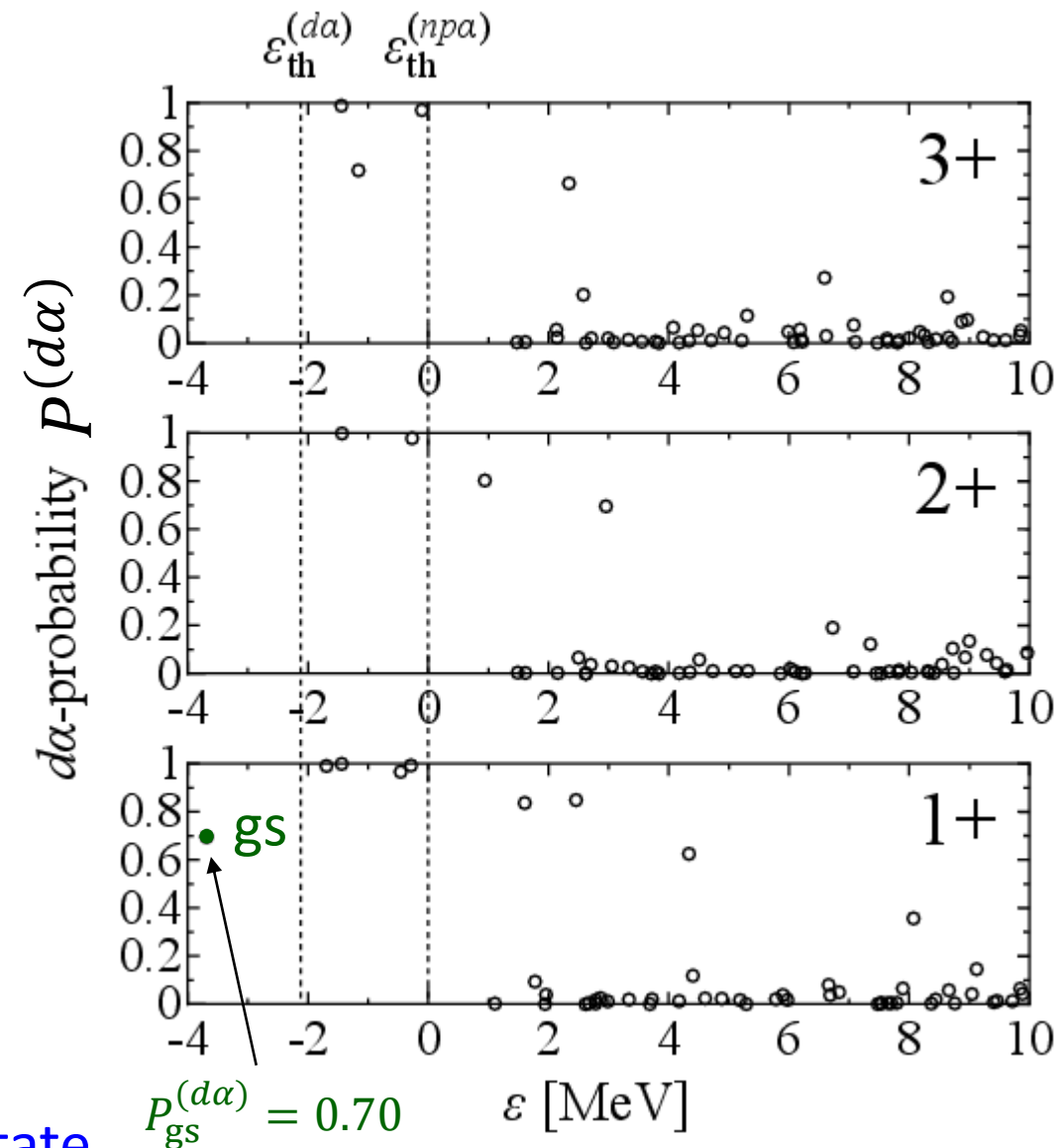
Details: S. Watanabe et al., PRC 92, 044611 (2015).

