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The IAV model for inclusive breakup: recent applications and perspectives

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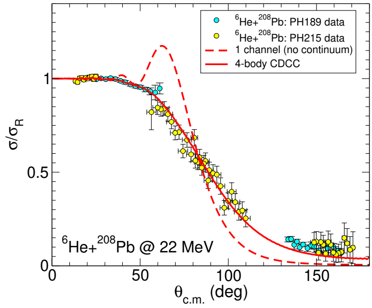
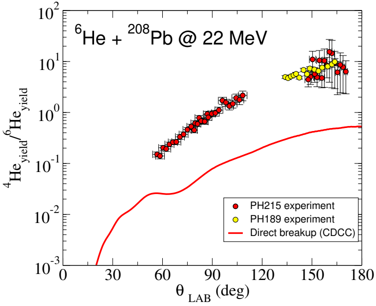
ENSAR2

MINISTERIO
DE ECONOMÍA
Y COMPETITIVIDAD

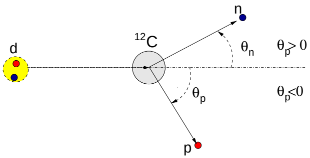
$$^6\mathrm{He} + ^{208}\mathrm{Pb} \rightarrow \alpha + \mathrm{X}$$





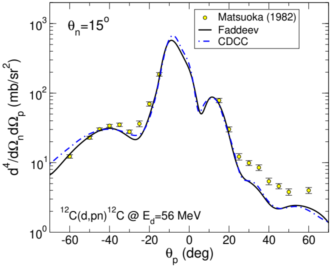
Data (LLN): Sánchez-Benítez et al, NPA 803, 30 (2008) L. Acosta et al, PRC 84, 044604 (2011), D. Escrig et al., NPA 792 (2007) 2
Calculations: Rodríguez-Gallardo et al, PRC 80, 051601 (2009)

☞ *CDCC reproduces elastic scattering, but not inclusive α's.*



Exclusive breakup:

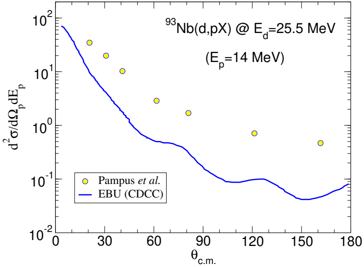
$d+^{12}$C$\rightarrow$ p+n+^{12}C(g.s.)



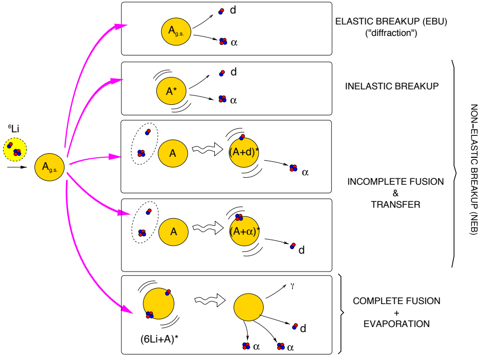
Deltuva et al,PRC76, 064602 (2007)

Inclusive breakup:

$d+^{12}$C$\rightarrow$ p+ X

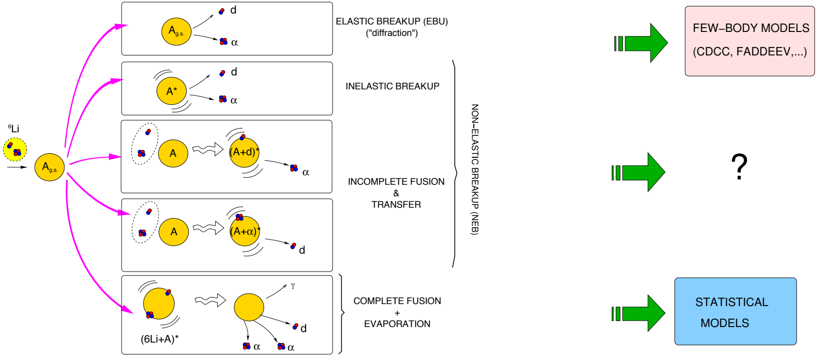


Data:Pampus et al, NPA311, 141 (1978)



➩ For a reaction of the form $a(= b + x) + A \to b + \text{anything}$

$$\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}$$



➩ For a reaction of the form $a(= b + x) + A \to b + \text{anything}$

$$\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}$$

- Inclusive breakup could in principle be evaluated computing all contributing processes using standard reaction methods, such as DWBA or CRC.

- This procedure has serious shortcomings:
  - Final $x+A$ states span a wide range of excitation energies and spins, so the number of populated states will in general be huge.
  - A significant part of the inclusive spectrum corresponds to x-A continuum.
  - An explicit calculation would require a detailed knowledge of the populated states: spin/parity, spectroscopic factors,..., which are poorly known above a few MeV of excitation energy
  - Final states will include, in addition to direct processes, partial fusion ("incomplete fusion"), which are not easily accounted for by standard direct reaction theories.

