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Potential roots of deep sub-barrier heavy-ion fusion hindrance phenomenon within the sudden approximation approach

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Reaction Seminar, May 13, 2021

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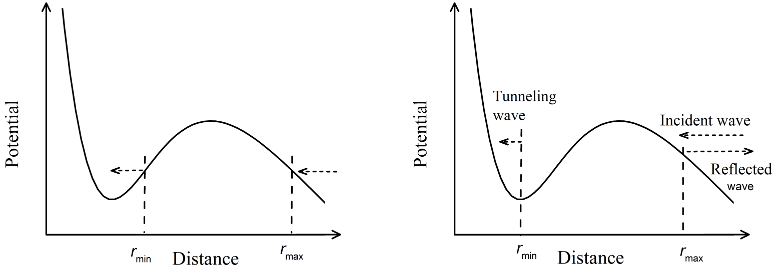
Time independent sub-barrier quantum tunneling



There are generally two ways to get the tunneling probability:

- Semi-classical approaches: WKB *et al.*

$$P_l^{\rm WKB}(E) = \exp[-2\int_{r_{\rm min}}^{r_{\rm max}} \sqrt{2\mu[V_l(r) - E]/\hbar^2} dr],$$

- Schrödinger equation under certain boundary conditions.

$$[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_N^{(0)}(r) + \frac{Z_P Z_T e^2}{r} - E]\ \psi(r) = 0$$

Physical idea of coupled-channels tunneling

Taking two energy levels as an example here

$$\left[\frac{\hbar^2}{2\mu}\nabla^2 + V_l(r) + \begin{pmatrix} 0 & F(r) \\ F(r) & \epsilon_1 \end{pmatrix}\right]\begin{pmatrix} u_0(r) \\ u_1(r) \end{pmatrix} = E\begin{pmatrix} u_0(r) \\ u_1(r) \end{pmatrix}$$

$F(r) = V_{01}(r) = V_{10}(r)$

The eigen-frequency:

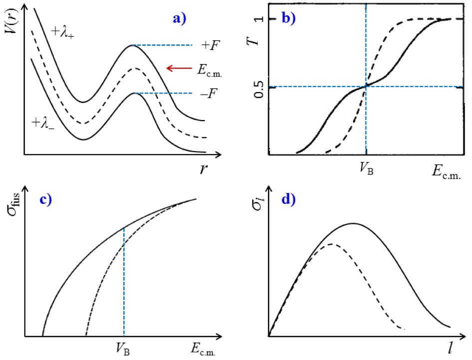
$$\lambda_\pm(r) = [\epsilon_1 \pm \sqrt{\epsilon_1^2 + 4F(r)^2}]/2$$

If $\epsilon_1 = 0$, then

$$\lambda_\pm(r) = \pm F(r),$$

and

$$P_l(E) = \frac{1}{2}\{P_l^0[E; V_l(r) + F(r)] + P_l^0[E; V_l(r) - F(r)]\}.$$



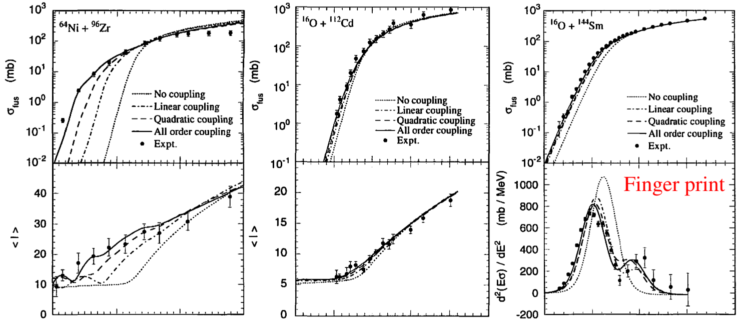
C. J. Lin, Heavy-ion nuclear reactions, (2015)

P. Fröbrich, Theory of Nuclear Reactions, (1996)

Multi-channels problem for heavy-ion reactions

Taking into full order coupling in V_{nm} is important

$$[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_PZ_Te^2}{r}+\epsilon_n-E]\,\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0$$



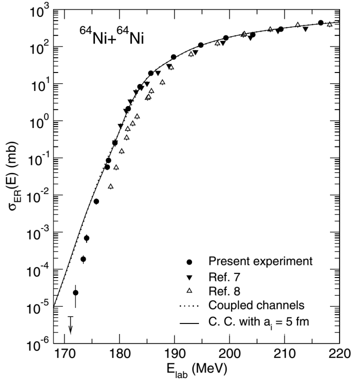
In CCFULL model, the full order couplings are considered.

H. Hagino *et al*, PRC. 55, 276 (1997).
M. Dasgupta *et al*, Annu. Rev. Nucl. Part. S 48, 401 (1998);
H. Hagino *et al*, *Comput. Phys. Commun.* **123** 143 (1999);

Discovery of deep sub-barrier fusion hindrance

B. B. Back, H. Esbensen, C. L. Jiang and K. E. Rehm (2014). Rev. Mod. Phys. 86: 317.

"The comparison with CC calculations using a Woods-Saxon potential allowed them to cleanly identify the fusion hindrance at the lowest energies."



Argonne National Laboratory Experiments:

C. L. Jiang, H. Esbensen et al,

Phys Rev Lett 89 (5), 052701 (2002);

Phys Rev Lett 93 (1), 012701 (2004);

Physical Review C 71(4): 044613 (2005)

Physics Letters B 640(1): 18-22. (2006)

Phys Rev Lett 113 (2), 022701 (2014).

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ANU Experiments:

M. Dasgupta, D. J. Hinde, A. Diaz-Torres, et al, Phys Rev Lett 99, 192701 (2007).

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INFN Experiments:

G. Montagnoli, A. M. Stefanini, et al,

Physical Review C 85(2): 024607. (2010);

Physics Letters B 728: 639. (2014)

Physical Review C 97(2): 024610.(2018)

Physical Review C 100(4): 044619. (2019).