$d\alpha$ -probability

$$P_n^{(d\alpha)} = \int d\mathbf{r} \left| \left\langle \Phi_d(\mathbf{y}) \left| \hat{\Psi}_{{}^6\text{Li}}^{(n)}(\mathbf{r}, \mathbf{y}) \right\rangle_{\mathbf{y}} \right|^2$$

Deuteron g.s. \uparrow

\uparrow Three-body pseudostate



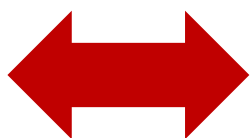
$d\alpha$ -BUX $\hat{\sigma}_{\text{BU}}^{(d\alpha)}$ and $np\alpha$ -BUX $\hat{\sigma}_{\text{BU}}^{(np\alpha)}$

${}^6\text{Li}+{}^{208}\text{Pb}$ scattering

	$\hat{\sigma}_{\text{BU}}^{(\text{tot})}$ [mb]	$\hat{\sigma}_{\text{BU}}^{(d\alpha)}$ [mb]	$\hat{\sigma}_{\text{BU}}^{(np\alpha)}$ [mb]
39 MeV	68.7	45.3	23.4
210 MeV	137	89.9	47.1

$$\hat{\sigma}_{\text{BU}}^{(d\alpha)} \approx 2\hat{\sigma}_{\text{BU}}^{(np\alpha)}$$

Almost comparable



This appears to contradict with the findings in the previous work:
“*four-body channel coupling is negligible in the elastic scattering*”

(${}^6\text{Li}+\text{T} \leftrightarrow \text{n}+\text{p}+\alpha+\text{T}$)

S. Watanabe et al., PRC 92, 044611 (2015).

P-separation (Approx.)

$$\hat{\sigma}_{\text{BU}}^{(\text{tot})} = \sum_n \hat{\sigma}_n$$

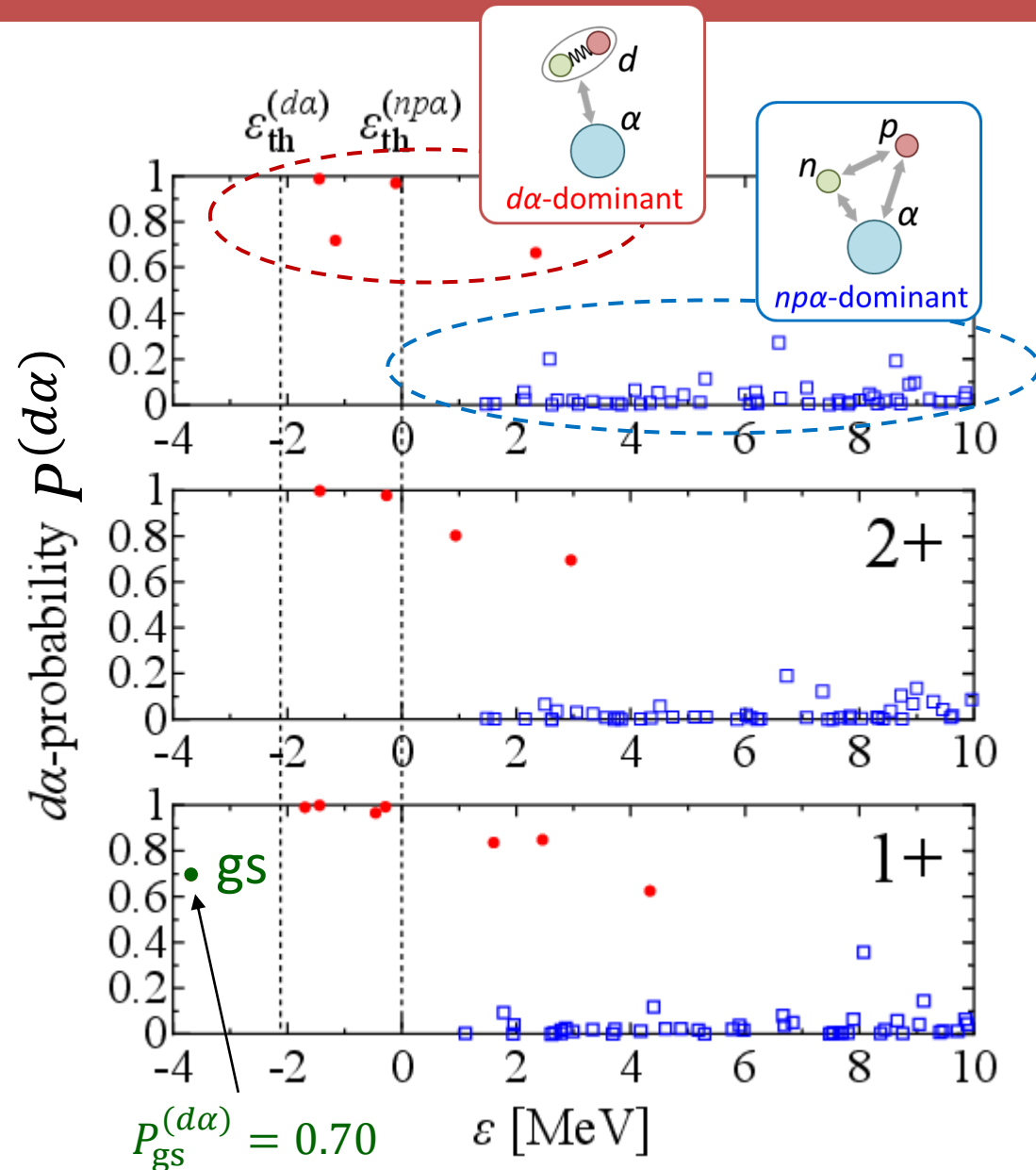
$$\hat{\sigma}_{\text{BU}}^{(d\alpha)} \approx \sum_n P_n^{(d\alpha)} \hat{\sigma}_n$$