Inclusive breakup models based on closed-form formulas provide an efficient (and elegant) alternative which exploit the fact that nonelastic $x-A$ processes are encoded in the imaginary part of the U_{xA} optical potential.

- **Problem:** describe $a + A$ scattering in a restricted (projected) modelspace (the P space) consisting of the projectile and target ground states without explicit inclusion other states:

$$\Psi = \underbrace{\Psi_P}_{\text{elastic}} + \underbrace{\Psi_Q}_{\text{non-elastic}}$$

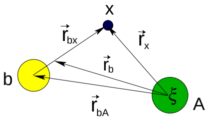
- Schrodinger equation in modelspace:

$$[T + \mathcal{V}]\,\Psi_P = E\Psi_P$$

$$\mathcal{V} = \underbrace{V_{PP}}_{\text{Bare interaction}} + \underbrace{V_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}V_{QP}}_{\text{"Polarization" potential}}$$

- $\mathcal{V}$ too complicated $\Rightarrow$ usually replaced by some energy-averaged optical potential U_a (complex)
- Asymptotically, Ψ_P contains only the elastic channel so it provides only $a + A$ elastic scattering
- However, in virtue of the optical theorem, it provides also the total reaction cross section:

$$\sigma_{\text{reac}} = -\frac{2}{\hbar v_a}\langle\Psi_P|\,\text{Im}[U_a]|\Psi_P\rangle$$



- **Problem:** We would like to solve the three-body scattering problem $a+A \to b+x+A$ in a projected modelspace including explicitly only the target ground state.
- Due to the A(g.s.) projection, the $x-A$ interaction becomes complex U_x.
- The final $x-A$ wavefunction in the projected modelspace, $\psi_x^0(\vec{k}_b,\vec{r}_x)$, must be consistent with the production of the b fragment with momentum $\vec{k}_b$.
- Asymptotically, the $x-A$ wavefunction contains only the A(g.s), i.e., EBU:

$$\psi_x^0(\vec{k}_b,\vec{r}_x) \to f(\vec{k}_b,\vec{k}_x)\,\frac{e^{ik_x r_x}}{r_x}$$

- The absorption taking place in the $x-A$ channel is, by definition, the NEB part of the b inclusive cross section:

$$\boxed{d^2\sigma = -\frac{2}{\hbar v_i}\langle\psi_x^0|W_x|\psi_x^0\rangle\,\frac{d\vec{k}_b}{(2\pi)^3}} \Rightarrow \boxed{\left.\frac{d^2\sigma}{dE_b\,d\Omega_b}\right|_{\rm NEB} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\psi_x^0|W_x|\psi_x^0\rangle}$$

- But, how to compute the x-channel wavefunction $\psi_x^0(\vec{k}_b,\vec{r}_x)$?

- Baur & co: DWBA sum-rule with surface approximation.
  - Baur et al, PRC21, 2668 (1980).
- Hussein & McVoy: extraction of singles cross section combining the spectator model with sum rule over final states.
  - Nucl. Phys. A445, 124 (1985).
- Ichimura, Austern, Vincent (IAV): Post-form DWBA.
  - Ichimura, Austern, Vincent, PRC32, 431 (1985).
  - Austern *al*, Phys. Rep.154, 125 (1987).
- Udagawa, Tamura (UT): prior-form DWBA.
  - Udagawa and Tamura, PRC24, 1348 (1981).
  - Udagawa, Lee, Tamura, PLB135, 333 (1984).

⇨*Most of these theories have fallen into disuse and are now being revisited by several groups*

Recent implementations of inclusive breakup theories:

- Carlson, Capote, Sin, FBS57, 307 (2016).
- Potel, Nunes, Thompson, PRC92, 034611 (2015).
- Lei, AMM, PRC92, 044616 (2015).

Brett Carlson, **Mahir Hussein**, A.M.M. and Gregory Potel at the workshop "Deuteron-induced reactions and beyond: inclusive breakup fragment cross sections, MSU, July 2016"
G. Potel, Eur. Phys. J. A (2017) 53: 178).

Calculation of the x-channel wavefunction $\psi_x^0(\vec{k}_b, \vec{r}_x)$ in the spectator model

- Spectator model: b scatters elastically by the target but does not influence the breakup; its motion is described by a distorted wave $\chi_b^{(-)}(\vec{k}_b, \mathbf{r}_b)$.

