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Theory of complete and incomplete fusion for weakly bound systems

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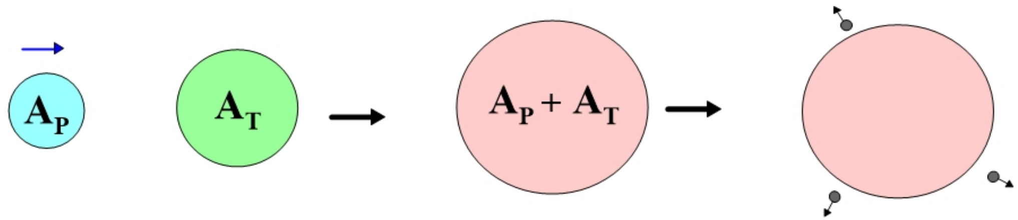
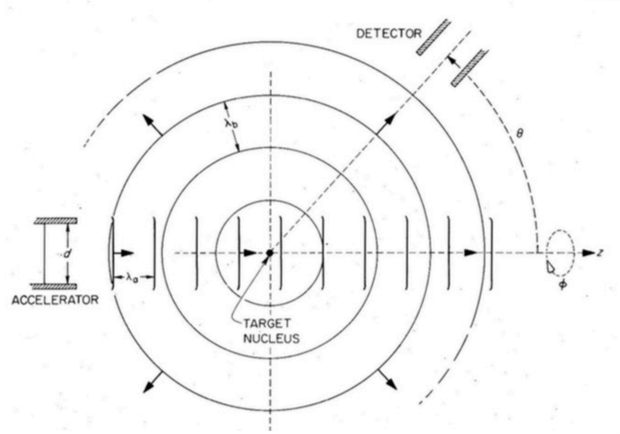
Fusion of tightly bound nuclei





Theoretical estimation

- Potential Scattering approach
- Fusion is not a channel

$$\sigma_F(E) = \frac{\pi}{K^2} \sum_{l=0} (2l+1) \; P_l^F(E)$$

$$P_l^F(E) = 1 - |S_l(E)|^2$$

- Alternative version (from continuity equation)

$$V(R) = U(R) - iW^F(R)$$

$$\Rightarrow \sigma_F = \frac{k}{E} \langle \psi | W^F | \psi \rangle$$

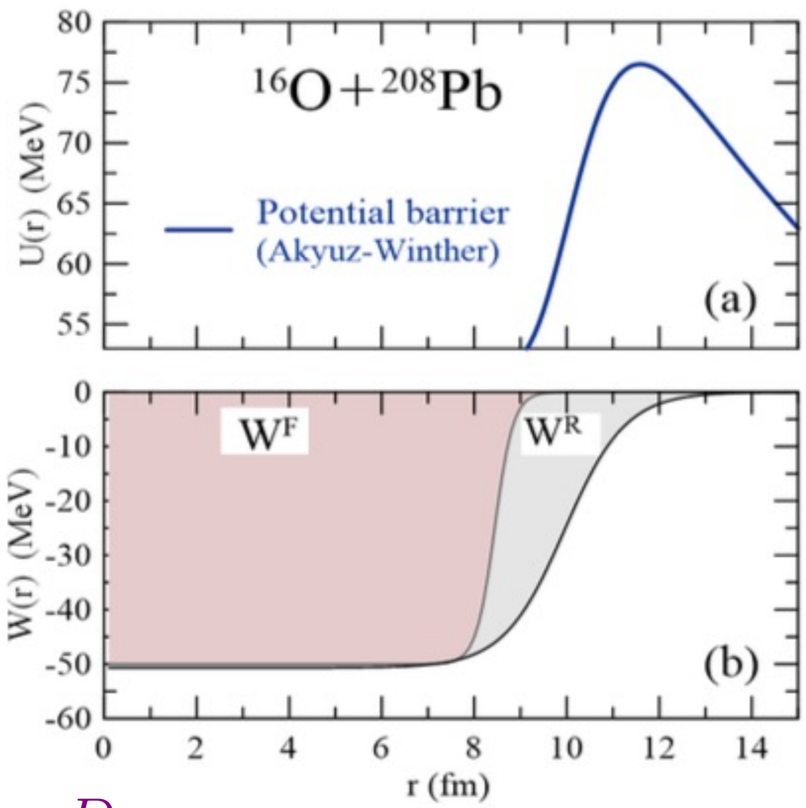
Theoretical estimation (cont.)

- Potential Scattering approach

It will only work when there is no relevant couplings at all !!!!!!!

Can the imaginary potential be divided into two parts when there are relevant couplings?

NO !!!!!!!



$$V(R) = U(R) - iW^F(R) - iW^D(R)$$

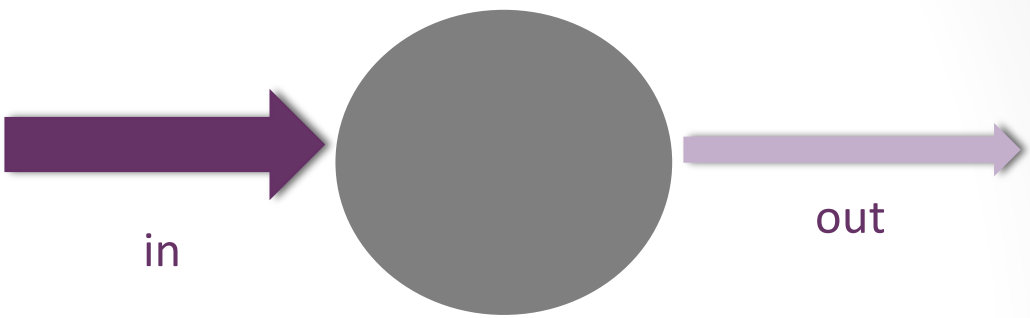
$$\Rightarrow \quad \sigma_{\mathrm{F}} = \frac{k}{E}\langle\psi|W^F|\psi\rangle$$

Theoretical estimation

- Potential Scattering approach (almost never works)

Alternative?

- Coupled Channels (inelastic, rearrangement)



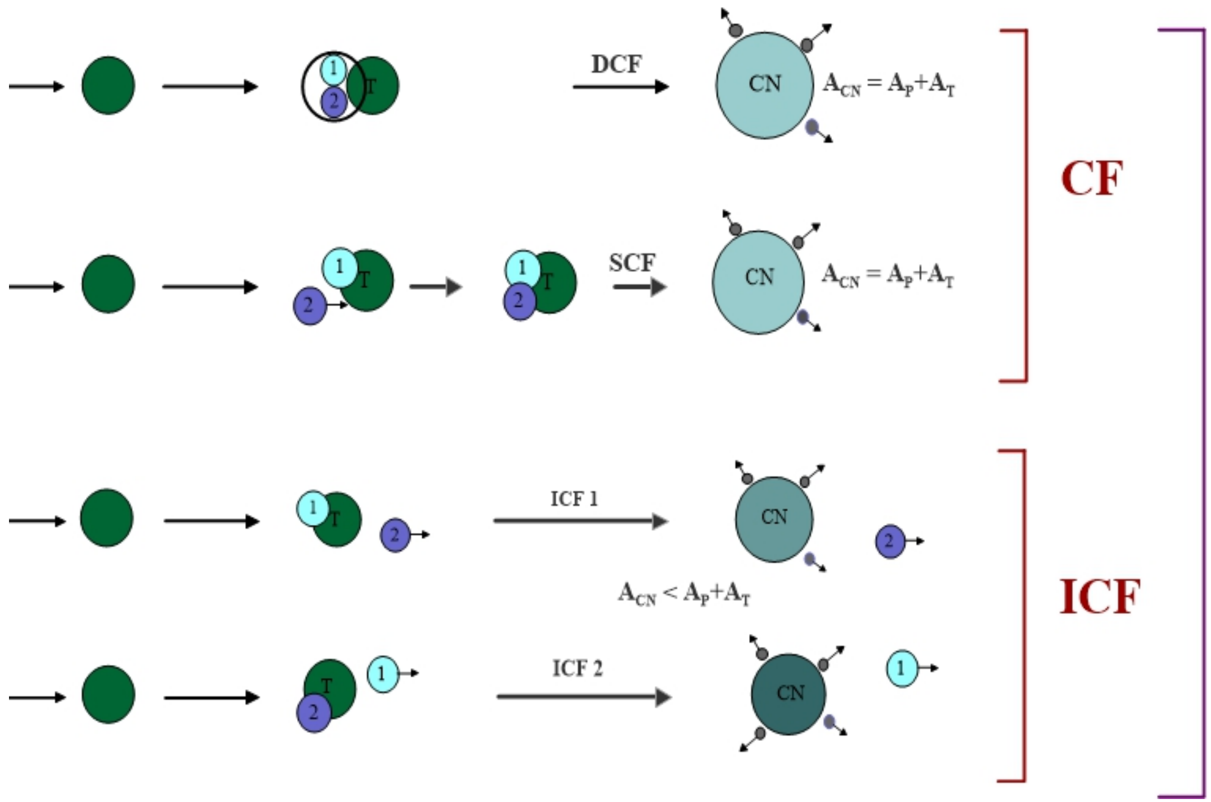
$$\Big[E_\alpha - H_\alpha\Big]\psi_\alpha^{(+)} = \sum_{\alpha' \neq \alpha} V_{\alpha\alpha'}\ \psi_{\alpha'}^{(+)}, \quad \alpha, \alpha' = 1, N$$

$$\Rightarrow \quad \sigma_{\rm F} = \frac{K}{E} \sum_{\alpha\alpha'=1} \langle \psi_\alpha | W_{\alpha\alpha'} | \psi_{\alpha'} \rangle$$

$$\sigma_{\rm R} = \frac{\pi}{k^2} \sum_l (2l+1)\ \Big[1 - |S_{0l}(E)|^2\Big]$$

$$\Rightarrow \quad \sigma_{\rm F} = \sigma_{\rm R} - \sum_{\alpha \neq 0} \sigma_\alpha$$

Collisions of weakly nuclei (different fusion processes)



$$\sigma_{\rm CF} = \sigma_{\rm DCF} + \sigma_{\rm SCF}$$

$$\sigma_{\rm TF} = \sigma_{\rm CF} + \sigma_{\rm ICF}$$

$$\sigma_{\rm ICF} = \sigma_{\rm ICF_1} + \sigma_{\rm ICF_2}$$

Other processes: elastic scattering, quasi-elastic scattering, transfer reactions, quasi-fission, deep inelastic, fission, break-up triggered by transfer .

Procedures used to answer: "Enhancement or suppression in relation to what?

a) **Comparison of data with theoretical predictions.**
b) **Comparison of data for weakly and tightly bound systems.**

1. Experiment vs. theory

$\Delta\sigma_F \equiv \sigma_F^{exp} - \sigma_F^{theo} \Rightarrow$ 'ingredients' missing in the theory

Theoretical possibilities:

a) Single channel - standard densities

$\Delta\sigma_F$ arises from all static and dynamic effects

b) Single channel - realistic densities

$\Delta\sigma_F$ arises from couplings to all channels

c) CC calculation with all relevant bound channels

$\Delta\sigma_F$ arises from continuum couplings

d) CDCC

no deviation expected

2. Compare with σ_F of a similar tightly bound system

Differences due to static effects:

1. Gross dependence on size and charge:

   Z_P, Z_T, A_P, A_T – affects V_B and R_B

   $V_B \sim Z_P Z_T e^2 / R_B; \quad \sigma_{geo} \sim \pi R_B^2, \quad R_B \propto (A_P^{1/3} + A_T^{1/3})$

2. Different barrier parameters due to diffuse densities

   (lower and thicker barriers)

Fusion data reduction required !

Fusion functions $F(x)$ (our reduction method)

$$E \rightarrow x = \frac{E - V_B}{\hbar\omega} \quad \text{and} \quad \sigma_F^{\exp} \rightarrow F_{\exp}(x) = \frac{2E}{\hbar\omega R_B^2}\,\sigma_F^{\exp}$$

Inspired in Wong's approximation

$$\sigma_F^W = R_B^2\,\frac{\hbar\omega}{2E}\,\ln\left[1 + \exp\left(\frac{2\pi\left(E - V_B\right)}{\hbar\omega}\right)\right]$$

If $\sigma_F^{\exp} = \sigma_F^{W}$ $\Rightarrow$ $F(x) = F_0(x) = \ln\left[1 + \exp\left(2\pi x\right)\right]$

$\boldsymbol{F_0(x)}$ **= Universal Fusion Function (UFF)**

system independent !