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Deep sub-barrier fusion hindrance & S factor

PHYSICAL REVIEW C **97**, 012801(R) (2018)

Rapid Communications

Reaction rate for carbon burning in massive stars

C. L. Jiang,[1] D. Santiago-Gonzalez,[1,2] S. Almaraz-Calderon,[1,3] K. E. Rehm,[1] B. B. Back,[1] K. Auranen,[1] M. L. Avila,[1]

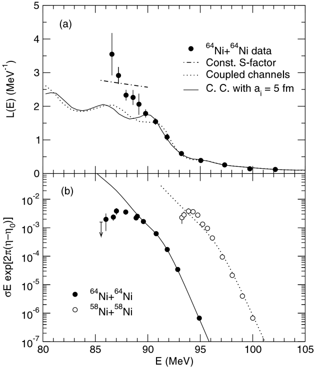
Carbon burning is a critical phase for nucleosynthesis in massive stars. The conditions for igniting this burning stage, and the subsequent isotope composition of the resulting ashes, depend strongly on the reaction rate for $^{12}C + ^{12}C$ fusion at very low energies. Results for the cross sections for this reaction are influenced by various backgrounds encountered in measurements at such energies. In this paper, we report on a new measurement of $^{12}C + ^{12}C$ fusion cross sections where these backgrounds have been minimized. It is found that the astrophysical S factor exhibits a maximum around $E_{cm} = 3.5$–4.0 MeV, which leads to a reduction of the previously predicted astrophysical reaction rate.

PRL **113,** 022701 (2014) PHYSICAL REVIEW LETTERS week ending 11 JULY 2014

Fusion Hindrance for a Positive-Q-Value System $^{24}Mg + ^{30}Si$

C. L. Jiang,[1,*] A. M. Stefanini,[2] H. Esbensen,[1] K. E. Rehm,[1] S. Almaraz-Calderon,[1] B. B. Back,[1] L. Corradi,[2] E. Fioretto,[2] G. Montagnoli,[3] F. Scarlassara,[3] D. Montanari,[3] S. Courtin,[4] D. Bourgin,[4] F. Haas,[4] A. Goasduff,[5] S. Szilner,[6] and T. Mijatovic[6]

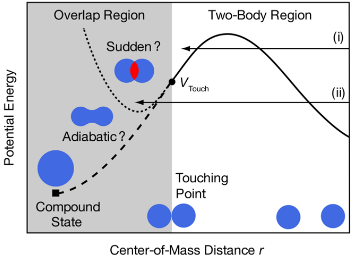
Measurements of the excitation function for the fusion of $^{24}Mg + ^{30}Si$ ($Q = 17.89$ MeV)have been extended toward lower energies with respect to previous experimental data. The S-factor maximum observed in this large, positive-Q-value system is the most pronounced among such systems studied thus far. The significance and the systematics of an S-factor maximum in systems with positive fusion Q values are discussed. This result would strongly impact the extrapolated cross sections and reaction rates in the carbon and oxygen burnings and, thus, the study of the history of stellar evolution.

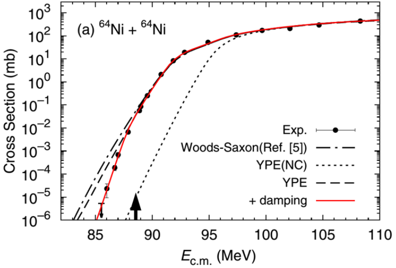


$$\langle \sigma v \rangle \approx \left(\frac{2}{\mu}\right)^{\frac{1}{2}} \frac{\Delta E_0}{(kT)^{3/2}} S(E_0) \exp\left(-\frac{3E_0}{kT}\right); \qquad S(E) = \sigma E \exp(2\pi\eta); \qquad \eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}$$

Fusion between light nuclei is of interest because its important roles in the late stages of massive star evolution.

Explanations: adiabatic approximation & deep potential





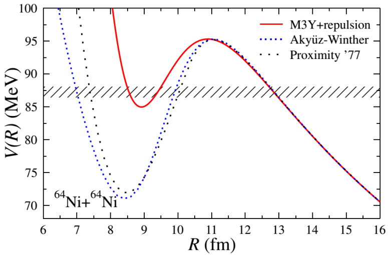
- T. Ichikawa, K. Hagino and A. Iwamoto, Phys Rev C 75, 064612 (2007); Phys Rev Lett 103, 202701 (2009); T. Ichikawa, Phys Rev C 92 (6), 064604 (2015).

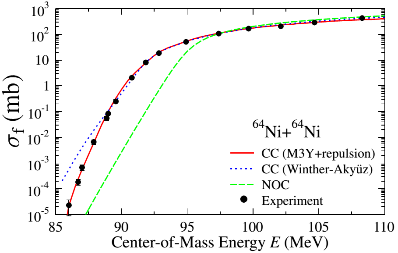
  On top of the conventional CC method, an extra one-dimensional adiabatic potential barrier is assumed after the reacting nuclei contact with each other, considering the formation of the composite system.

- K. Hagino, A. B. Balantekin, N. W. Lwin et al, Phys Rev C 97, 034623 (2018).

  Two Woods-Saxon potentials with different slopes.

Explanations: sudden approximation & shallow potential





- Ş. Mişicu and H. Esbensen, Phys Rev Lett 96 (11), 112701 (2006); Phys Rev C 75, 034606 (2007); ....

  Hindrance of Heavy-Ion Fusion due to Nuclear Incompressibility. Double-folding potential with M3Y forces supplemented by a repulsive core.

- C. Simenel, A. S. Umar, K. Godbey, et al, Phys Rev C 95, R031601 (2017).

  Density constrained time dependent Hartree-Fock model. It is concluded that: " ...to explain experimental fusion data at deep sub-barrier energies, then cannot be justified by an effect of incompressibility. It is more likely that it simulates other effects such as Pauli repulsion."

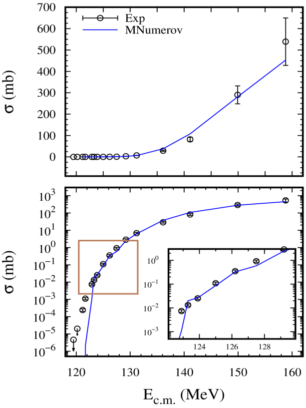
- V. V. Sargsyan, G. G. Adamian, N. V. Antonenko et al, Eur Phys J A 56, 19 (2020).

  Extended quantum diffusion approach + Double folding potential.