- Within the spectator model and assuming 3-body model of the reaction, Austern *et al.* (Phys. Rep. 154 , 125 (1987)) derived:

$$\varphi_x^{3B}(\vec{r}_x) = \langle \vec{r}_x \chi_b^{(-)}(\vec{k}_b) | \Psi^{3B(+)} \rangle$$

  with $\Psi^{3B(+)}$ a 3-body scattering wavefunction.

- In practice, no straightforward choice for $\Psi^{3B(+)}$, e.g.:
  - ✖ Faddeev: accurate, but difficult to compute
  - ✖ CDCC: more feasible, but only accurate for small b-x separations

Alternative expressions for $\psi^0_x(\vec{k}_b, \vec{r}_x)$

- The 3-body model of Austern *et al* (3B):

$$(E_x^+ - K_x - U_x)\varphi_x^{3B}(\vec{r}_x) = \langle \vec{r}_x\, \chi_b^{(-)} |\, V_{\rm post} | \Psi^{3B(+)} \rangle \qquad V_{\rm post} \equiv V_{bx} + U_{bA} - U_{bB}$$

- The Ichimura, Austern, Vincent post-form DWBA formula (IAV):

$$(E_x^+ - K_x - U_x)\varphi_x^{\rm post}(\vec{r}_x) = \langle \vec{r}_x\, \chi_b^{(-)} |\, V_{\rm post} | \chi_a^{(+)} \phi_a \rangle \qquad V_{\rm post} \equiv V_{bx} + U_{bA} - U_{bB}$$

- The Udagawa and Tamura prior-form formula DWBA (UT):

$$(E_x^+ - K_x - U_x)\psi_x^{\rm prior}(\vec{r}_x) = \langle \vec{r}_x\, \chi_b^{(-)} |\, V_{\rm prior} | \chi_a^{(+)} \phi_a \rangle \qquad V_{\rm prior} \equiv U_{xA} + U_{bA} - U_a$$

- The Hussein-McVoy formula (HM):

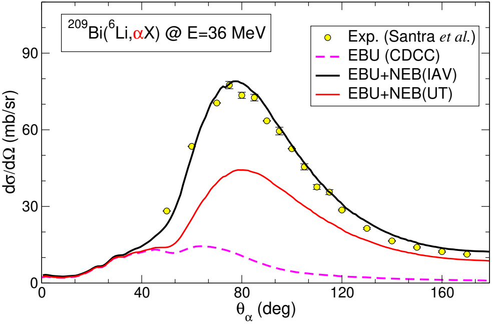
$$\varphi_x^{\rm HM}(\vec{r}_x) = \langle \vec{r}_x \chi_b^{(-)} | \chi_a^{(+)} \phi_a \rangle$$

Relation among IAV, HM and UT models

IAV, UT and HM are related by:

$$\varphi_x^{\rm post}(\vec{r}_x) = \varphi_x^{\rm prior}(\vec{r}_x) + \varphi_x^{\rm HM}(\vec{r}_x)$$

Extensive comparisons with experimental data clearly favour the IAV model over the UT and HM theories:



The U_{xA} potential

$$(E_x - K_x - U_x)\varphi_x^{\rm post}(\vec{r}_x) = \langle \vec{r}_x \chi_b^{(-)} | V_{\rm post} | \chi_a^{(+)} \phi_a \rangle$$

- For $E_x > 0$, U_x is the usual optical model potential describing $x - A$ elastics
  ☞ $\mathrm{Im}[U_x]$ accounts for $x - A$ **absorption**.
- For $E_x < 0$, U_x represents the distribution of single-particle (s.p.) states
  ☞ $\mathrm{Im}[U_x]$ accounts for **s.p. fragmentation** (spreading widths).

$$(E_x - K_x - U_x)\varphi_x^{\rm post}(\vec{r}_x) = \langle \vec{r}_x \, \chi_b^{(-)} | V_{\rm post} | \chi_a^{(+)} \phi_a \rangle$$

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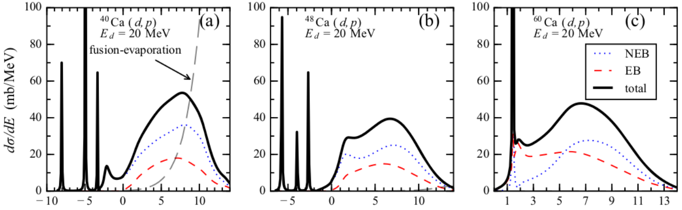
  ☞ $\mathrm{Im}[U_x]$ accounts for **s.p. fragmentation** (spreading widths).