Direct use of the reduction method

Compare $F_{\exp}(x)$ with UFF for x values where $\sigma_{\mathrm{F}}^{\mathrm{opt}} = \sigma_{\mathrm{F}}^{\mathrm{W}}$

Deviations are due to couplings with bound channels and breakup

Refining the method

Eliminate the failure of the Wong model for light systems at sub-barrier energies

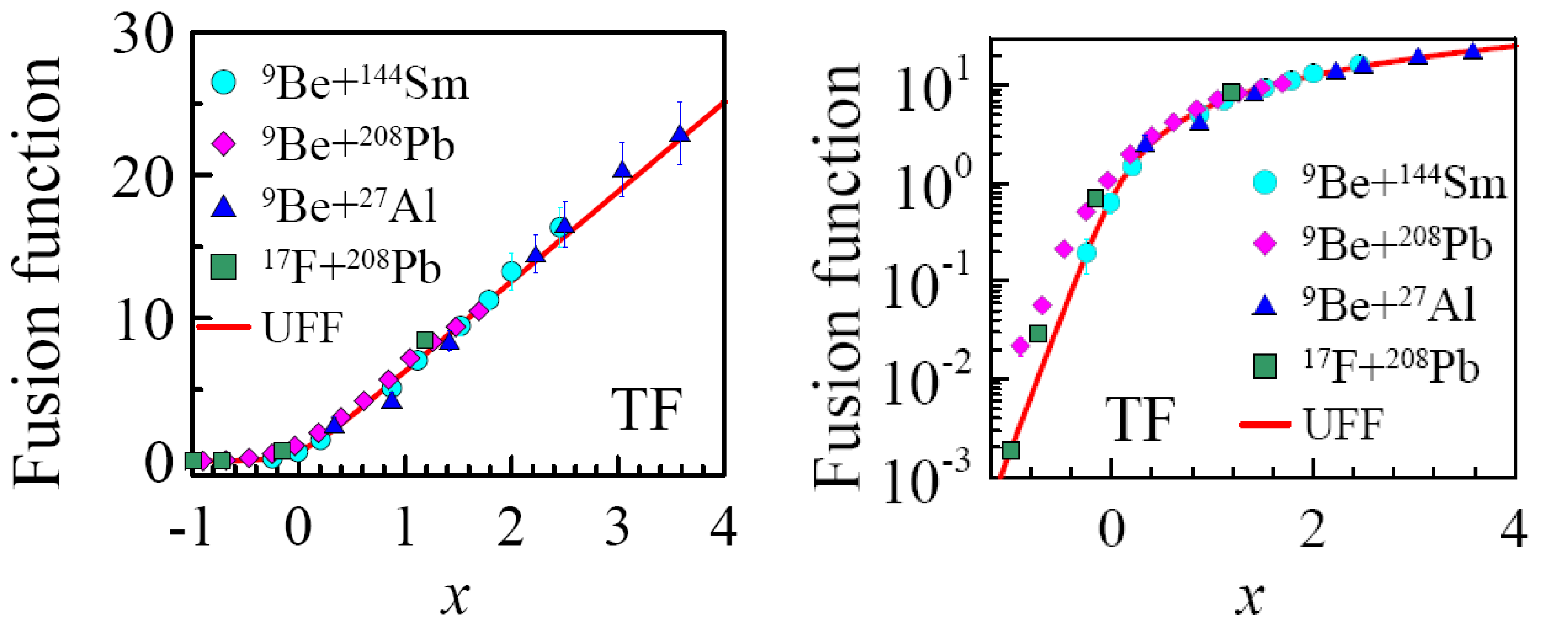
Eliminate influence of couplings with bound channels

Renormalized fusion function

$$F_{\exp}(x) \rightarrow \bar{F}_{\exp}(x) = \frac{F_{\exp}(x)}{R(x)}, \quad \text{with} \;\; R(x) = \frac{\sigma_{\mathrm{F}}^{\mathrm{CC}}}{\sigma_{\mathrm{F}}^{W}} = \frac{\sigma_{\mathrm{F}}^{\mathrm{CC}}}{\sigma_{\mathrm{F}}^{\mathrm{opt}}}$$

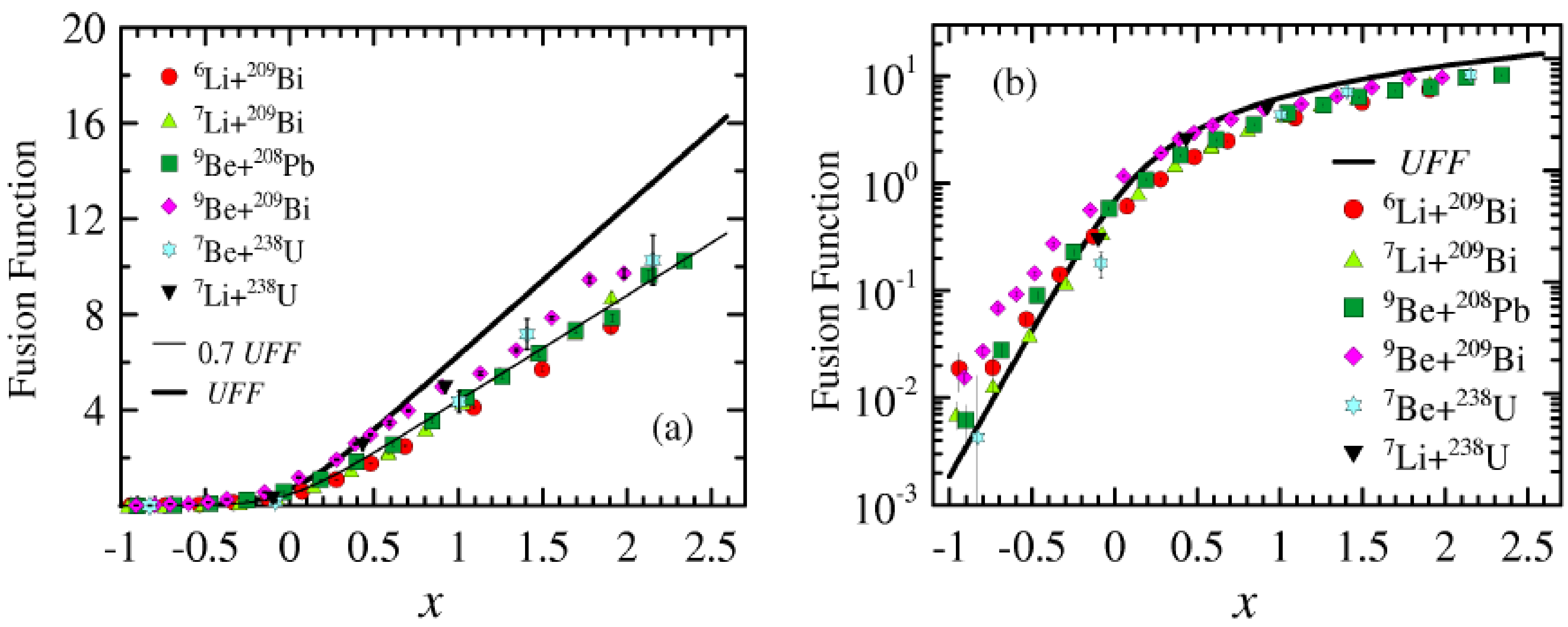
If CC calculation describes data $\rightarrow \bar{F}_{\exp} = \mathrm{UFF}$

Use of UFF for investigating the role of BU dynamical effects on the total fusion of heavy weakly bound systems



No effect above the barrier- enhancement below the barrier

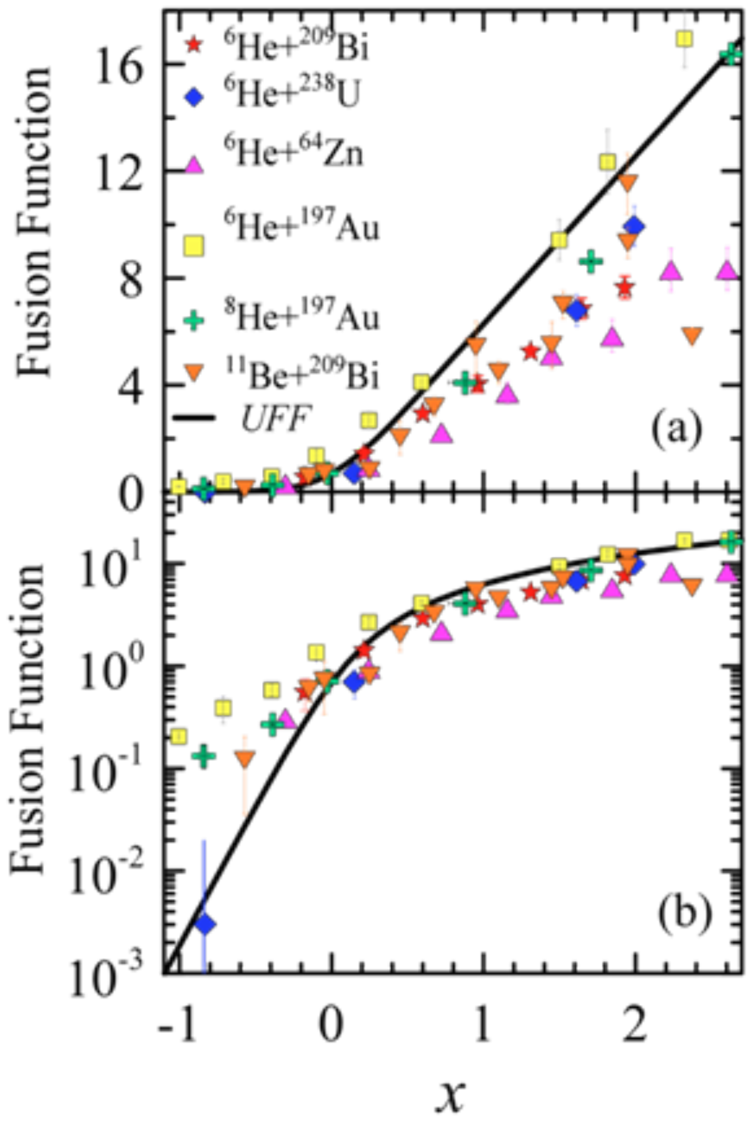
Use of UFF for investigating the role of BU dynamical effects on the complete fusion of stable weakly bound heavy systems



We did not include any resonance of the projectiles in CC calc.

Suppression above the barrier- enhancement below the barrier

Fusion of neutron halo ^{6,8}He, ^{11}Be



Conclusion from the systematic (several systems): CF enhancement at sub-barrier energies and suppression above the barrier, when compared with what it should be without any dynamical effect due to breakup and transfer channels.

How to measure and calculate CF, and ICF?