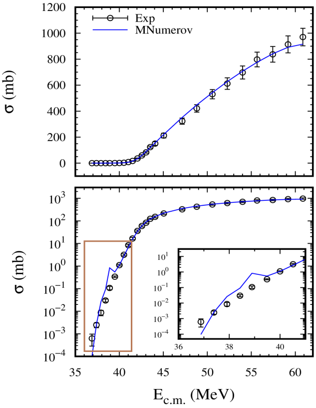
The instability of the coupled channels model

There are fluctuations at deep sub-barrier energy region. “For shallow pocket potentials, however, the IWBC should be replaced by an imaginary potential at the potential pocket to avoid numerical instabilities."

^{64}Ni+^{100}Mo



^{36}S+^{48}Ca



C. Simenel, et al, Phys Rev C 95, R031601 (2017).

V.I. Zagrebaev *et al*, 2004 *Phys. Atom. Nucl.* **67** 1462

Some open questions

About deep sub-barrier fusion hindrance:

- Whether could the CC calculation of the fusion cross section be stable at the deep sub-barrier energy region?

  Some works used an extra imaginary potential around the potential minimum to eliminate the fluctuations of the conventional CC calculation. However, one has to add more parameters.

- Is Woods-Saxon potential able to describe the deep sub-barrier fusion hindrance phenomenon well enough?

  It is said that it is not able to describe it in many works. And hybrid potential model, other potential models, and reaction mechanisms are widely used now.

- What's the mechanism of the fusion hindrance?

  The shallow potential or deep potential.

Contents

Import gradients for solving the coupled-channels equation

There are several parts to construct the coupled-channels approach:

1. Nuclear potential:

   real potential (double folding, proximity, Woods-Saxon potential), complex potential

2. Coupled potential:

   full order coupling, linear coupling, or the quadratic coupling

3. Boundary condition:

   regular boundary condition, incoming wave boundary condition

4. Numerical method:

   finite difference method (Numerov , three-point difference), finite element method (KANTBP), R-matrix method.

O. Chuluunbaatar, A. A. Gusev, *et al*, CPC. 177, 649 (2007)
A. A. Gusev, O. Chuluunbaatar, S. I. Vinitsky *et al*, CPC 185, 3341 (2014)

The nuclear potential

The Akyüz-Winther (AW) type Woods-Saxon potential as starting point:

$$\begin{aligned} V_N^{(0)}(r) &= -\frac{V_0}{1+\exp((r-R_0)/a_0)}. \\ &= \frac{-16\pi\gamma a_0 \bar{R}}{1+\exp[(r-R_P-R_T)/a_0]}, \end{aligned}$$

A. Winther, Nucl. Phys. A 594, 203 (1995)

with

$$\begin{aligned} \frac{1}{a_0} &= 1.17[1+0.53(A_P^{-1/3}+A_T^{-1/3})] \\ \bar{R} &= \frac{R_P R_T}{R_P+R_T} \qquad R_i = 1.2A_i^{1/3} - 0.09,\ i = P, T \\ \gamma &= 0.95\left(1 - 1.8\frac{(N_P-Z_P)(N_T-Z_T)}{A_P A_T}\right) \end{aligned}$$

No free parameters and widely used for fusion reaction.

The coupled potential (full order coupling)

The nuclear coupling Hamiltonian can be generated by changing the potential radius to a dynamical operator $R_0 + \hat{O}$ with $\hat{O}|\alpha\rangle = \lambda_\alpha|\alpha\rangle$

$$\hat{O} = \frac{\beta_\lambda}{\sqrt{4\pi}} r_{\rm coup}\, A_T^{1/3}(a^\dagger_{\lambda 0} + a_{\lambda 0})$$

Bohr, A. and Mottelson, B. R. Nuclear Structure II, (1969)

The nuclear coupling potential is given on top of the potential as

$$V'_N(r, \hat{O}) = -\frac{V_0}{1 + \exp((r - R_0 - \hat{O})/a_0)}.$$

It is considered with full order by diagonalizing the matrix $\hat{O}$

$$O_{nm} = \frac{\beta_\lambda}{\sqrt{4\pi}} r_{\rm coup}\, A_T^{1/3}(\sqrt{m}\delta_{n,m-1} + \sqrt{n}\delta_{n,m+1})$$

The nuclear coupling matrix elements between phonon state $|n\rangle$ and $|m\rangle$ is

$$\begin{aligned} V^{(N)}_{nm} &= \langle n|V'_N(r,\hat{O})|m\rangle - V^{(0)}_N \delta_{n,m} \\ &= \sum_\alpha \langle n|\alpha\rangle\langle\alpha|m\rangle V'_N(r,\lambda_\alpha) - V^{(0)}_N \delta_{n,m} \end{aligned}$$

H. Hagino *et al*, *Comput. Phys. Commun.* **123** 143 (1999);

The incoming wave boundary condition

The incoming wave boundary conditions (IWBC)

$$\psi_n(r) = \begin{cases} T_n \exp\left(-ik_n(r_{\min})r\right), & r \leq r_{\min} \\ H_l^-(k_n r)\delta_{n,0} - R_n H_l^+(k_n r), r \geq r_{\max} \end{cases}$$

Here $k_n = k_n(r \to +\infty)$, and $k_n(r)$ is the local wave number for n-th channel

$$k_n(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - \epsilon_n - \frac{l(l+1)\hbar^2}{2\mu r^2} - V_N^{(0)}(r) - \frac{Z_P Z_T e^2}{r} - V_{nn}(r)\right)}$$

There are problems in the previous boundary condition.

- The plane wave boundary condition at the left boundary $r_{\min}$ involves only the diagonal part. This requires that the off-diagonal matrix elements tend to zero.
- However, at $r_{\min}$, the distance between two nuclei is so short that the off-diagonal matrix elements are usually not zero. There can be sudden noncontinuous changes in the left boundary.
- A linear transformation should be done at the left boundary.

V.V. Samarin, V.I. Zagrebaev, 2004 *NPA* **734** E9;
V.I. Zagrebaev, V.V. Samarin, 2004 *Phys. Atom. Nucl.* **67** 1462;

The new method KANTBP

The coupled-channels Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\psi_{nn_o}+\sum_{n'=1}^{N}V_{nn'}(r)\psi_{n'n_o}(r)=0, \quad (1)$$

with

- n_o is a number of the open entrance channel with a positive relative energy $E_{n_o}=E-\epsilon_{n_o}>0$, $n_o=1,\ldots,N_o\le N$.
- $\{\psi_{nn_o}(r)\}_{n=1}^{N}$ are components of a desirable matrix solution.