$$\hat{\sigma}_{\text{BU}}^{(np\alpha)} \approx \sum_n \left(1 - P_n^{(d\alpha)}\right) \hat{\sigma}_n$$

$$P_n^{(d\alpha)} = 1 \text{ for } \varepsilon_n \leq \varepsilon_{\text{th}}^{(np\alpha)}$$

Three- and four-body channel-coupling effect on the elastic scattering

S. Watanabe, T. Matsumoto, K. Ogata, and M. Yahiro, *PRC* **92**, 044611 (2015).



■ Categorize BU states

- $d\alpha$ -dominant state $|d\alpha\rangle_i$ 15 states
 $|\text{BU}\rangle_i$ with $P_i^{(d\alpha)} > 0.5$
- $np\alpha$ -dominant state $|np\alpha\rangle_j$ 140 states
 $|\text{BU}\rangle_j$ with $P_j^{(d\alpha)} \leq 0.5$

Note

The number of $np\alpha$ -dominant states is much more than that of $d\alpha$ -dominant states.

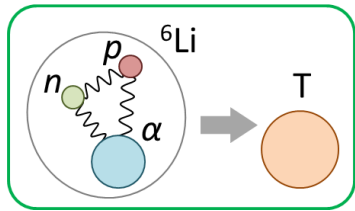
We investigate the channel-coupling effects by switching on and off

- three-body channel (${}^6\text{Li}+T \leftrightarrow n+p+\alpha+T$)
- four-body channel (${}^6\text{Li}+T \leftrightarrow n+p+\alpha+T$)