Finding CF and ICF cross section is a great challenge (both for experimentalists and theorists)

Experiment:

- σ_{CF} absorption of all projectile charge (^{11}Be = ^{10}Be +n)
- Most experiments determine only σ_{TF}
- Individual σ_{CF} and/or σ_{ICF} have been measured for some particular stable P-T combinations:

Some examples: ^{6}Li: B = 1.47 MeV $(\alpha + d)$

^{7}Li: B = 2.45 MeV $(\alpha + t)$

^{9}Be: B = 1.65 MeV $(^{8}\mathrm{Be} + n)$

$^{6,7}\mathrm{Li} + {}^{209}\mathrm{Bi}$

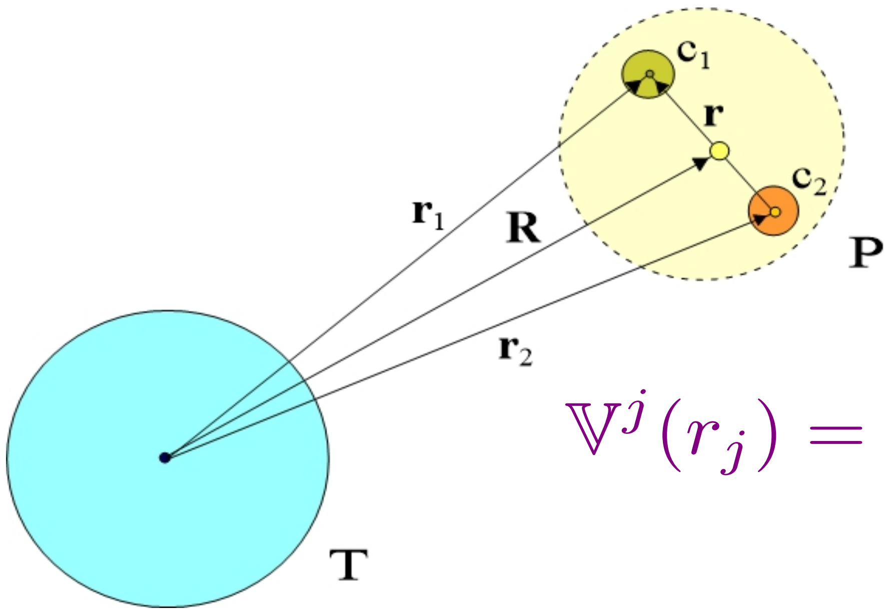
$^{159}\mathrm{Tb}$

$^{197}\mathrm{Au}$

$^{9}\mathrm{Be} + {}^{208}\mathrm{Pb}$

Theory (quantum mechanic):

Projectiles of two-fragment



$$\mathbb{V}(\mathbf{r},\mathbf{R}) = \mathbb{V}^1(r_1) + \mathbb{V}^2(r_2)$$

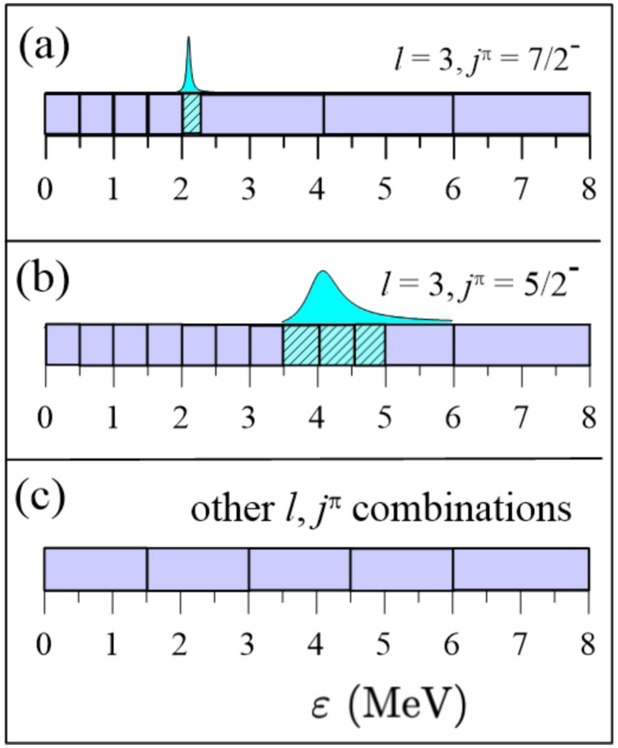
$$\mathbb{V}^j(r_j) = \mathbb{U}^j(r_j) - i\mathbb{W}^j(r_j), \quad j = 1,2$$

Difficulty:

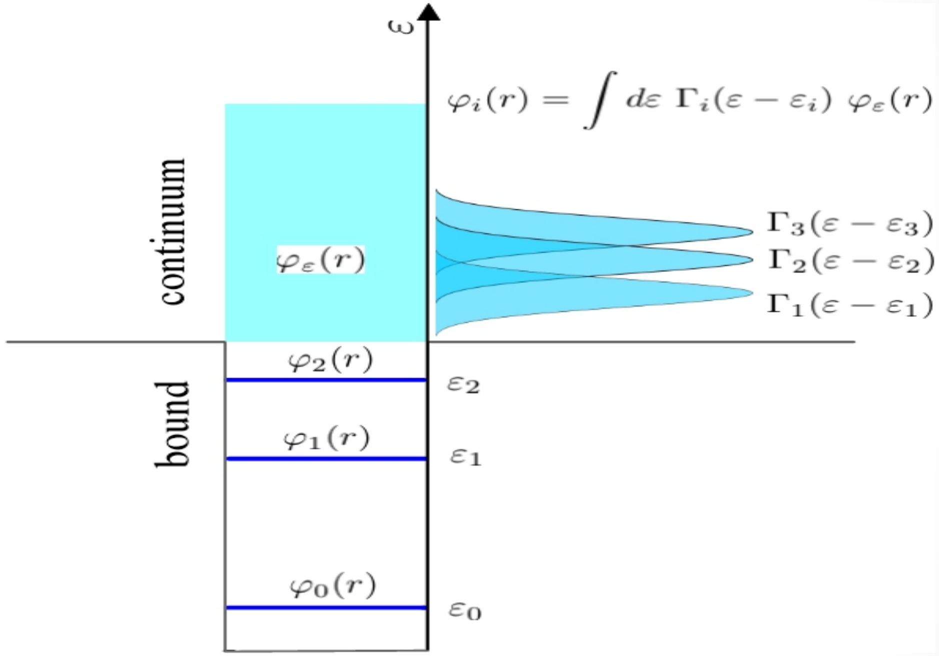
Continuous energy label $\Longrightarrow$ **Infinite set of equations (even with truncation)**

Solution: discretize the continuum

CDCC method with bins $\{\varphi_\varepsilon\} \Longrightarrow \{\varphi_i\}$



Bins adopted for ^{7}Li



Reduces to a standard CC problem,
(finite number of coupled equations)

- Project angular momentum
- Solve CC equations, get S-matrices and radial w.f.

Calculation of fusion cross sections

- Indirect calculation:

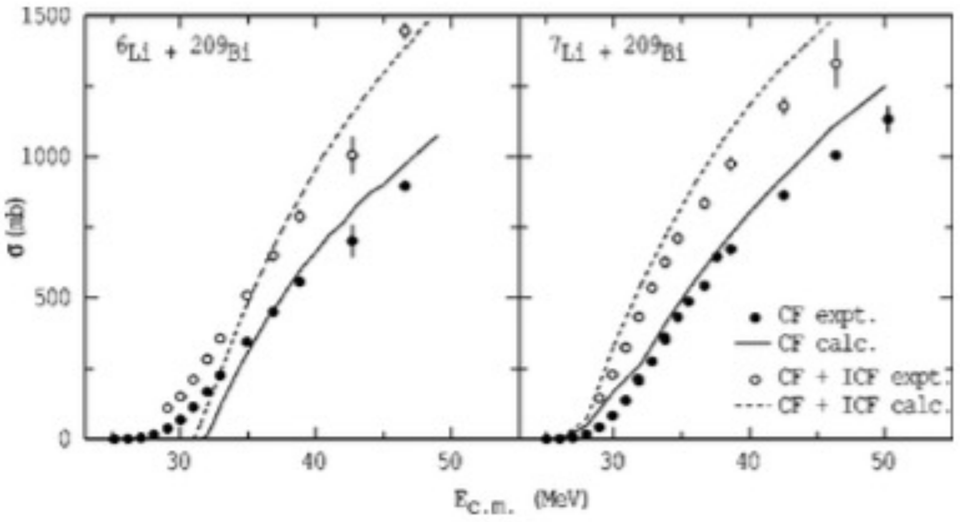
$$\sigma_{\mathrm{R}} = \frac{\pi}{k^2} \sum_l (2l+1) \left[1 - |S_{0l}(E)|^2\right] \Longrightarrow \sigma_{\mathrm{F}} = \sigma_{\mathrm{R}} - \sum_{\alpha \neq 0} \sigma_\alpha$$

- Direct calculation using radial wave functions

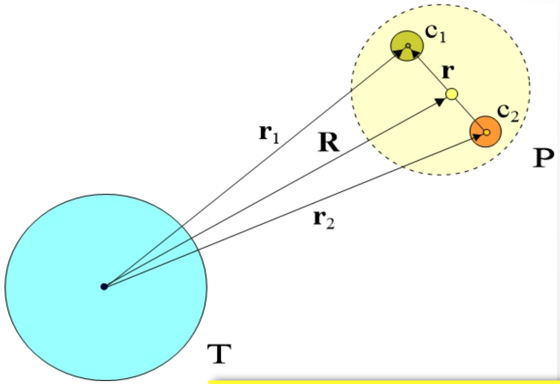
$$\sigma_{\mathrm{F}} = \frac{k}{E} \sum_{\alpha\alpha'=1} \langle \psi_\alpha | W^1_{\alpha,\alpha'} + W^2_{\alpha,\alpha'} | \psi_{\alpha'} \rangle$$

Fusion Estimations: classical picture

Hagino et al., NPA **238**, 475 (2004), Dasgupta et al., PRC **66**, 041602 (2002),

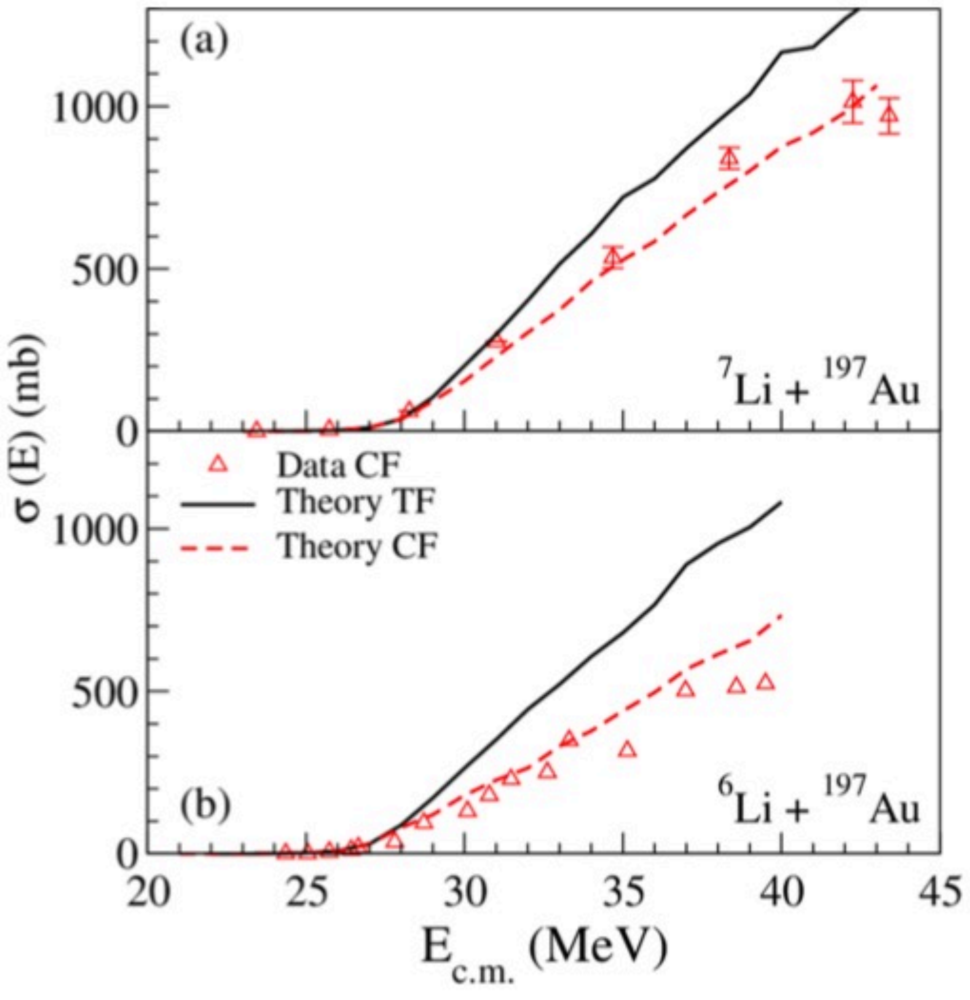


- Classical picture with stochastic parameters.