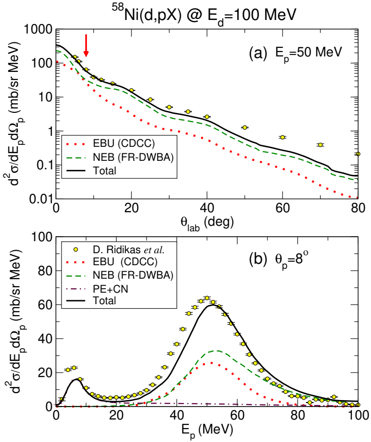
⇨ *These properties are naturally accommodated in dispersive optical model (DOM) potentials*

⇨ *Both "transfer" to unbound states and bound states described on an equal footing*



Applications of the IAV model to inclusive breakup

- EBU calculated with CDCC.
- NBU calculated with DWBA IAV model



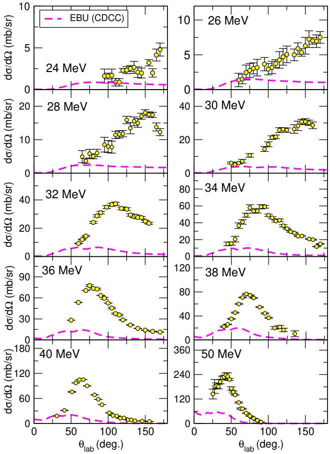
Data:

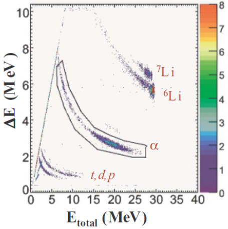
Ridikas et al, PRC63, 014610 (2000)

Calculations

J. Lei, A.M.M., PRC 92, 044616 (2015)

- Large α yield ($\sigma_\alpha \gg \sigma_d$) $\Rightarrow$ evidence of NEB channels
- EBU alone cannot explain the data.

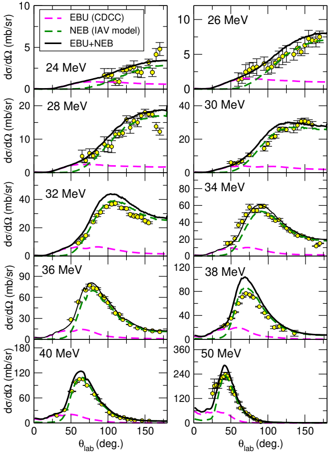




Santra et al, PRC85, 014612 (2012)

Assume: $\sigma_\alpha \simeq \sigma^{\text{EBU}} + \sigma^{\text{NEB}}$:

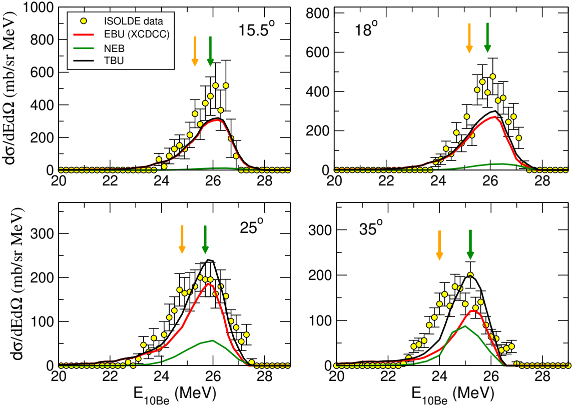
- EBU ➩ CDCC
- NEB ➩ IAV



- Inclusive data well accounted for by EBU+NEB
- Inclusive α's dominated by NEB
- EBU (^{6}Li $\rightarrow \alpha + d$) only relevant for small scattering angles.

J. Lei and AMM, PRC92, 044616 (2015)

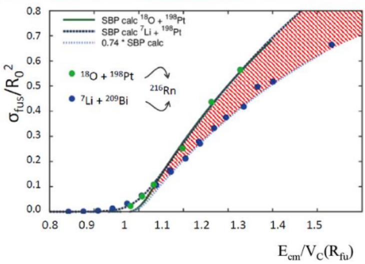
^{64}Zn(^{11}Be,^{10}Be)X @ 28.7 MeV



Di Pietro et al, PLB 798 (2019) 134954

Applications of the IAV model to fusion

CF of weakly bound nuclei suppressed at energies above the Coulomb barrier:



- Observed for weakly bound projectiles (^{6,7,8}Li,^{9}Be)
- CF reduced by $\sim$**30%** with respect to BPM or CC calculations.

M. Dasgupta et al., PRC 70, 024606 (2004)

Common interpretation:

- ⇨ CF is mostly reduced by **breakup** and **incomplete fusion (ICF)**.
- ⇨ ICF is modeled as a **two-step** process: **elastic breakup** followed by **capture** of one charged fragment (breakup-fusion, BF).