Four-body channel-coupling effect

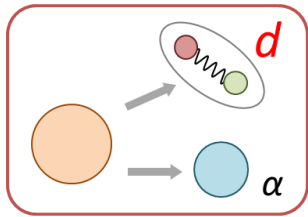
Channel coupling

Elastic channel

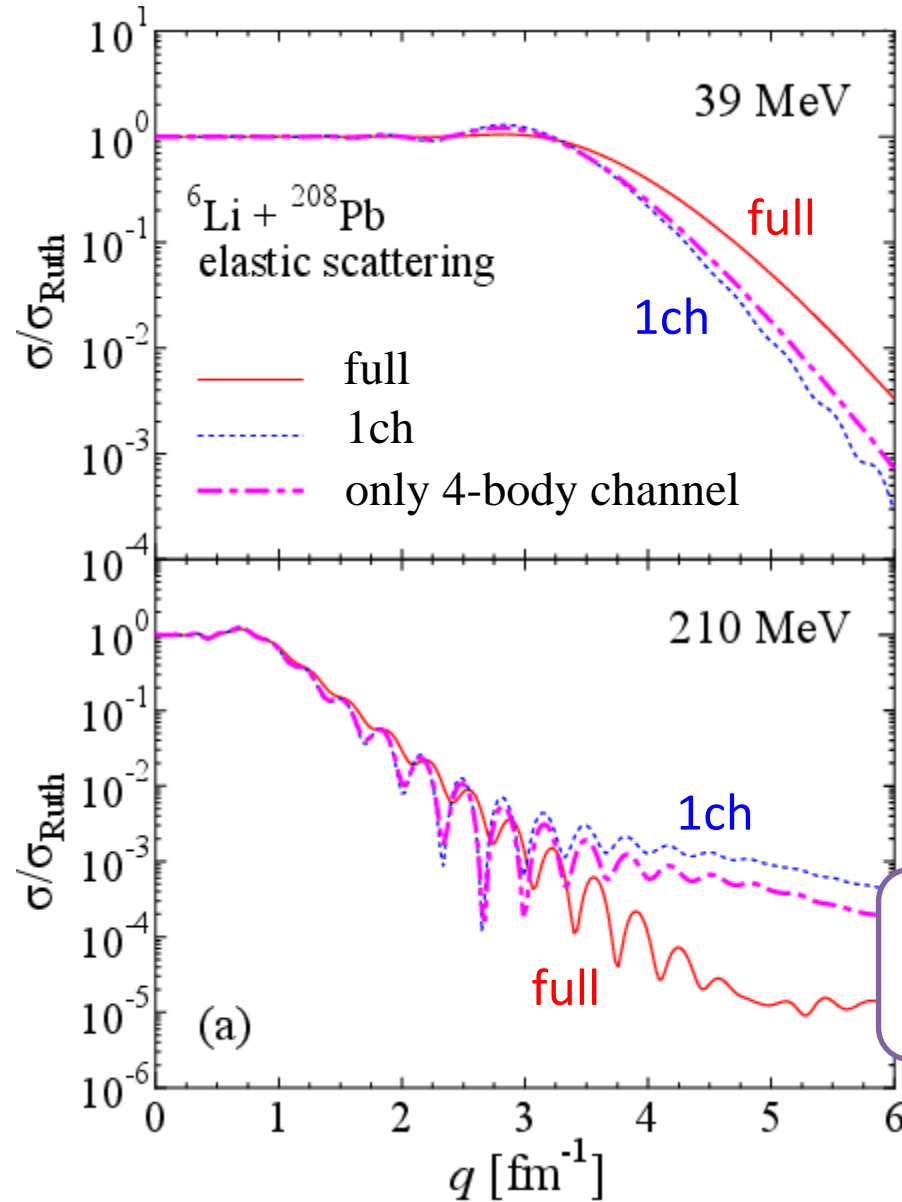
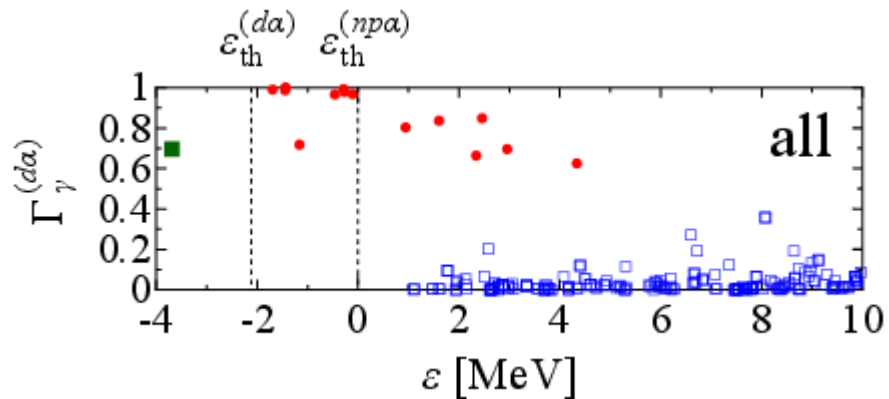
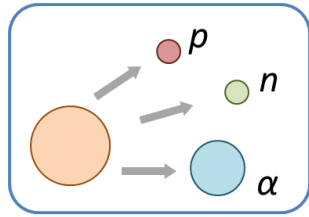