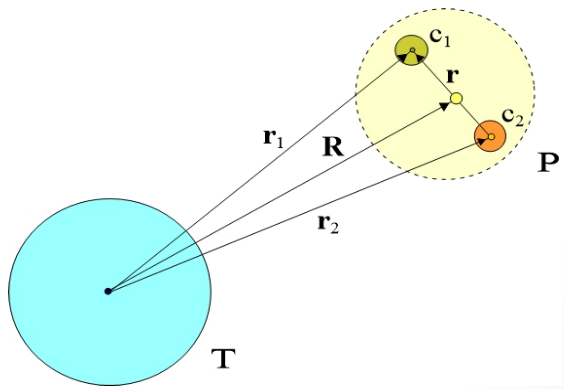


Fusion Estimations: semi-classical models

* Marta et al., PRC **89**, 034625 (2014), Kolinger et al., PRC **98**, 044604 (2018)



- Classical trajectory
- Intrinsic dynamic: time dependent Schrodinger equation
- Fusion: tunnelling trough the barrier

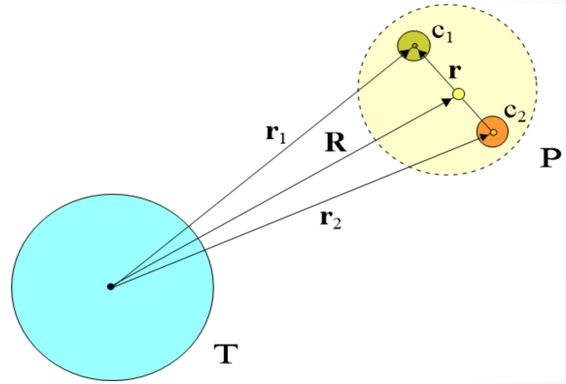


The method of Hagino, Vitturi, Dasso and Lenzi (HVDL)

Hagino et al., PRC **61**, 037602 (2000)
A. Diaz-Torres and I. J. Thompson, PRC **65**, 024606 (2002).

- **P-T imaginary potential** (instead of $W^{(1)} + W^{(2)}$)

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2) \rightarrow W(R) = W_\alpha \delta_{\alpha,\alpha'}$$



Then,

$$\sigma_{\rm TF} = \frac{k}{E} \sum_{\alpha=1}^{N} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle = \sum_{\alpha=1}^{N} \sigma_\alpha$$

Or,

$$\sigma_{\rm TF} = \sigma_{\rm B} + \sigma_{\rm C}$$

$$With\ \sigma_{\rm B} = \frac{k}{E} \sum_{\alpha\ \epsilon\ bound} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle$$

Contributions from
from bound channels

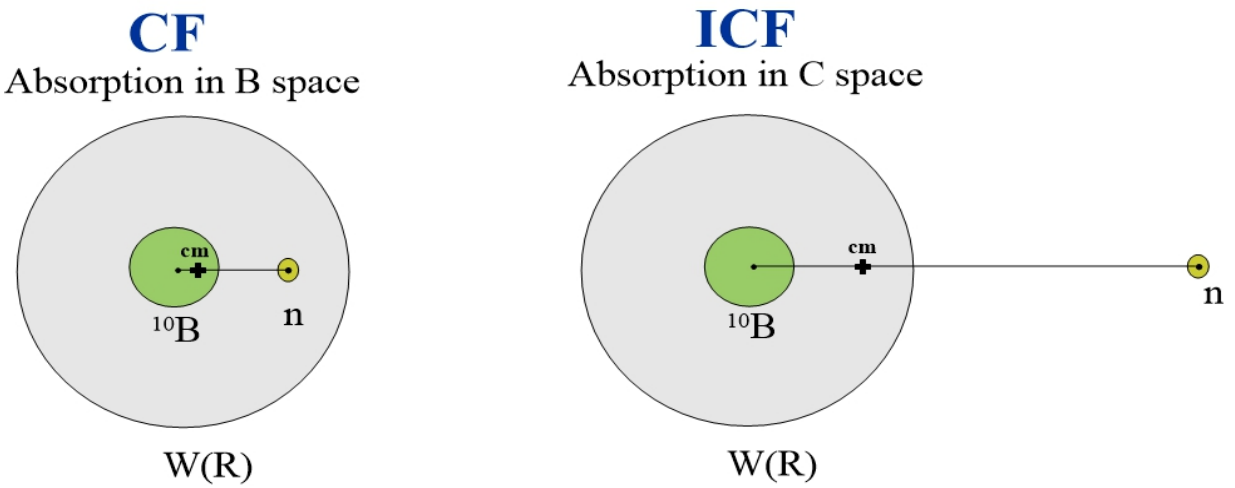
$$And\ \sigma_{\rm C} = \frac{k}{E} \sum_{\alpha\ \epsilon\ cont.} \langle \psi_\alpha \,|\, W_\alpha \,|\, \psi_\alpha \rangle$$

From continuum channels (*bins*)

Basic Assumption: $\sigma_{CF} = \sigma_B \;, \sigma_{ICF} = \sigma_C$

Limitation: works for a fragment much heavier than the other

$^{11}Be\ (^{10}Be - n) + {}^{208}Pb$

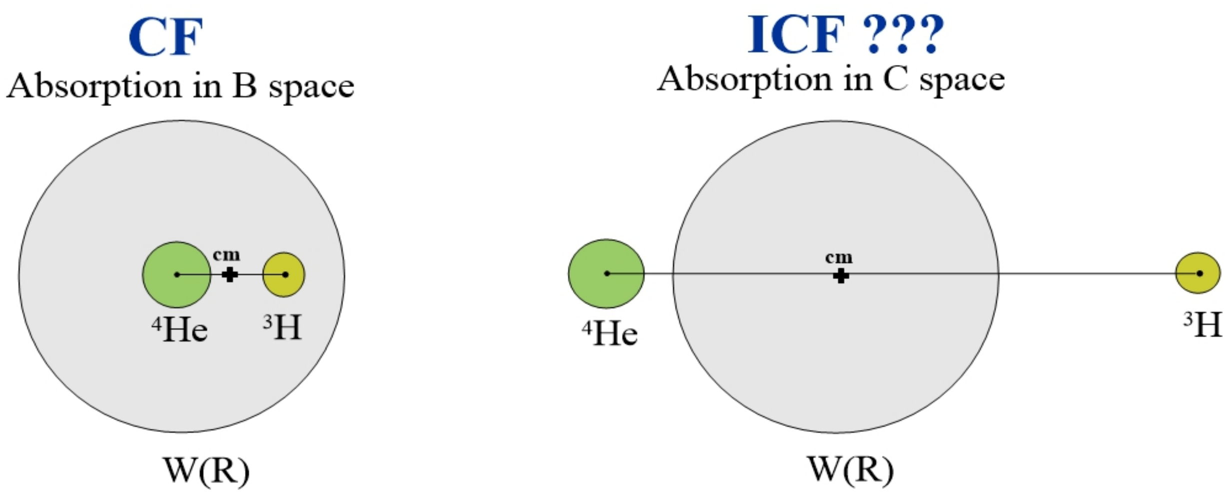


Works fine !

Basic Assumption: $\sigma_{\rm CF} = \sigma_{\rm B}$, $\sigma_{\rm ICF} = \sigma_{\rm C}$

Limitation: works for a fragment much heavier than the other

^{7}Li (^{4}He - ^{3}H) + ^{209}Bi



Does not work !

Indirect determination of CF using the spectator model*

* Lei and Moro, PRL 122, 042503 (2019)

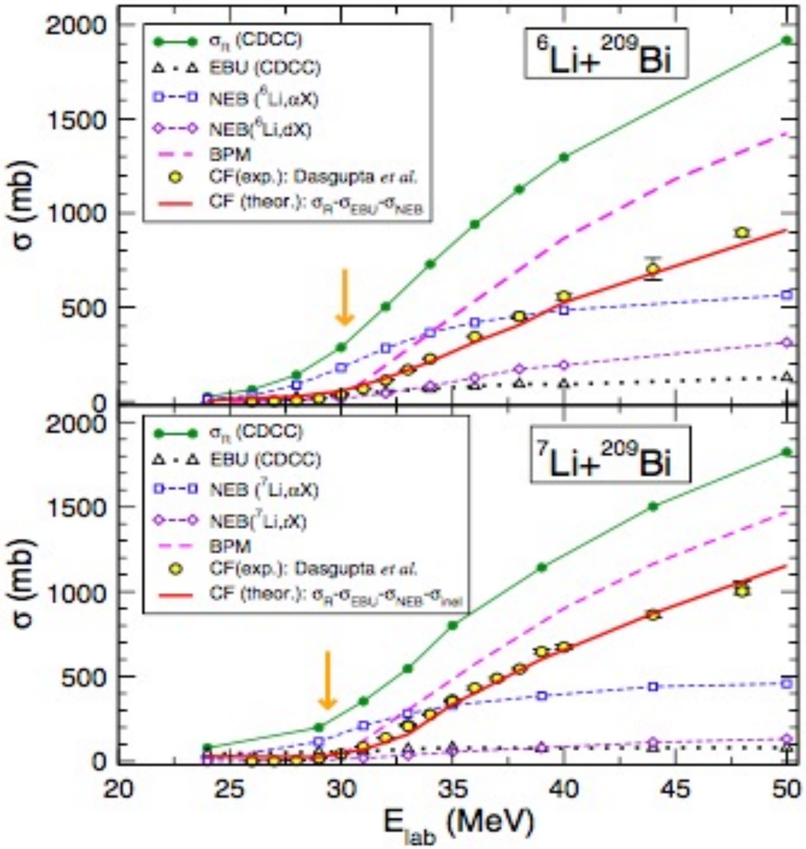
Extract σ_{CF} from the relation:

$$\sigma_{\rm R} = \sigma_{\rm CF} + \sigma_{\rm inel} + \sigma_{\rm EBU} + \sigma_{\rm NBU}^{(1)} + \sigma_{\rm NBU}^{(2)}$$

- σ_{R} : from CDCC calculation or opt. model analysis
- σ_{inel} :from standard CC calculation (only bound channels)
- σ_{EBU} : from CDCC calculation:
- σ_{NEB1} ,σ_{NEB2} : from inclusive spectator- participant model (IAV)

Quantum mechanical methods

1. **Indirect determination of CF using the spectator model***



Nice model, … but cannot evaluate ICF

* Lei and Moro, PRL 122, 042503 (2019)

Other methods found in the literature

- S. Hashimoto et al., Prog. Theor. Phys. **122**, 1291 (2009): Radial integrals of the imaginary potentials with CDCC w.f.s over the coordinates of the fragments, r1 and r2. They picked contribution from proper regions to determine individual cross sections for each fusion process. ICF the neutron and the proton in the d + ^{7}Li collision.

- M. Boseli and Diaz-Torres, JPG 41 (2014) 094001, PRC 92 (2015) 044610: Used position projection operators to describe the time-evolution of wave packets. Used to estimate CF and ICF cross sections for the ^{6}Li +^{209}Bi system. The method is promising but so far it has not been used in realistic calculations involving weakly bound projectiles.