Let $\mathbf{W}$ is the symmetric matrix of dimension $N\times N$

$$W_{nm}=W_{mn}=\frac{2\mu}{\hbar^2}\left[\left(\frac{l(l+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n\right)\delta_{nm}+V_{nm}(r)\right]. \quad (2)$$

Then the equation can be expressed as

$$-\psi''_{nm}(r)+\sum_{m'}W_{nm'}\psi_{m'm}(r)=\frac{2\mu E}{\hbar^2}\psi_{nm}(r), \quad (3)$$

The new method KANTBP

Diagonalize the matrix at $r = r_{\min}$

$$\mathbf{W}\mathbf{A} = \mathbf{A}\tilde{\mathbf{W}}, \quad \{\tilde{\mathbf{W}}\}_{nm} = \delta_{nm}\tilde{W}_{mm}, \quad \tilde{W}_{11} \leq \tilde{W}_{22} \ldots \leq \tilde{W}_{NN}. \tag{4}$$

The functions $y_m(r)$ are solutions of the uncoupled equations

$$y''_m(r) + K_m^2 y_m(r) = 0, \quad K_m^2 = \frac{2\mu E}{\hbar^2} - \tilde{W}_{mm}. \tag{5}$$

In open channels at $K_m^2 > 0$, $m = 1, \ldots, M_o \leq N$ the solutions $y_m(r)$ have the form:

$$y_m(r) = \frac{\exp(-\imath K_m r)}{\sqrt{K_m}}. \tag{6}$$

In this case $\psi_{nn_o}(r)$ expressed by the linear combinations of the linear independent solutions

$$\psi_{nn_o}(r) = \sum_{m=1}^{M_o} A_{nm} y_m(r) \hat{T}_{mn_o}, \quad r = r_{\min}. \tag{7}$$

In this way, the off-diagonal matrix elements have been considered in the calculation.

The new method KANTBP

Summary of the boundary conditions for open channels

$$\psi^{as}_{nn_o}(r) = \begin{cases} \sum_{m=1}^{M_o} A_{nm} \frac{\exp(-\imath K_m r)}{\sqrt{K_m}} \hat{T}_{mn_o}, & r = r_{\min}, \\ \hat{H}_l^-(k_n r)\delta_{n,n_o} + \hat{H}_l^+(k_n r)\hat{R}_{nn_o}, & r = r_{\max}. \end{cases} \quad (8)$$

In this case the partial tunneling probability from the ground state ($n_o = 1$) is

$$P_l(E) \equiv T^{(l)}_{n_o n_o}(E). \quad (9)$$

At fixed orbital momentum l, it is given by summation over all possible intrinsic states:

$$T^{(l)}_{n_o n_o}(E) = \sum_{m=1}^{M_o} \left|\hat{T}_{mn_o}\right|^2, \quad R^{(l)}_{n_o n_o}(E) = \sum_{n=1}^{N_o} \left|\hat{R}_{nn_o}\right|^2, \quad T^{(l)}_{n_o n_o}(E) = 1 - R^{(l)}_{n_o n_o}(E) \quad (10)$$

The condition $T^{(l)}_{n_o n_o}(E) + R^{(l)}_{n_o n_o}(E) - 1 = 0$ fulfills with ten significant digits by the element method KANTBP.

O. Chuluunbaatar, A. A. Gusev, A.G. Abrashkevich *et al*, CPC. 177, 649 (2007)
A. A. Gusev, O. Chuluunbaatar, S. I. Vinitsky *et al*, CPC 185, 3341 (2014)
A. A. Gusev, O. Chuluunbaatar, S. I. Vinitsky *et al*, Math. Mod. Geom. 3, 2 22 (2015)
V. I. Zagrebaev, Phys. Rev. C 78 047602 (2008)

Contents

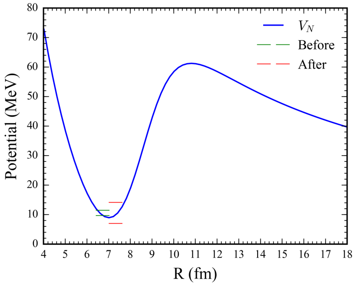
^{16}O+^{144}Sm: A Benchmark calculation

When $l = 0$

$$\frac{\hbar^2}{2\mu} W_{nm} = \left[\left(V_N + \epsilon_n \right) \delta_{nm} + V_{nm}(r) \right]$$

"Before": diagonal elements of $\frac{\hbar^2}{2\mu} W$

"After": diagonal elements of $\frac{\hbar^2}{2\mu} \tilde{W}$

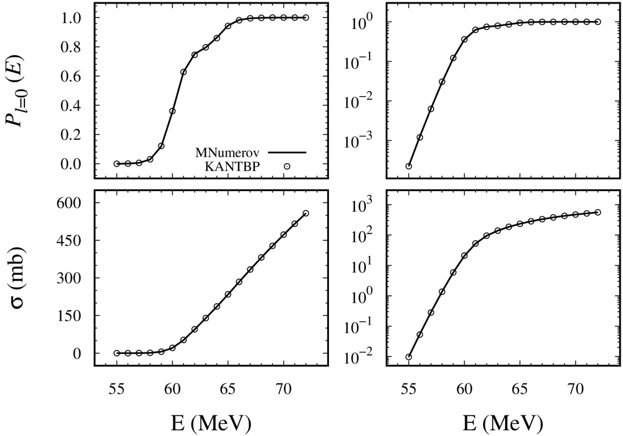


H. Hagino *et al*, 1999 *CPC* **123** 143;

Ecm (MeV)	sigma (mb)	<l>
55.00000	0.97449E-02	5.87031
56.00000	0.05489	5.94333
57.00000	0.28583	6.05134
58.00000	1.36500	6.19272
59.00000	5.84375	6.40451
60.00000	20.59856	6.86092
61.00000	52.14435	7.81887
62.00000	94.62477	9.18913
63.00000	139.58988	10.65032
64.00000	185.55960	11.98384
65.00000	234.04527	13.13045
66.00000	283.93527	14.18620
67.00000	333.26115	15.21129
68.00000	381.21017	16.20563
69.00000	427.61803	17.16333
70.00000	472.48081	18.08211
71.00000	515.83672	18.96273
72.00000	557.73621	19.80734

$^{16}O+^{144}Sm$: A Benchmark calculation

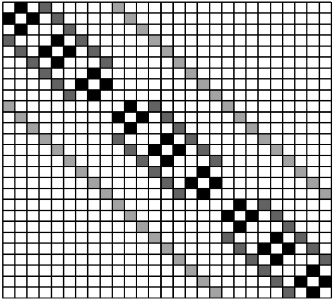
Tunneling probability and fusion cross sections at linearizion and logarithmic scale for $^{16}O+^{144}Sm$.