Weak

3-body channel



4-body channel

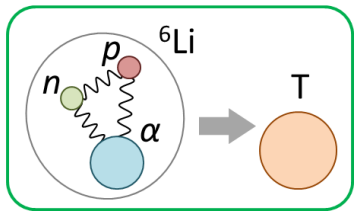


The results differ little from 1ch calculations.

Three-body channel-coupling effect

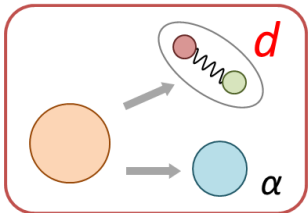
Channel coupling

Elastic channel

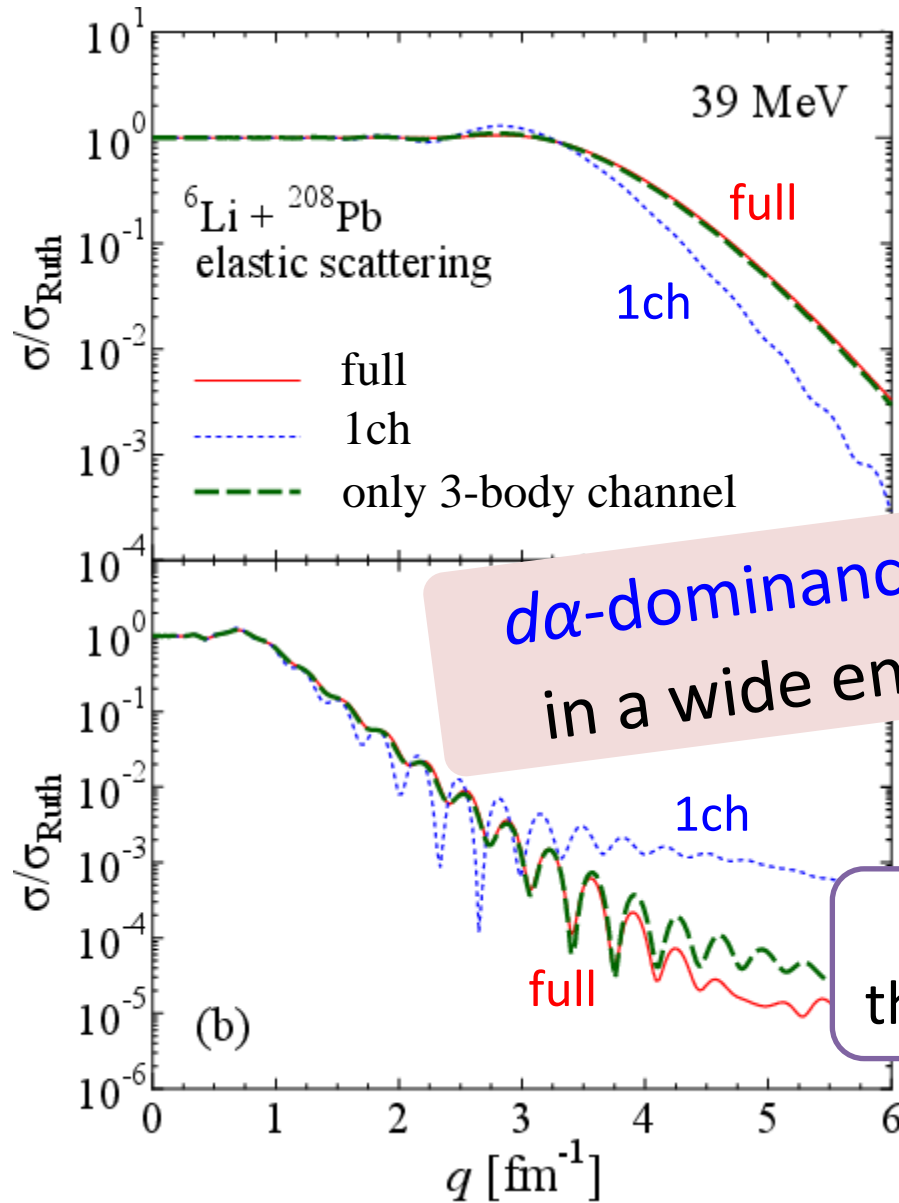
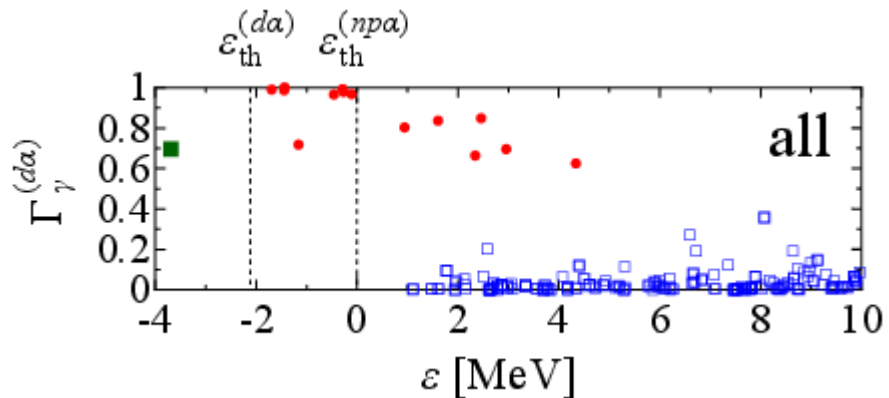
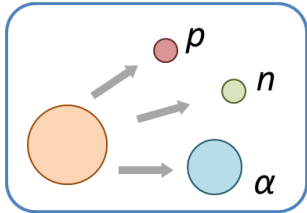