- V.V. Parkar et al., PRC 94, 024606 (2016): Performed separate CDCC calculations with short-range W to determine CF, ICF, TF (no self-consistent) ^{6,7}Li +^{209}Bi,^{198}Pt

A new QM method to evaluate CF and ICF*

(Based on the HVDL method, but with abs. of each fragment)

Instead of absorption of the cm of the projectile:

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2) \rightarrow W(R) = W_\alpha \delta_{\alpha,\alpha'}$$

Individual absorption of each fragment:

$$W(\mathbf{R},\mathbf{r}) = W^1(r_1) + W^2(r_2), \quad W_{\alpha,\alpha'} \neq W_\alpha$$

Assumption:

W^i does not connect spaces B and C

$$\mathbb{W}^{(i)}(r_i) = \frac{W_0}{1 + \exp[(r_i - R_w)/a_w]}, \quad i = 1, 2, \qquad (44)$$

with the following parameters:

$$W_0 = 50 \text{ MeV}, \quad R_w = 1.0\left[A_i^{1/3} + A_T^{1/3}\right] \text{ fm}; \quad a_w = 0.2 \text{ fm}.$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

Contribution from the B-space: $\sigma_{\mathrm{B}} = \sigma_{\mathrm{DCF}}$

(as in the HVDL method)

$$\sigma_{\mathrm{B}} = \frac{k}{E} \sum_{\alpha \,\epsilon\, bound} \left\langle \psi_\alpha \middle| W^1(r_1) + W^2(r_2) \middle| \psi_\alpha \right\rangle$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

Contribution from channels in the continuum to TF

(here is the difference to HVDL)

$$\sigma_{\rm C} = \frac{k}{E} \sum_{\alpha\alpha' \,\epsilon\, C} \left[\left\langle \psi_\alpha \,\middle|\, W^1_{\alpha,\alpha'}(r_1) \middle|\, \psi'_\alpha \right\rangle + \left\langle \psi_\alpha \,\middle| W^2_{\alpha,\alpha'}(r_2) \middle|\, \psi_{\alpha'} \right\rangle \right]$$

Performing ang. mom. projection and the summing over α and α' (in C),

$$\sigma_{\rm C} = \frac{\pi}{k^2} \sum_J (2J+1) \left[P^1(J) + P^2(J) \right]$$

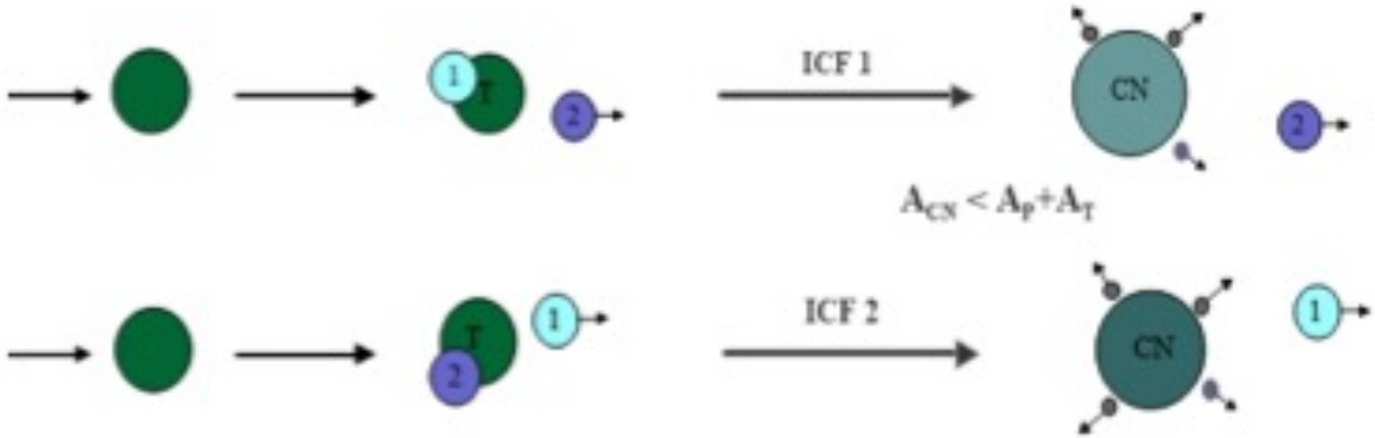
$P^{(i)}(J)$ = abs. probability of fragment *i* in the C-space

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

ICF (ICF1, ICF2), SCF cross sections

$$\sigma_{\mathrm{ICF1}} = \frac{\pi}{k^2}\sum_J (2J+1)P^1(J)\left[1 - P^2(J)\right]$$

$$\sigma_{\mathrm{ICF2}} = \frac{\pi}{k^2}\sum_J (2J+1)P^2(J)\left[1 - P^1(J)\right]$$

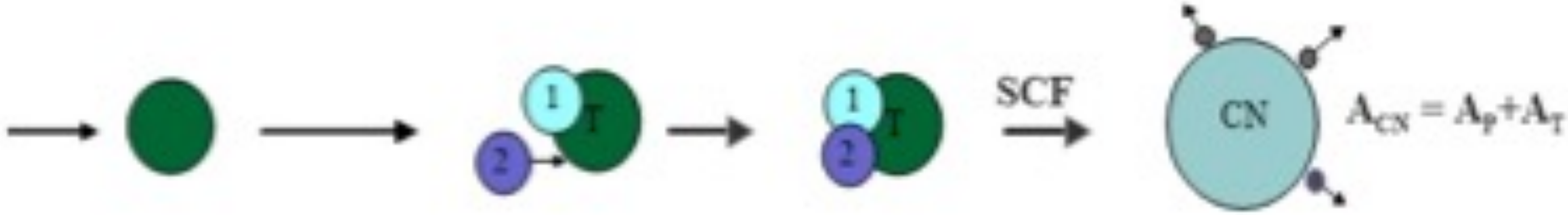


$$\sigma_{\mathrm{ICF}} = \sigma_{\mathrm{ICF1}} + \sigma_{\mathrm{ICF2}}$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

ICF (ICF1, ICF2), SCF cross sections

$$\sigma_{\mathrm{SCF}} = \sigma_{\mathrm{C}} - \sigma_{\mathrm{ICF}} = \frac{\pi}{k^2} \sum_J \left(2P^1(J) \times P^2(J)\right)$$



$$\sigma_{\mathrm{CF}} = \sigma_{\mathrm{DCF}} + \sigma_{\mathrm{SCF}}$$

$$\sigma_{\mathrm{TF}} = \sigma_{\mathrm{CF}} + \sigma_{\mathrm{ICF}}$$

* J. Rangel, M. Cortes, J. Lubian, LFC (Phys. Let. B, 803- 2020)

Application: **Fusion cross sections in ^{7}Li + ^{209}Bi**

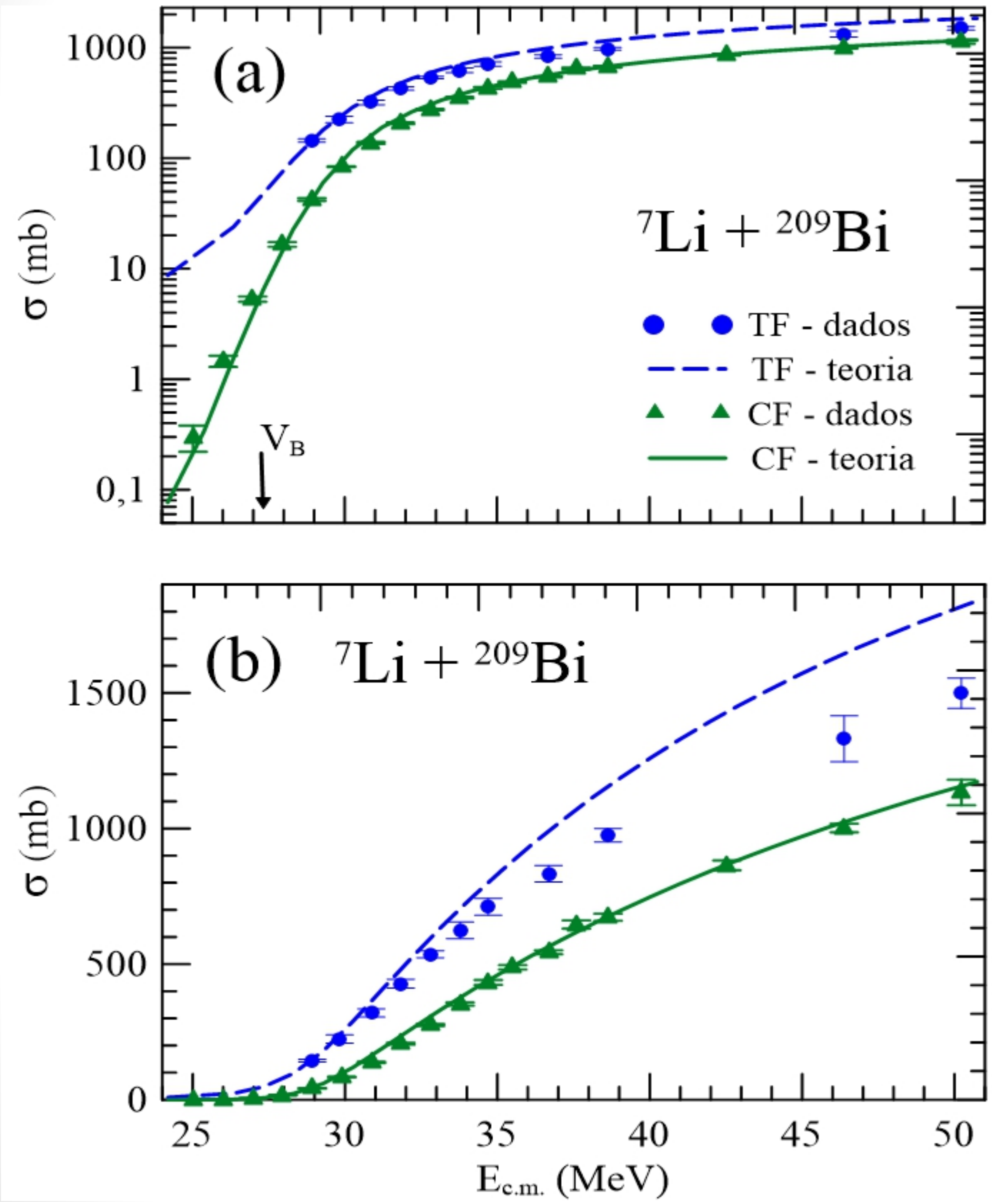
^{4}He + t BE= 2.47 MeV

Procedure:

- Perform CDCC calculations running FRESCO, with options to export intrinsic and radial w.f.
- !!! We need radial w.f. converged inside V_B too. Hard task!!!
- Use them in the the angular momentum projected expressions for the cross sections (Code CF-ICF, unpublished)

- J. Rangel, M.R. Cortes, J. Lubian, L.F.Canto (Phys. Let. B, 803- 2020)
- M.R Cortes, J. Rangel. J.L. Ferreira, J. Lubian., L.F Canto (PRC 102, 06428 (2020)

$^{7}Li + ^{209}Bi$ fusion – theory vs. experiment*

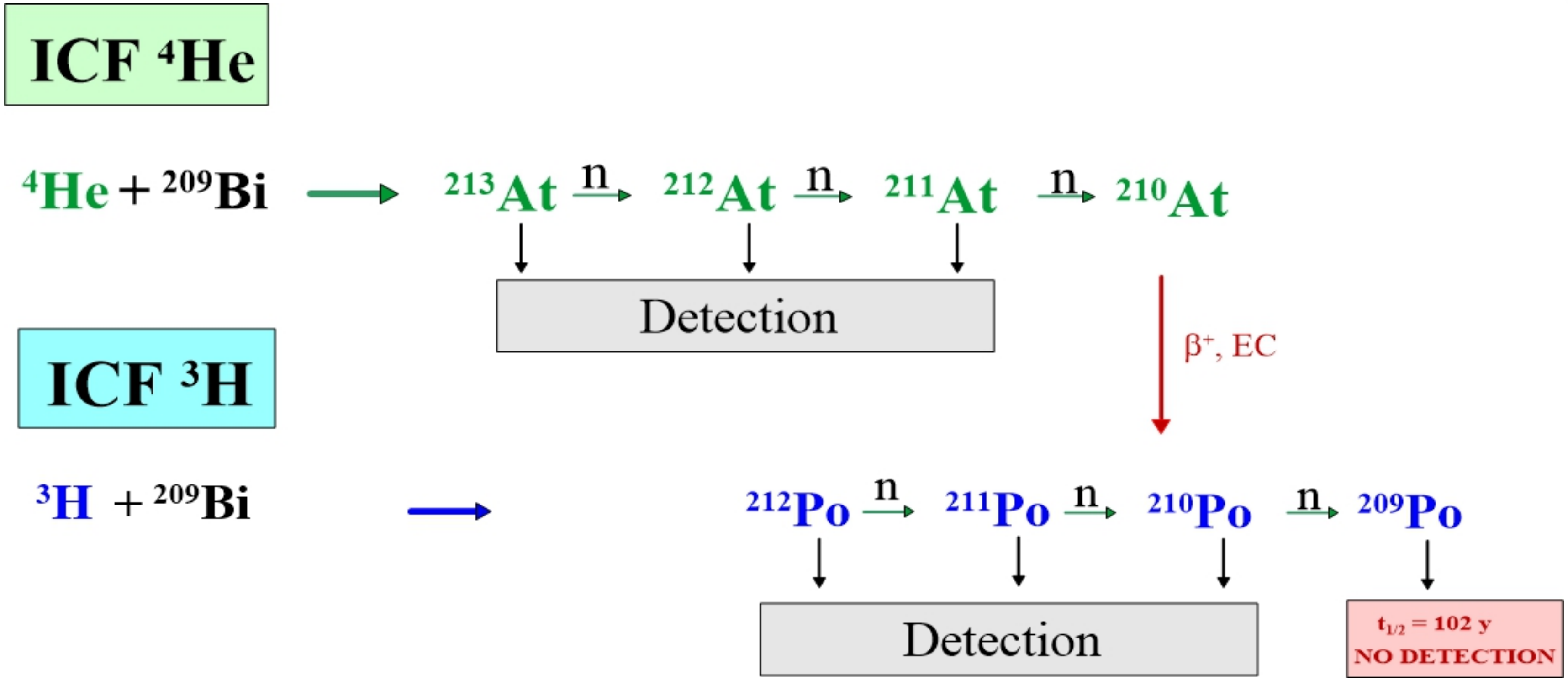


- TF and CF predictions are in excellent agreement with data below V_B up to E ~ 36 MeV. CF is well described in the whole energy inertval