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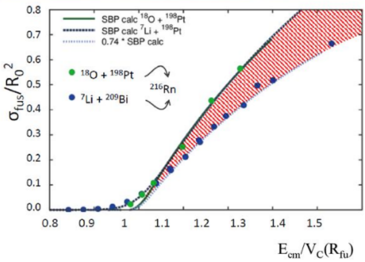

- Observed for weakly bound projectiles (^{6,7,8}Li, ^{9}Be)
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⇨ ICF is modeled as a **two-step** process: **elastic breakup** followed by **capture** of one charged fragment (breakup-fusion, BF).

...*but, calculations based on this two-step picture give only a modest reduction of CF (eg. Cook et al. , PRC 93, 064604 (2016)).*

- CF is related to other nonelastic channels by the **reaction cross section**. For a two-body projectile ($a = b + x$):

$$\sigma_R \approx \sigma_{\rm CF} + \sigma_{\rm inel} + \sigma_{\rm EBU} + \sigma^{(b)}_{\rm NEB} + \sigma^{(x)}_{\rm NEB}$$

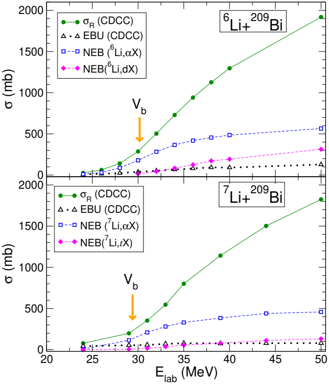
- CF can be deduced from

$$\sigma_{\rm CF} \approx \underbrace{\sigma_R}_{\text{CDCC/OM}} - \underbrace{\sigma_{\rm inel}}_{\text{CC/CDCC}} - \underbrace{\sigma_{\rm EBU}}_{\text{CDCC}} - \underbrace{(\sigma^{(b)}_{\rm NEB} + \sigma^{(x)}_{\rm NEB})}_{\text{IAV model}}$$

N.b. Since ICF is part of the NEB, the calculated CF takes into account the flux going to ICF

What is the relative contribution of the different nonelastic channels?

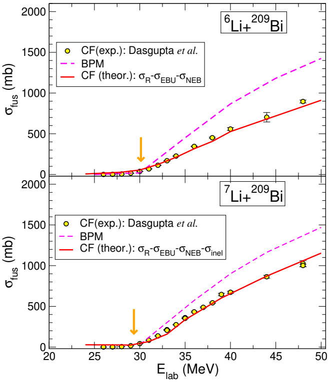
- EBU: computed with CDCC
- NEB: computed with IAV model



- Dominant nonelastic channel is ^{209}Bi(^{6,7}Li,α)X
- EBU ($\alpha + d$) mechanism gives a minor contribution to σ_{reac}.

J. Lei, AMM, PRL122, 042503 (2019)

Effect of nonelastic channels on fusion

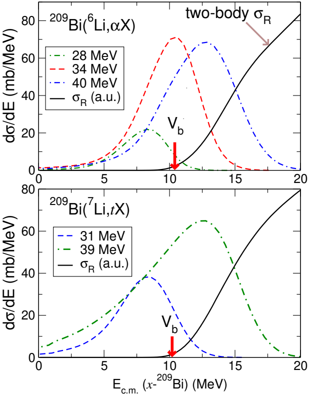


- CF well reproduced
- Suppression mostly due to NEB (eg. ICF)
- Dominant channel reducing CF is ^{209}Bi(^{6,7}Li,α X)
- The EBU mechanism plays a minor role

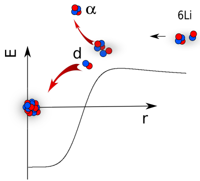
Data: Dasgupta et al., PRC 70, 024606 (2004)

J. Lei, AMM, PRL122, 042503 (2019)

In ^{209}Bi(^{6,7}Li,α)X near the barrier, the d/t+^{209}Bi are below their Coulomb barrier. What is the mechanism permitting the large absorption probability of these fragments? Trojan Horse effect.



- d/t + ^{209}Bi absorption enhanced in three-body reaction with respect to free two-body case.
- The ^{6,7}Li brings the light fragment (d/t) inside its Coulomb barrier Trojan Horse mechanism.



- This is the mechanism behind the TH method used in nuclear astrophysics.

Extraction of the ICF component of the NEB

- NEB includes $x - A$ ICF, but also other nonelastic processes such as target excitation.