Strong

3-body channel



4-body channel



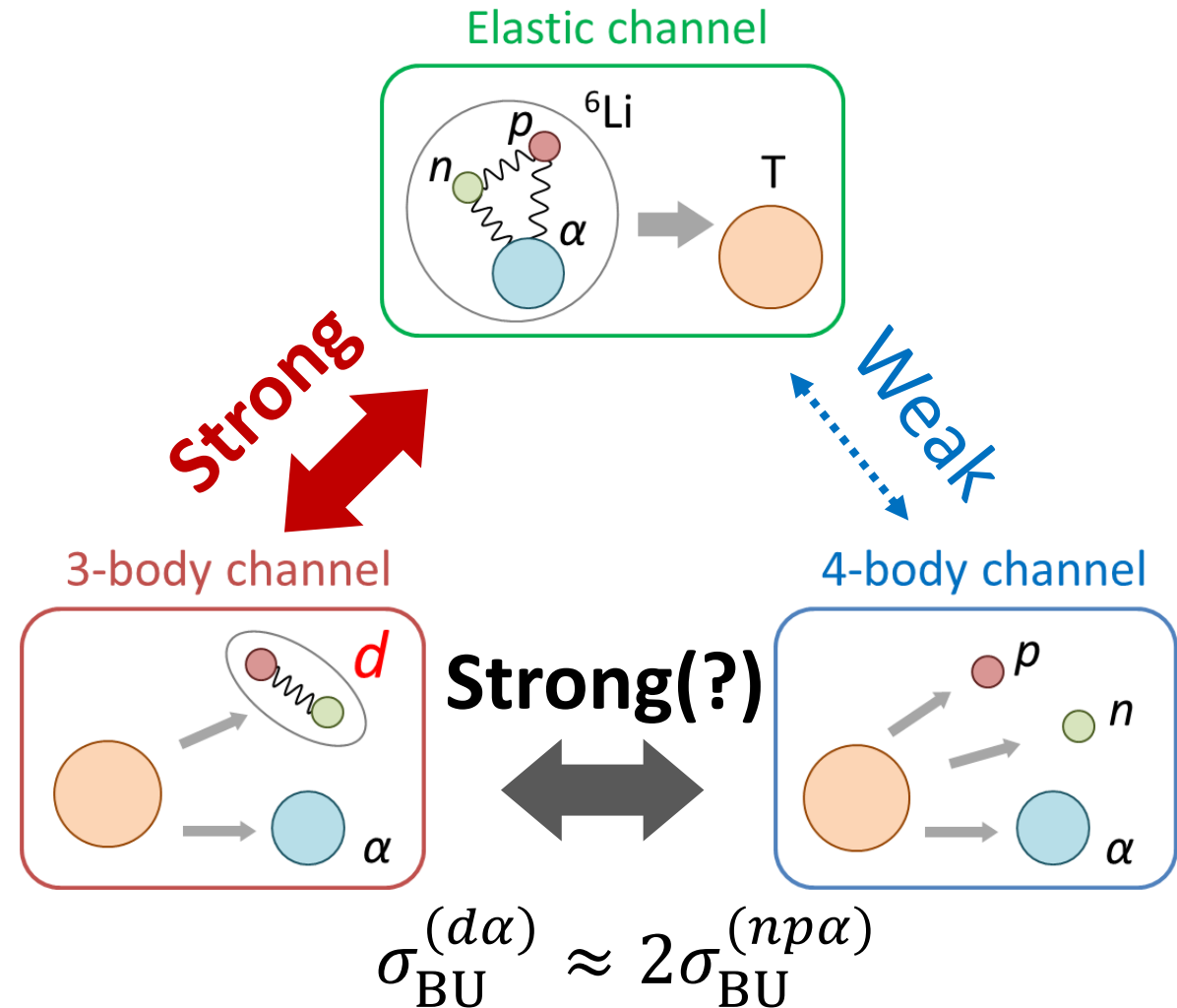
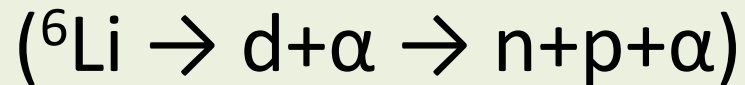
dα-dominance is realized
in a wide energy range.

The results simulate
the full calculations well.

Channel-coupling strength

What is happening in ${}^6\text{Li}$ scattering?

${}^6\text{Li}$ may be broken up into three particles after breaking up into two clusters.



Summary

We have proposed an approximate treatment (**P-separation**) for decomposing discretized BUXs.

S. Watanabe, K. Ogata, and T. Matsumoto, PRC 103, L031601 (2021).

- ✓ We applied the P-separation to ^{11}Be scattering with core excitation.
 - $^{11}\text{Be}+T \rightarrow ^{10}\text{Be}(\text{gs})+n+T \rightarrow ^{10}\text{Be}(2+)+n+T$
- ✓ The **P-separation reproduces the exact BUXs well** regardless of the configurations and/or the resonance positions of ^{11}Be .
- ✓ We also found that $P_n^{(0+)} \approx \Gamma_n^{(0+)}$ is realized.

This method can be an **alternative approach for decomposing discretized BUXs** into components in four- or five-body scattering where **the strict decomposition is hard to perform.**