- Predictions for TF above ~ 36 MeV overestimate the experiment. But… the 4 data points with highest energies are only lower bounds (according to the authors of the experiment)

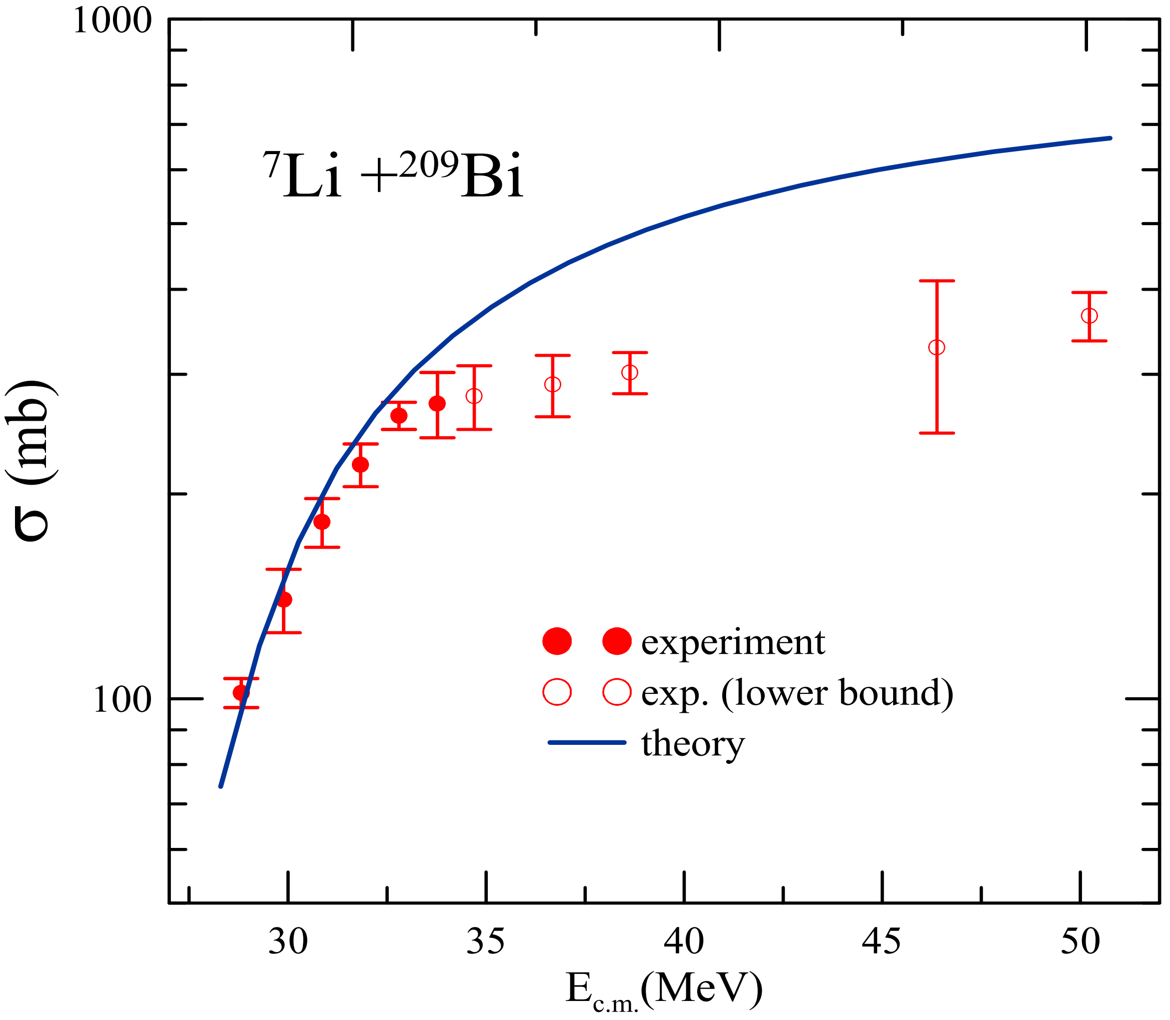
* Dasgupta *et al.*, PRC **66**, 041602(R) (2002); PRC **70**, 024606 (2004)

Decay schemes of nuclei produced by ICF



- ^{209}Po is the decay chain of both ICF processes
- Contribution from ^{209}Po is not detected
- Estimates with PACE: ^{209}Po is important above 36 MeV.
- **Above $E_{c.m.}$ ~ 36 MeV, data is only a lower bound**

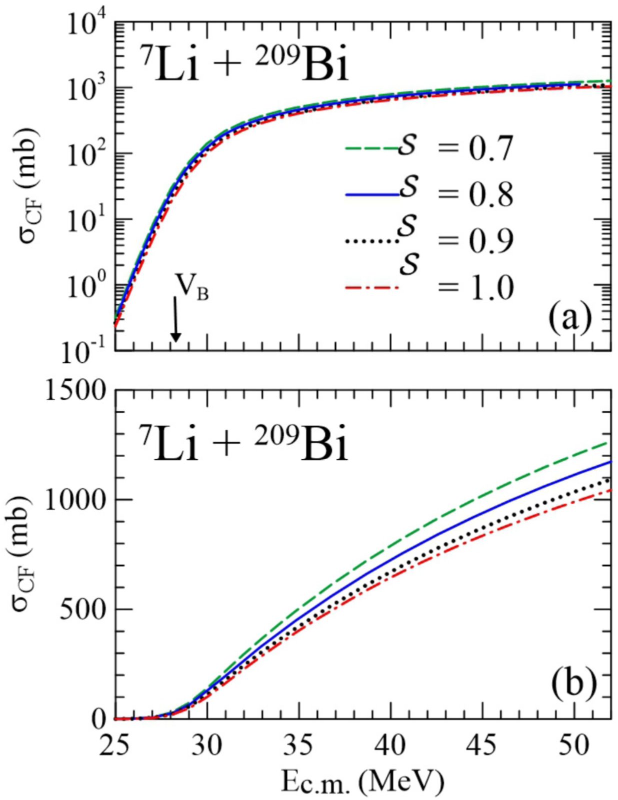
Conclusion of authors of the experiment



- Excellent agreement where all relevant decay channels are measured

- Consistent with data where they give a lower bound

Specroscopic factors for bound-continuum couplings



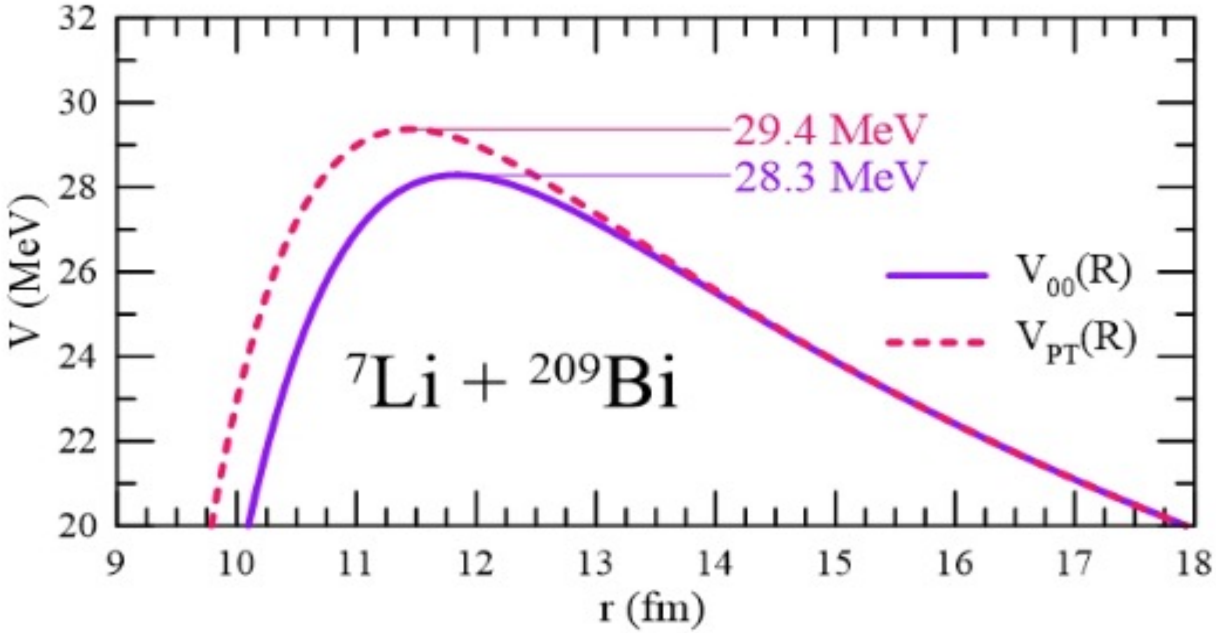
- Not relevant for qualitattive studies
- Relevant for quantitative studies
- Lower S=> higer CF , lower ICF

- No microscopic results fro S in cluster models => free parameter

S. Watanabe et al. PRC 92, 044611 (2015) suggested cluster configuration of ~ 70% for ^{6,7}Li.

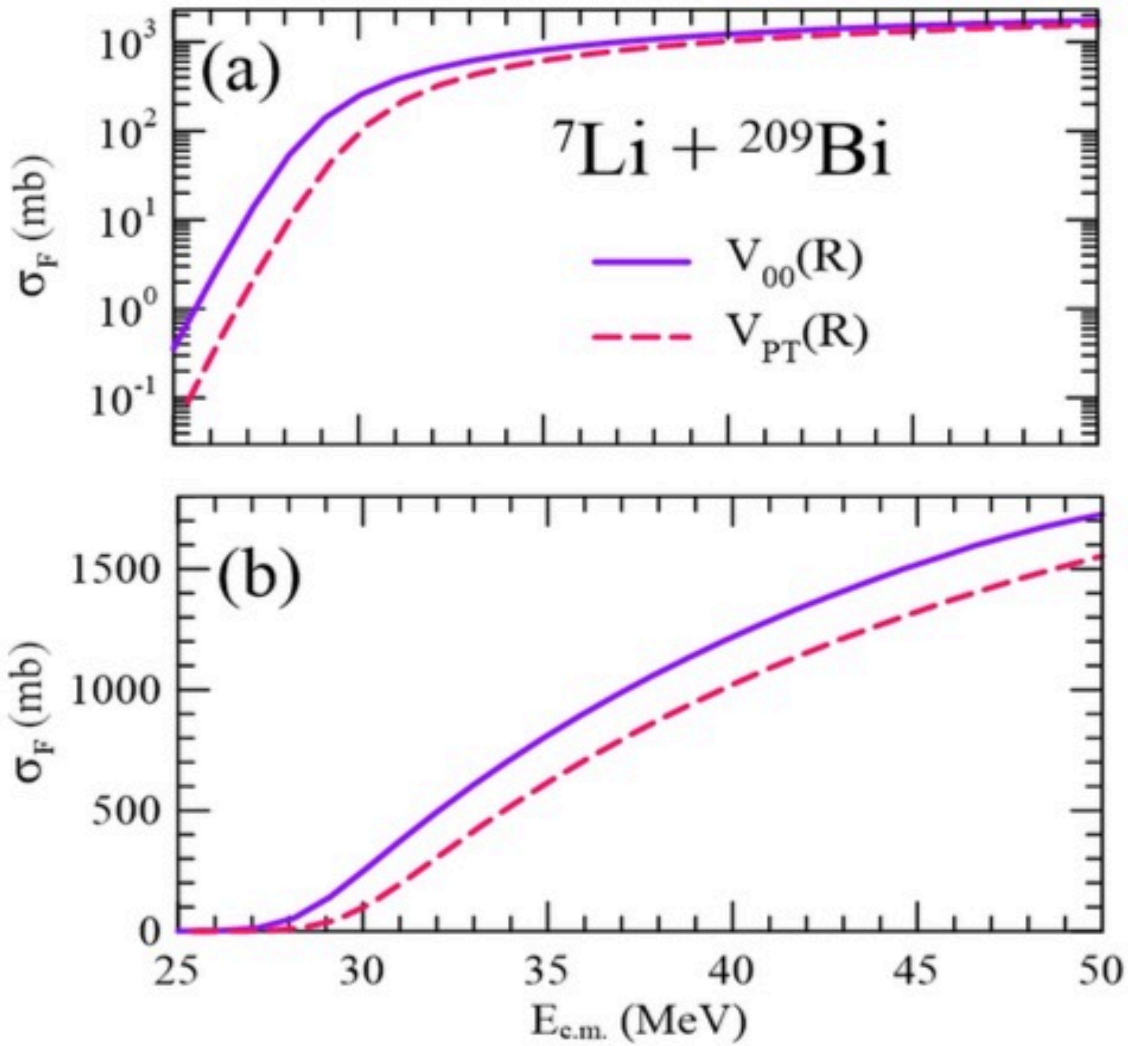
We used S=0.8 in our calculations for ^{7}Li