- **Heuristic approach:** associate ICF with absorption inside the Coulomb barrier due to short-range imaginary potential $W_{xA}^{\rm fus}$:

$$\boxed{\frac{d\sigma^{\rm ICF}}{d\Omega_b dE_b} \approx -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x| W_{xA}^{\rm fus} |\varphi_x\rangle}$$

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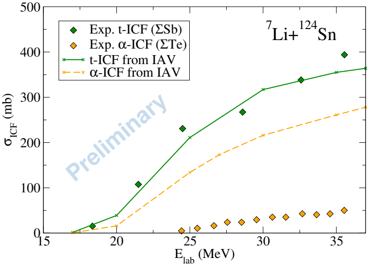
Caveats:

- Energy dependence of $W_{xA}^{\rm fus}$?
- Dependence of ICF on $W_{xA}^{\rm fus}$ parameters?
- Presence of surface absorption in W_{aA} results in unphysically small ICF cross section

See alternative approaches by:

- ⇨ Parkar, Jha, Kailas, PRC 94, 024609 (2016).
- ⇨ Rangel, Cortés, Lubian, Canto, PLB 803 (2020) 135337; Cortés et al ,PRC 102, 064628 (2021)

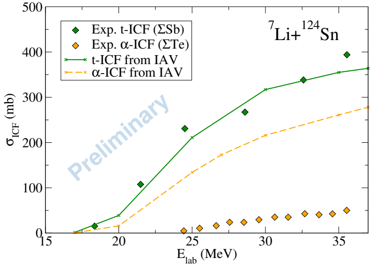
ICF in $^7Li+^{124}Sn$

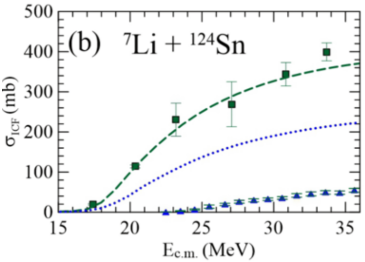


✔ t-ICF well reproduced

✖ ...but α-ICF largely overpredicted

Data: Parkar et al, PRC 97, 014607 (2018)





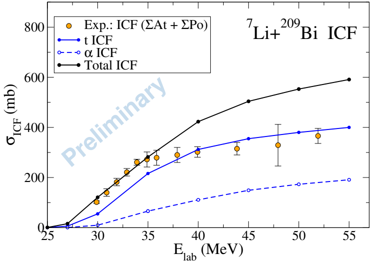
Cortés et al, PRC102, 064628 (2021)

✔ t-ICF well reproduced

✖ ...but α-ICF largely overpredicted

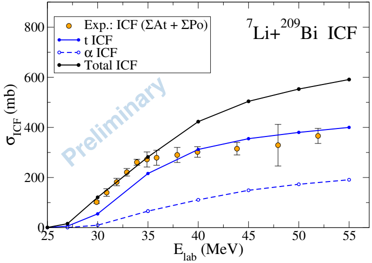
Data: Parkar et al, PRC 97, 014607 (2018)

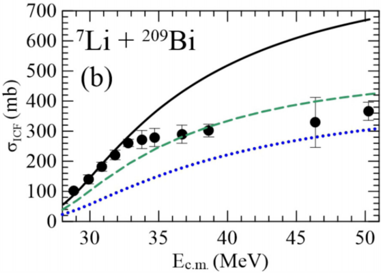
ICF in $^7Li+^{209}Bi$



- ✔ ICF well reproduced below $E_{\rm lab} < 36$ MeV.
- ✖ ...but overpredicted at higher incident energies

Data: Dasgupta et al, PRC 66, 041602(R) (2004); PRC 70, 024606 (2004)





Cortés et al, PRC102, 064628 (2021)

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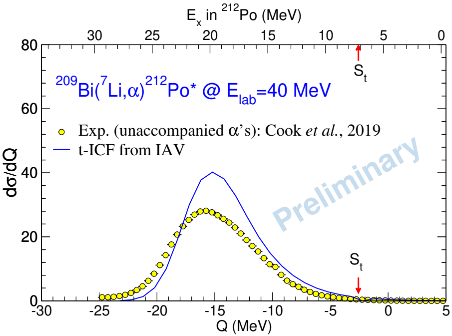
☞ Since the theory provides the differential cross section

$$\frac{d\sigma^{\rm ICF}}{d\Omega_b dE_b}$$

angular and energy distributions can also be obtained.



Data: Cook et al, PRL122, 102501 (2019)

ICF cross sections are also needed for a meaningful analysis of surrogate reactions aimed at indirect measurements of neutron induced reactions

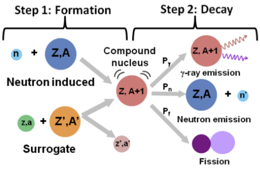
Goal:

- Determine the A(n,χ) cross section from A(d,pχ) cross section

(figure by B. Jurado)

(aimed) $$\sigma_{(n,\chi)} = \sum_{J,\pi} \sigma^{\rm CN}_{n+A}(E,J,\pi) G^{\rm CN}_{\chi}(E,J,\pi)$$

(measured) $$P_{(d,p\chi)}(E) = \sum_{J,\pi} F^{\rm CN}_{(d,p)}(E,J,\pi) G^{\rm CN}_{\chi}(E,J,\pi)$$