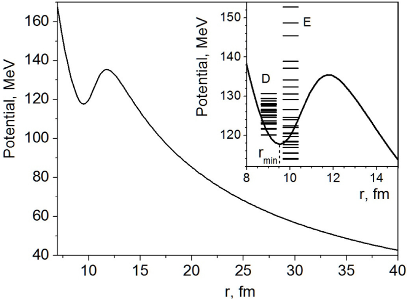


Calculations by the KANTBP method agree well with the MNumerov method.

$^{32}S + ^{182}W$: the coupled potential

S. I. Vinitsky, P. W. Wen, A. A. Gusev, O. Chuluunbaatar, R. G. Nazmitdinov, A. K. Nasirov, C. J. Lin, H. M. Jia and A. Góźdź, Acta Phys. Pol. B Proc. Suppl. 13 (3), 549 (2020).

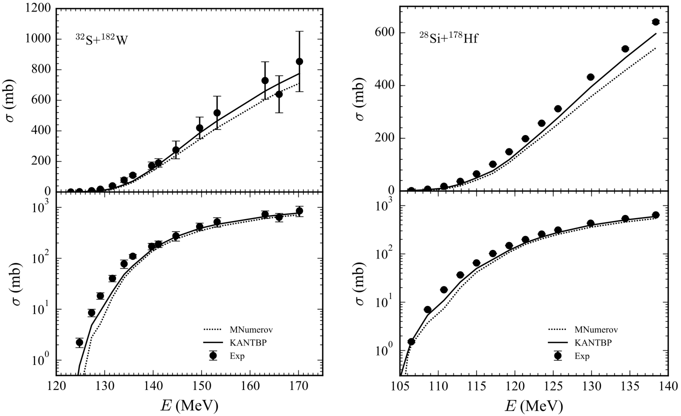




There are many non-diagonal elements of $\hat{O}_{nm}(r)$ at $r_{\min}$.

$^{32}S+^{182}W$, $^{28}Si+^{178}Hf$: Near barrier fusion

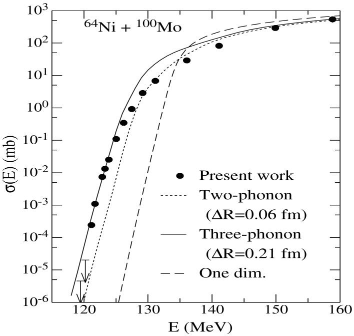
S. I. Vinitsky, P. W. Wen, A. A. Gusev, O. Chuluunbaatar, R. G. Nazmitdinov, A. K. Nasirov, C. J. Lin, H. M. Jia and A. Góźdź, Acta Phys. Pol. B Proc. Suppl. 13 (3), 549 (2020).



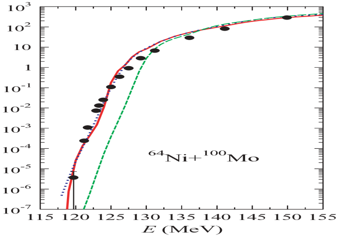
There are obvious differences in sub-barrier energy region.

^{64}Ni+^{100}Mo: Deep sub-barrier fusion

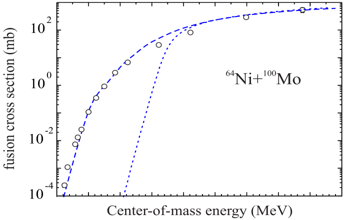
Jiang, C. L, et al, 2005 PRC 71 044613



about 7 MeV (for the same $\Delta R = 0.21$ fm). This is a general feature of coupled-channels calculations; and it is, therefore, very unlikely that any minor adjustment in the coupled-channels calculations would reproduce the steep falloff that the data exhibit at extreme sub-barrier energies. Thus, it appears that the fusion hindrance behavior, which now has been observed for many systems, is also present in the new data for ^{64}Ni + ^{100}Mo. This will be shown more convincingly in the next section, where other representations of the fusion cross section are discussed.



Mişicu *et al*, 2007 *PRC* **75** 034606;



A.V. Karpov *et al*, 2015 *PRC* **92** 064603;

The CC calculations with the M3Y+repulsion potential is usually used.

^{64}Ni+^{100}Mo, ^{28}Si+^{64}Ni: Deep sub-barrier fusion

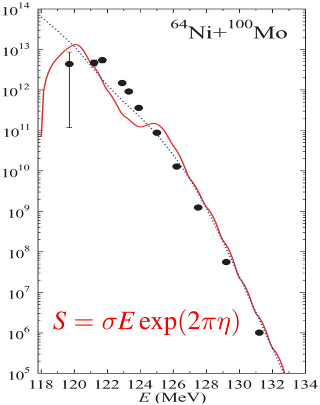
PRL **96,** 112701 (2006) PHYSICAL REVIEW LETTERS week ending 24 MARCH 2006

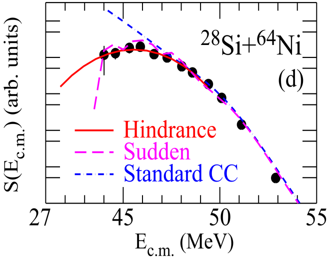
Hindrance of Heavy-Ion Fusion due to Nuclear Incompressibility

Ş. Mişicu* and H. Esbensen

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

(Received 26 January 2006; published 21 March 2006)





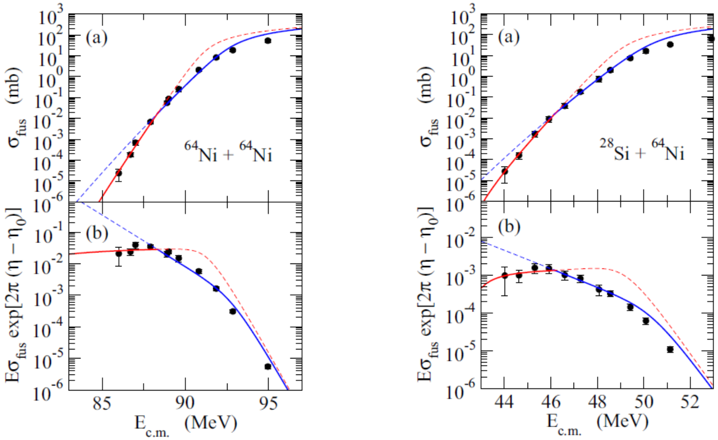
The M3Y+repulsion potential is usually used.