Cluster model for weakly bound nuclei: Static effect



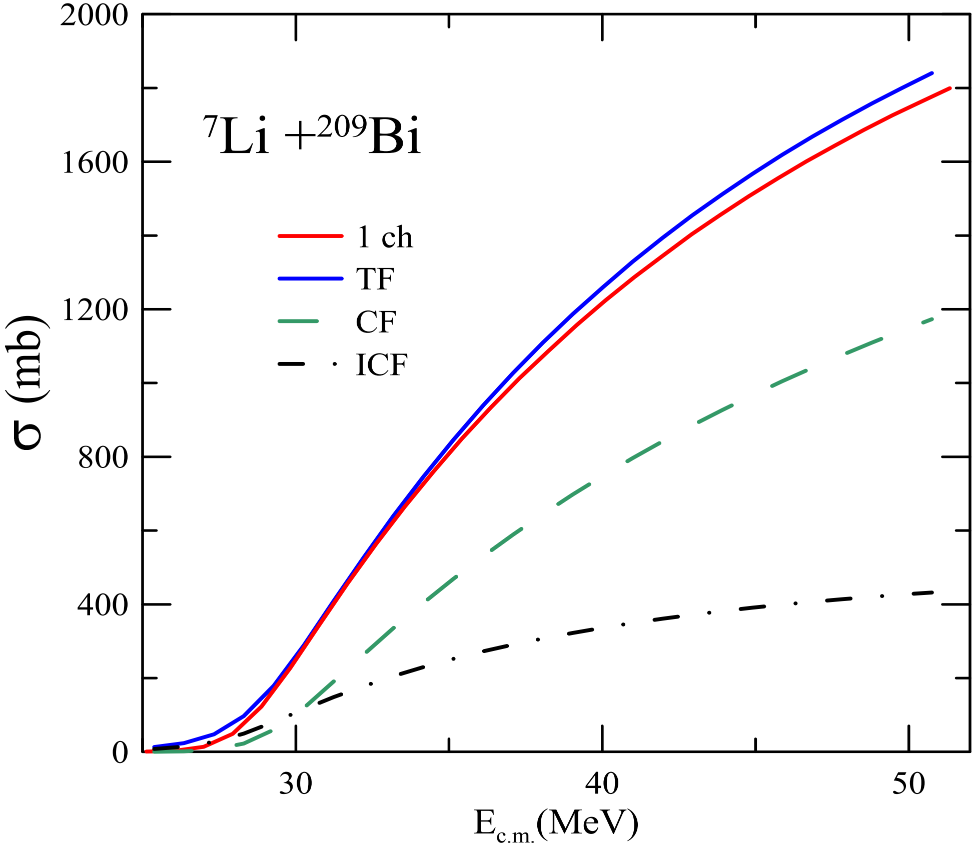
Potencial	R_B (fm)	V_B (MeV)	$\hbar\omega$ (MeV
V_{PT}	11.4	29.4	4.3
V_{00}	11.9	28.3	4.0

Cluster model for weakly bound nuclei: Static effect



Effects of the breakup couplings

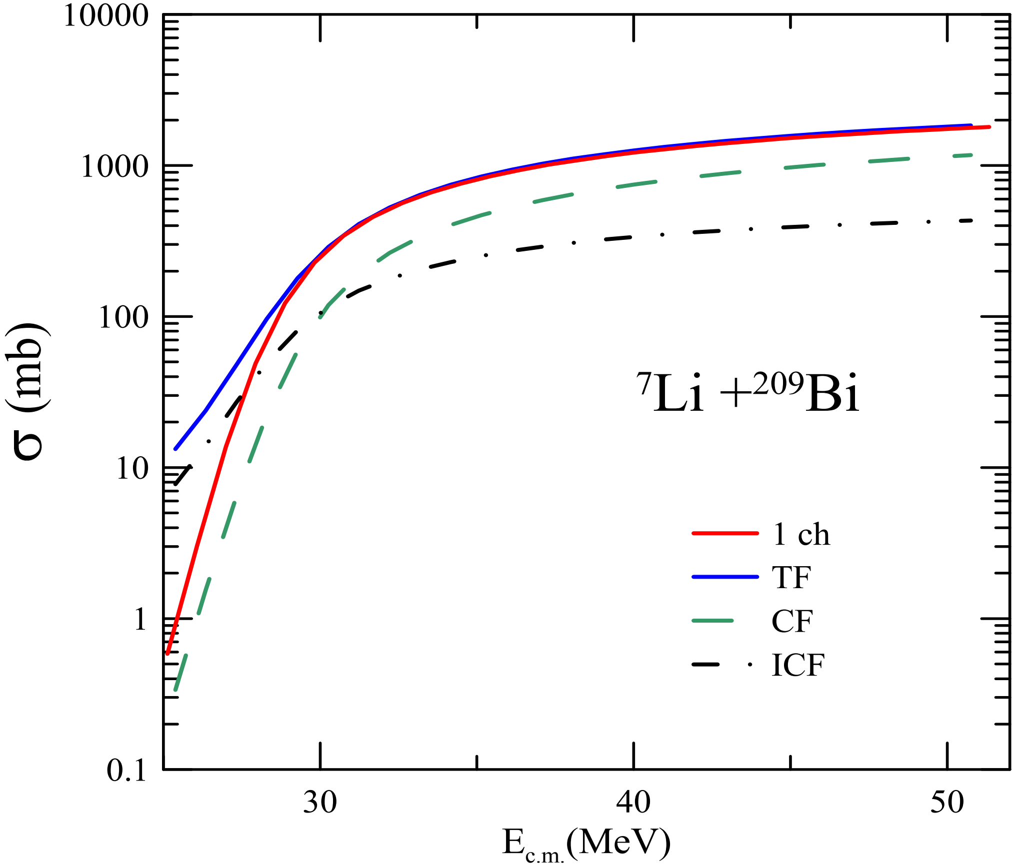
Above V_B



- TF almost identical do 1-channel
- Bu-couplings redistribute σ_{TF} between CF and ICF, without changing the sum

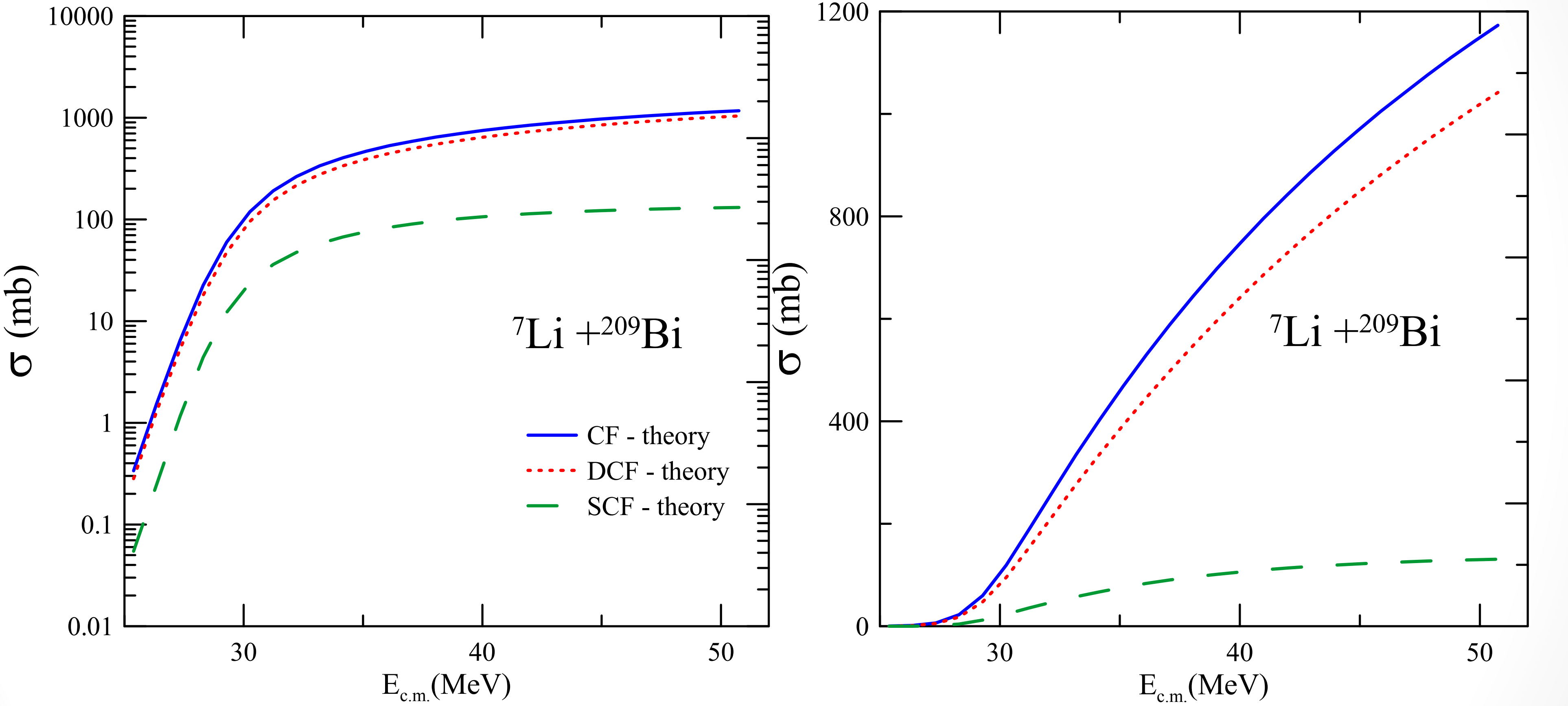
Effects of the breakup couplings

Below V_B

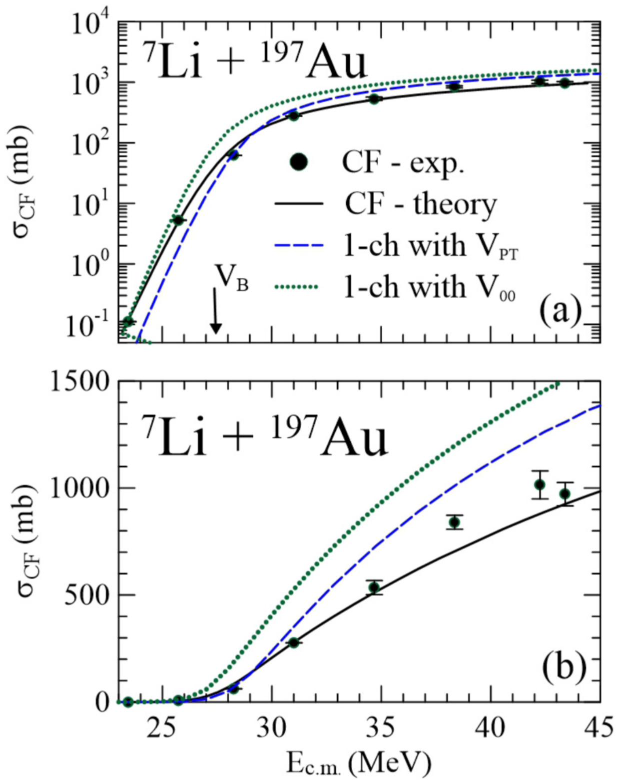


- CF is suppressed - weaker suppression as E decreases below V_B (28.2 MeV)
- ICF becomes larger than CF for E < 32 MeV
- Owing to ICF, TF is enhanced below V_B (lighter fragments fuses easily)