Theory requirements:

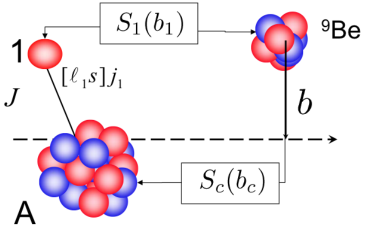
- A reaction model for the formation process: d+ A ⇒ p+ B*, encoded in the function $F^{\rm CN}_{(d,p)}(E,J,\pi)$
  - Angular/energy distribution of "p" fragments
  - Spin distribution of B* compound
  - Understanding of competing channels not leading to CN (e.g. deuteron breakup)
- A realistic n+A OMP for relevant range of energies

Ichimura-Austern-Vincent (IAV) model [PRC32, 431 ('85)]
(recently implemented by Lei et al, Potel et al, Carlson et al)

Dispersive optical model (Mahaux & Sartor, 1991)
(and modern implementations: Dickhoff, Charity, etc)

Application to intermediate energy knockout reactions

- Fast-moving projectile on a (typically) light target.
- One nucleon suddenly removed (absorbed) due to its interaction with the target.
- The remaining residue remain unchanged and is detected.
- The momentum of the core is related to that of the removed nucleon because in the rest frame of the projectile $\vec{P} = 0$



$$\vec{P} = \vec{p}_c + \vec{p}_1 = 0$$

The Eikonal HM formula

- Start from the Hussein-McVoy prescription for the x-channel WF:

$$\varphi_x^{\mathrm{HM}}(\vec{r}_x) = \langle \vec{r}_x \chi_b^{(-)} | \chi_a^{(+)} \phi_a \rangle$$

- Approximations:
  - $U_{aA} = U_{xA} + U_{bA}$
  - $\chi_a^{(+)}$ and $\chi_b^{(-)}$ treated in the Glauber (eikonal) limit

- Leads to the popular Eikonal Hussein-McVoy formula for NEB ("stripping"):

$$\sigma_{\mathrm{NEB}}^{\mathrm{EHM}} = \frac{2}{v_{\mathrm{a}}} (2\pi)^3 \frac{E_x}{\hbar k_x} \int \mathrm{d}^3\vec{r}_b \mathrm{d}^3\vec{r}_x \, |\phi_a(\vec{r}_{bx})|^2 \, |S_{bA}(b_b)|^2 \left[1 - |S_{xA}(b_x)|^2\right]$$

with:

  - $|S_{bA}(b_b)|^2$=probability of survival of the core.
  - $1 - |S_{xA}(b_x)|^2$=probability of absorption of the x particl .

- A similar formula can be obtained for the EBU part ("diffraction")

- Agreement theory vs experiment quantified with the reduction factor:

$$R_s = \frac{\sigma_{\text{theor}}}{\sigma_{\text{exp}}}$$

with

$$\sigma_{\text{theor}} = \sum_{n\ell j} S^a_{bx}(I;n\ell j)\sigma_{\text{sp}}(I;n\ell j) \qquad \sigma_{\text{sp}}(I;n\ell j) = \sigma^{\text{EBU}}_{\text{sp}} + \sigma^{\text{NEB}}_{\text{sp}}$$

☞ $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in σ_{theor}?

☞ R_s strongly dependent on $\Delta S = S_p - S_n$.

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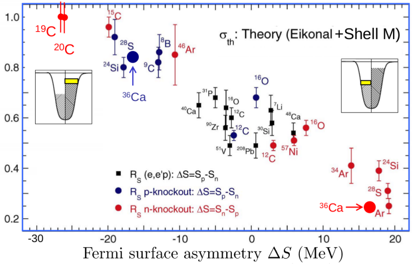
Measurements at the two Fermi surfaces



-Gade et al, PRC 77, 044306 (2008)
Tostevin, PRC90,057602(2014)

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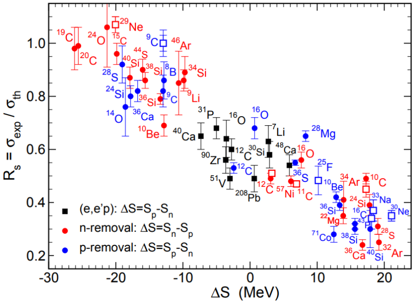
$$\sigma_{\rm sp}(I; n\ell j) = \sigma^{\rm EBU}_{\rm sp} + \sigma^{\rm NEB}_{\rm sp}$$



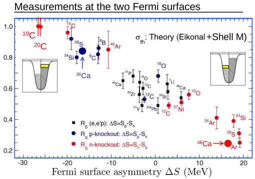
Tostevin and Gade, arXiv:2103.13133 (2021)

(Posted today!)