Ş. Mişicu, *et al*, 2007 *PRC* **75** 034606; C.L. Jiang *et al*, 2018 *PRC* **97** 012801

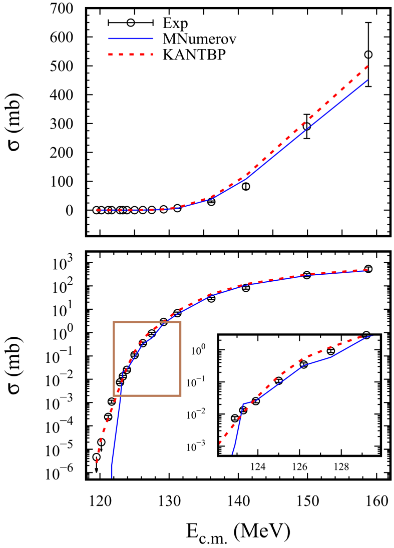
^{64}Ni+^{100}Mo, ^{28}Si+^{64}Ni: Deep sub-barrier fusion

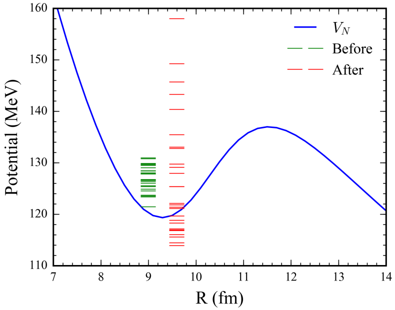
Two potentials including a larger (smaller) logarithmic slope at energies lower (higher) than the threshold energy

HAGINO, BALANTEKIN, LWIN, AND THEIN PHYSICAL REVIEW C 97, 034623 (2018)



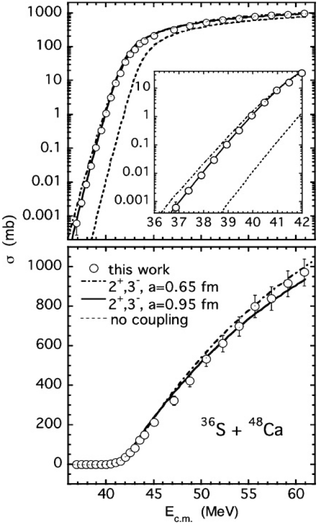
^{64}Ni+^{100}Mo: Deep sub-barrier fusion

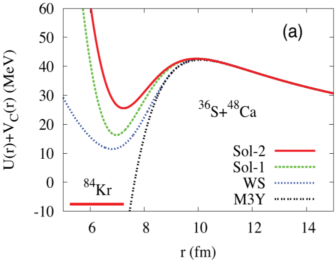


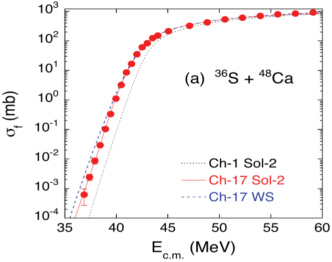


New calculations are more stable and agree with experimental data better

$^{36}S+^{48}Ca$: Deep sub-barrier fusion



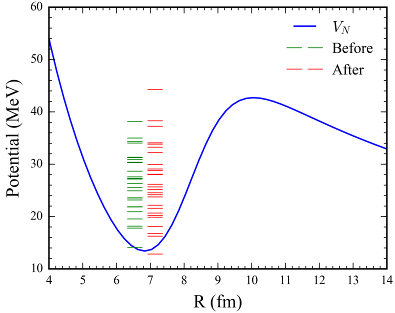


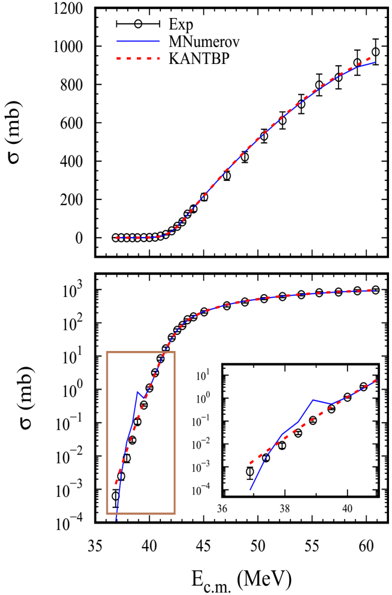


The M3Y+repulsion potential is usually used. The weak imaginary potential is adopted to eliminate some unwanted fluctuations.

A.M. Stefanini, *et al*, 2008 *PRC* **78** 044607; G. Montagnoli *et al*, 2013 *PRC* **87** 014611

$^{36}S+^{48}Ca$: Deep sub-barrier fusion





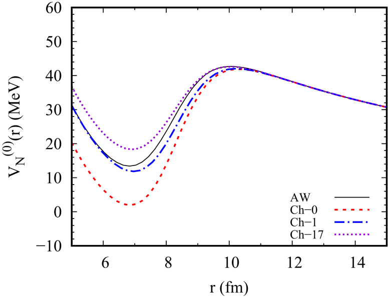
New calculations are more stable, and are higher than experimental data at deep sub-barrier energy.

P. W. Wen, O. Chuluunbaatar, et al, Phy. Rev. C, 101:014618, 2020.

S. I. Vinitsky, P. W. Wen, et al, Acta Phys. Pol. B Proc. Suppl., 13:549, 2020.

$^{36}S+^{48}Ca$: Deep sub-barrier fusion

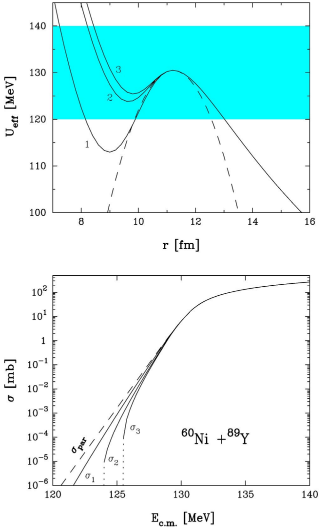
	AW	Ch-0	Ch-1	Ch-17
$N_{P_{3-}}$	-	0	1	1
$N_{P_{2+}}$	-	0	0	2
$N_{T_{2+}}$	-	0	0	2
V_0 (MeV)	61.338	72.325	61.462	55.911
a_0 (fm)	0.654	0.636	0.662	0.676
R_0 (fm)	8.143	8.272	8.208	8.167
V_B (MeV)	42.706	41.885	42.305	42.617
R_B (fm)	10.052	10.296	10.146	10.042
$\hbar\omega$ (MeV)	3.285	3.315	3.237	3.196

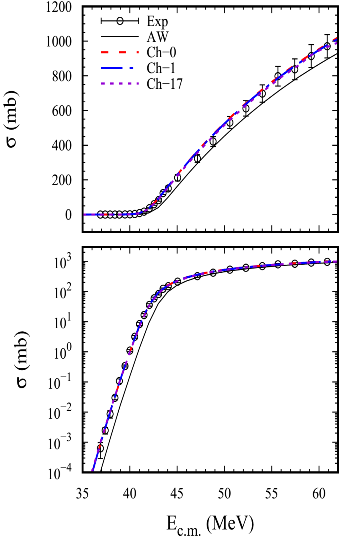


- This reaction can be fitted well by different set of WS parameters.
- The parameters are not far from AW potential parameters.

$^{36}S+^{48}Ca$: Deep sub-barrier fusion

C.H. Dasso, *et al*, 2003 *PRC* **68** 054604

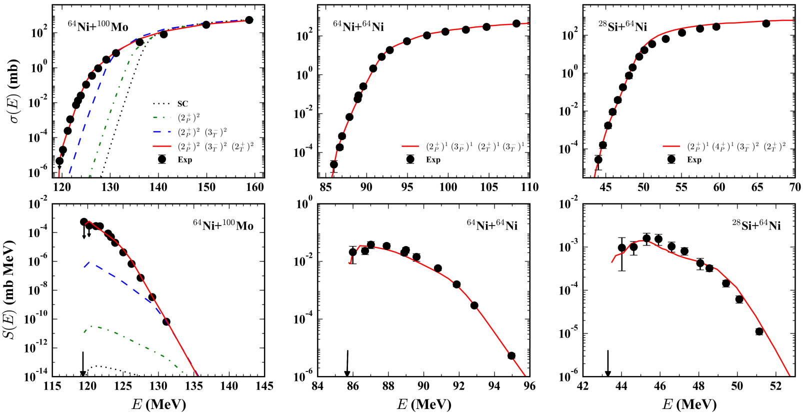




The deep sub-barrier cross sections are sensitive to the potential pocket.

^{64}Ni+^{100}Mo, ^{64}Ni+^{64}Ni, ^{28}Si+^{64}Ni

P. W. Wen, C. J. Lin, R. Nazmitdinov, S. I. Vinitsky, et al. *PRC*, 103, 054601, 2021.

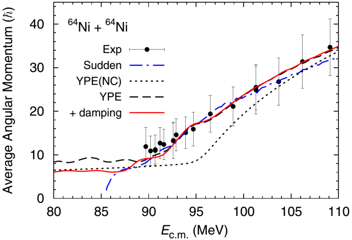


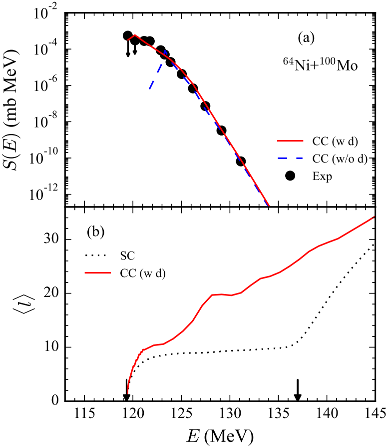
Woods-Saxon potential and multiphonon coupling are enough.

^{64}Ni+^{100}Mo: Potential details

TABLE I. Woods-Saxon potential parameters V_0 (MeV), a_0 (fm), and R_0 (fm) for ^{64}Ni + ^{100}Mo, ^{64}Ni + ^{64}Ni, and ^{28}Si + ^{64}Ni reaction systems. The potential barrier V_B and the minimum of the potential pocket V_P are also listed.

	^{64}Ni + ^{100}Mo	^{64}Ni + ^{64}Ni	^{28}Si + ^{64}Ni
V_0 (MeV)	79.938	65.829	53.529
a_0 (fm)	0.686	0.801	0.944
R_0 (fm)	10.190	9.239	7.790
V_B (MeV)	136.993	96.389	51.946
V_P (MeV)	119.344	85.699	43.298

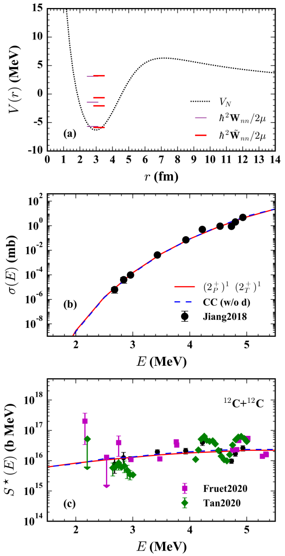




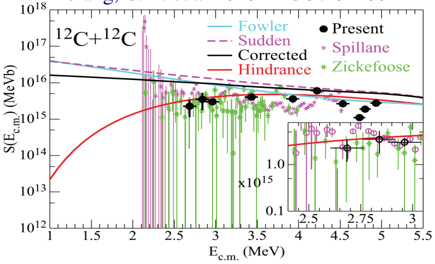
$\langle l \rangle$ could be used as a probe to seperate these two mechanisms.

Ichikawa, T. (2015). Phys Rev C 92: 064604.