- $R_s < 1 \Rightarrow$ possible correlations (long-range, short-range, tensor,...) not included in $\sigma_{\rm theor}$?
- R_s strongly dependent on $\Delta S = S_p - S_n$.

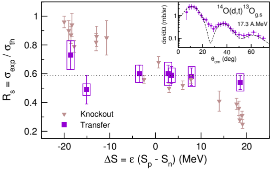
...however, this behaviour have not been corroborated by other probes, such as transfer or proton-induced knockout reactions (p, pN)

HI knockout (∼100 MeV/u)
Tostevin, PRC90,057602(2014)



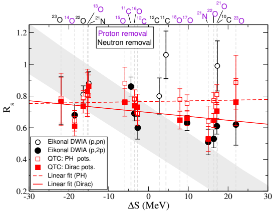
- Reaction model: eikonal + adiabatic
- R_s strongly dependent on $S_p - S_n$.

Low-energy transfer
Flavigny, PRL110, 122503(2013)



- Reaction model: ADWA, DWBA, CRC
- $R_s \sim$ constant.

(p, pN) @ 200-400 MeV/u
Aumann, PPNP118,103847(2021)



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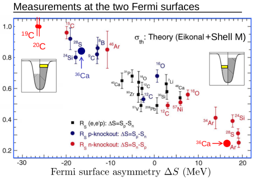

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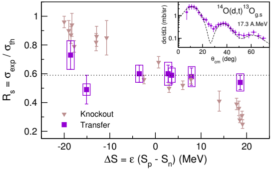

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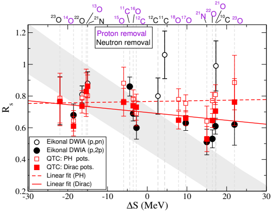

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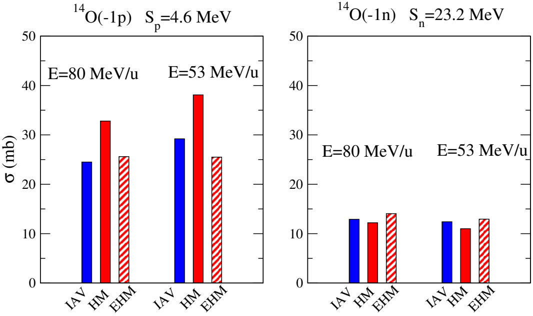
R_s from knockout disagree with those from transfer and (p, pN) ⇒ description of reaction mechanism?

Test case: ^{14}O(-1n) and ^{14}O(-1p) on ^{9}Be target with the same (energy-independent) potentials and structure model



- The Eikonal HM model (EHM) compares very well with the IAV result, even at relatively low incident energies ($\sim$50 MeV/u)
- The quantum-mechanical HM model (HM) works better for well bound nuclei, but deviates from IAV for weakly-bound nuclei.
- Other effects relevant for the comparison with data (e.g. energy dependence of OMPs) not considered here (see Flavigny, PRL 108, 252501 (2012), J. Lei and Bonaccorso, PLB813 (2021) 136032)

Why the EHM is apparently so accurate?

$$\boxed{\varphi_x^{\rm post} = G_x\langle \vec{r}_x\, \chi_b^{(-)} |\, V_{\rm prior} | \psi_{xb}^{3B(+)} \rangle + \langle \vec{r}_x \chi_b^{(-)} | \psi_{xb}^{3B(+)} \rangle}$$

with $G_x = (E_x^+ - K_x - U_x)^{-1}$

1. HM formula: $|\Psi^{\rm 3B(+)}\rangle \approx |\chi_a^{(+)}\phi_a\rangle$

$$\boxed{\varphi_x^{\rm post}(\vec{r}_x) = \varphi_x^{\rm prior}(\vec{r}_x) + \varphi_x^{\rm HM}(\vec{r}_x)}$$

2. Eikonal HM (EHM) formula: $U_a = U_{bA} + U_{xA} \Rightarrow V_{\rm prior} = 0$

$$\boxed{\varphi_x^{\rm post} = \langle \vec{r}_x \chi_b^{(-)} | \psi_{xb}^{3B(+)} \rangle}$$

- Inclusive breakup processes are a commonplace in nuclear reactions studies.
- Closed-form formulae, such as that by IAV, provides an accurate and efficient tool to compute inclusive breakup cross sections.
- NEB mechanisms (including ICF) provide a quantitative description of CF suppression of weakly-bound projectiles.
- ICF can be approximately isolated from the NEB cross section using short-range absorptive potentials
- Other promising applications in progress by several groups: knockout reactions, surrogate method,...

Special thanks to Jin Lei for the collaboration and numerous calculations

And to Mahir Hussein for his contribution to the subject and stimulating discussions