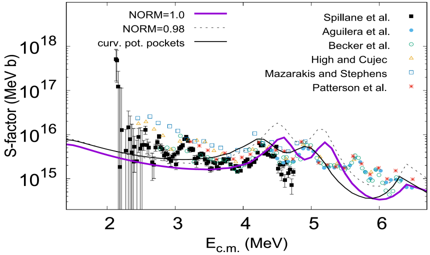
$^{12}C+^{12}C$



Jiang, C. L. et al 2018 PRC 97 012801

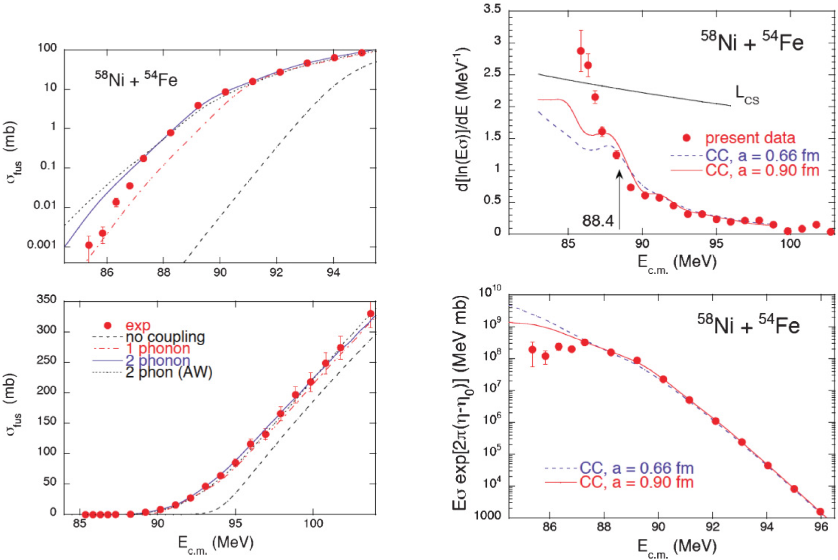


A. Diaz-Torres, et al, 2018 PRC 97 055802



Time dependent tunneling treatment is important.

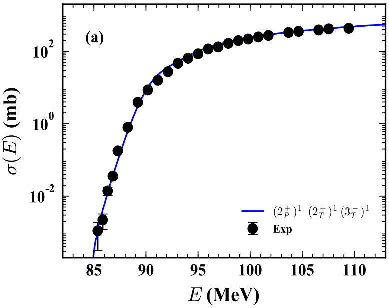
$^{58}Ni + ^{54}Fe$ experiment



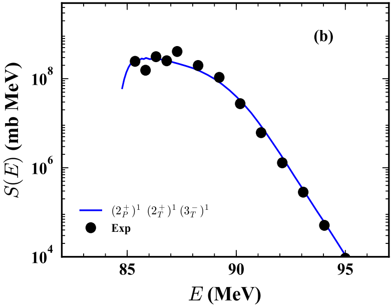
A. M. Stefanini, G. Montagnoli, et al, Phys Rev C 82, 014614 (2010)

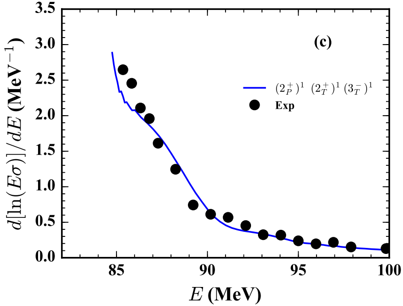
^{58}Ni+^{54}Fe calculations (preliminary results)

The fitted WS potential parameters are V_0= 51.312 MeV, R_0= 9.268 fm, and a_0 = 0.688 fm.



The diffuseness parameter is normal.





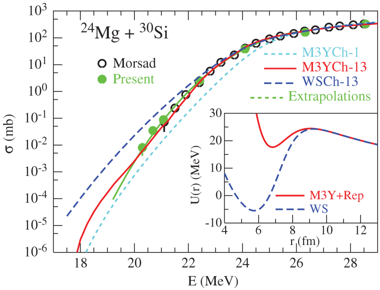
$^{24}Mg+^{30}Si$ (preliminary results)

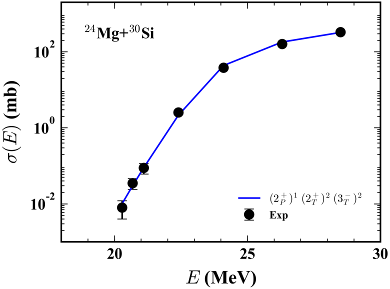
PRL **113,** 022701 (2014) PHYSICAL REVIEW LETTERS week ending 11 JULY 2014

Fusion Hindrance for a Positive-Q-Value System $^{24}Mg + ^{30}Si$

C. L. Jiang,[1,*] A. M. Stefanini,[2] H. Esbensen,[1] K. E. Rehm,[1] S. Almaraz-Calderon,[1] B. B. Back,[1] L. Corradi,[2] E. Fioretto,[2] G. Montagnoli,[3] F. Scarlassara,[3] D. Montanari,[3] S. Courtin,[4] D. Bourgin,[4] F. Haas,[4] A. Goasduff,[5] S. Szilner,[6] and T. Mijatovic[6]

Measurements of the excitation function for the fusion of $^{24}Mg + ^{30}Si$ ($Q = 17.89$ MeV)have been extended toward lower energies with respect to previous experimental data. The S-factor maximum observed in this large, positive-Q-value system is the most pronounced among such systems studied thus far. The significance and the systematics of an S-factor maximum in systems with positive fusion Q values are discussed. This result would strongly impact the extrapolated cross sections and reaction rates in the carbon and oxygen burnings and, thus, the study of the history of stellar evolution.





Contents

Summary

Potential answers to the previous questions based on sudden approximation:

- Whether could the calculation of the fusion cross section be stable at the deep sub-barrier energy region?

  The calculations are stable now with the coupled-channels approach adopting the finite element method KANTBP with the improved boundary condition.

- Is Woods-Saxon potential able to describe the deep sub-barrier fusion hindrance phenomenon well enough?

  The deep sub-barrier fusion cross sections, as well as the S facotr, of several typical reactions have been successfully described by using the most simple 3 parameter WS potential and multiphonon couplings.

- What's the mechanism of the fusion hindrance?

  $\langle l \rangle$ could be used to clarify shallow or deep potential.

Perspective

- What's the systematics of the maximum of fusion hindrance and S factor with respect to different reaction systems?

  We have fitted several reactions with hindrance feature at deep sub-barrier energy region, and are trying to see the systematics by fitting more reactions.

- The impact of the finite elements method on complex potential and regular boundary condition?

  The current version of the high accuracy KANTBP is only suitable for real potential yet. Prof. S. I. Vinitsky, O. Chuluunbaatar, A. A. Gusev are working on this aspect.

- The role of other mechanisms like transfer or decoherence on deep sub-barrier fusion hindrance?

Thank you for your attention !