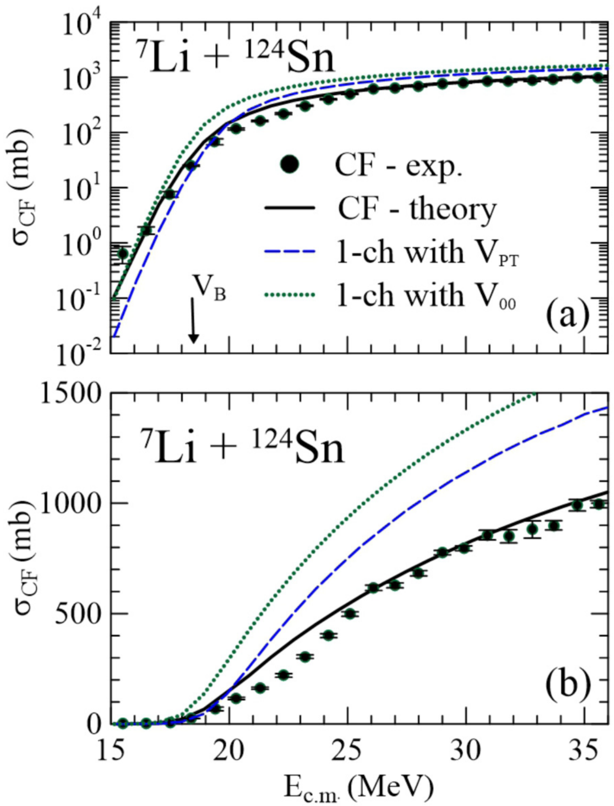
Complete fusion



CF theory vs experimental data



C. S. Palshetkar et al., PRC 89, 024607 (2014).



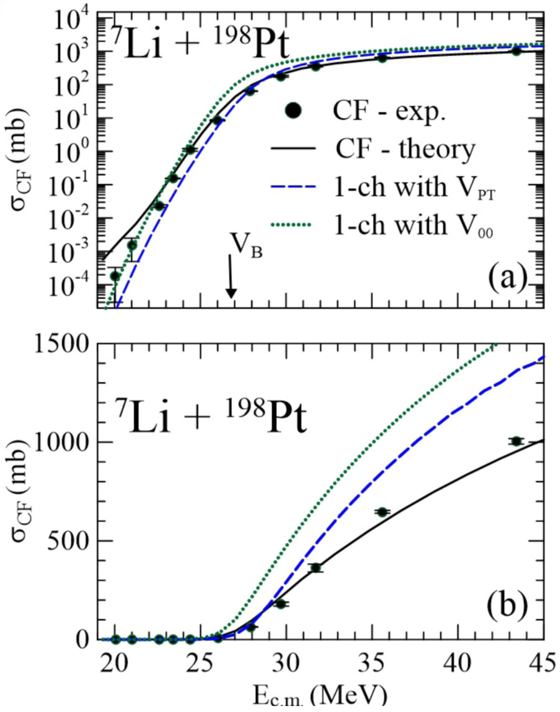
V. V. Parkar, et al. PRC 97, 014607 (2018)

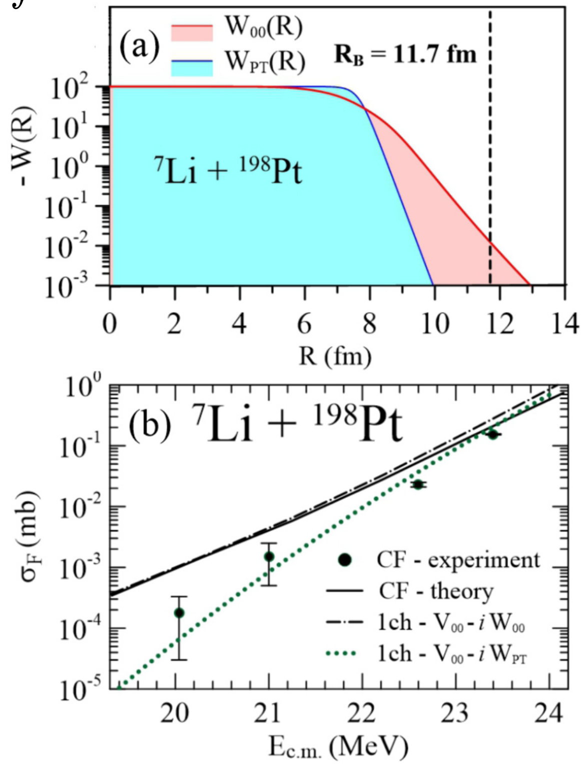
V_{PT} - without cluster good agreement with the data

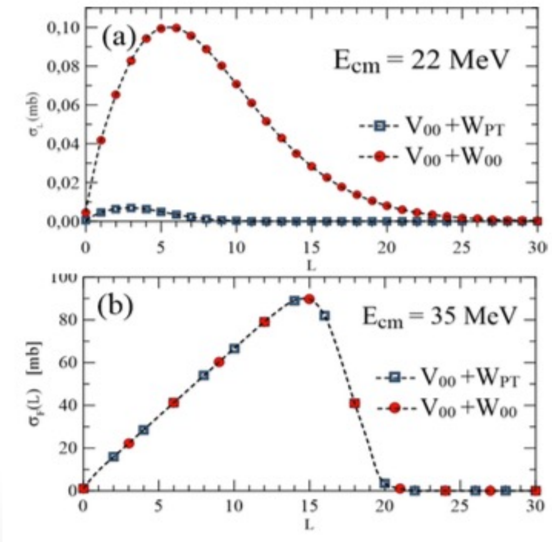
V_{00} – with cluster

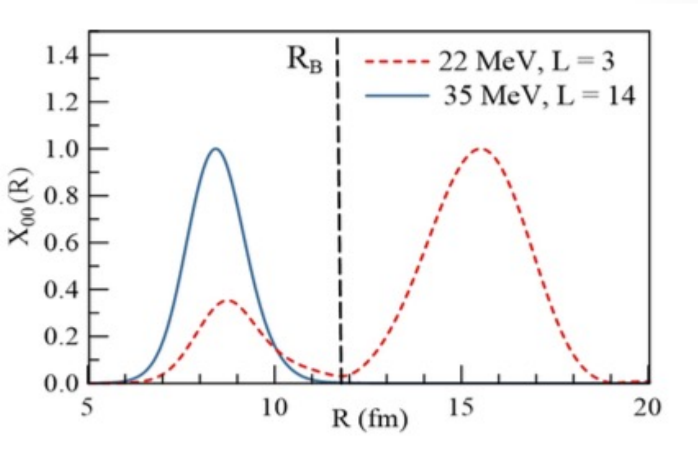
^{7}Li + ^{198}Pt fusion – theory vs. experiment*

$$W_{00}(R) = \int d^3\mathbf{r}\,|\phi_0(\mathbf{r})|^2[\mathbb{W}^{(1)}(r_1) + \mathbb{W}^{(2)}(r_2)]$$



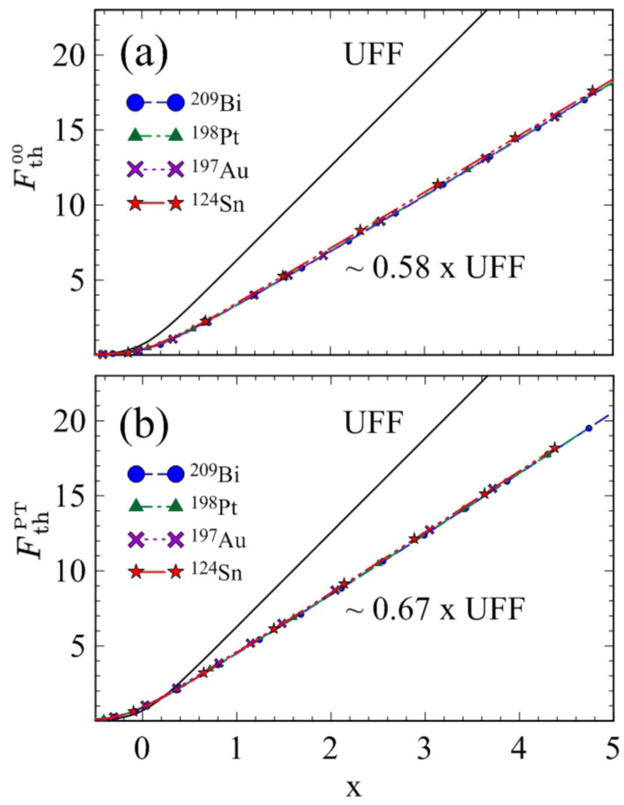






* A. Shrivastava, et. Al, Phys. Rev. Lett. , 103:232701,(2009)

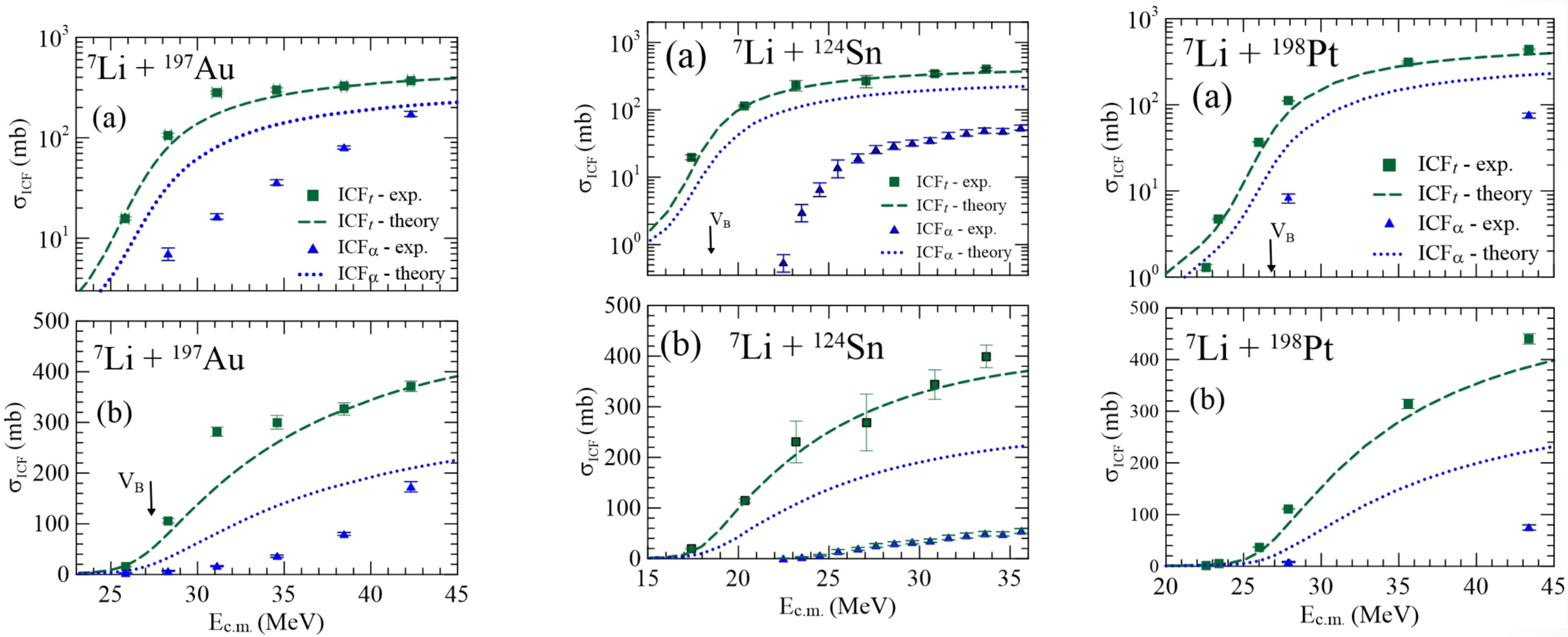
Reduced CF



$$F_{th}^{00}(x) \simeq 0.58 \times F_0(x); \quad F_{th}^{PT}(x) \simeq 0.67 \times F_0(x),$$

Different results depending pf the real potential used to reduce the data

ICF: theory versus experiment



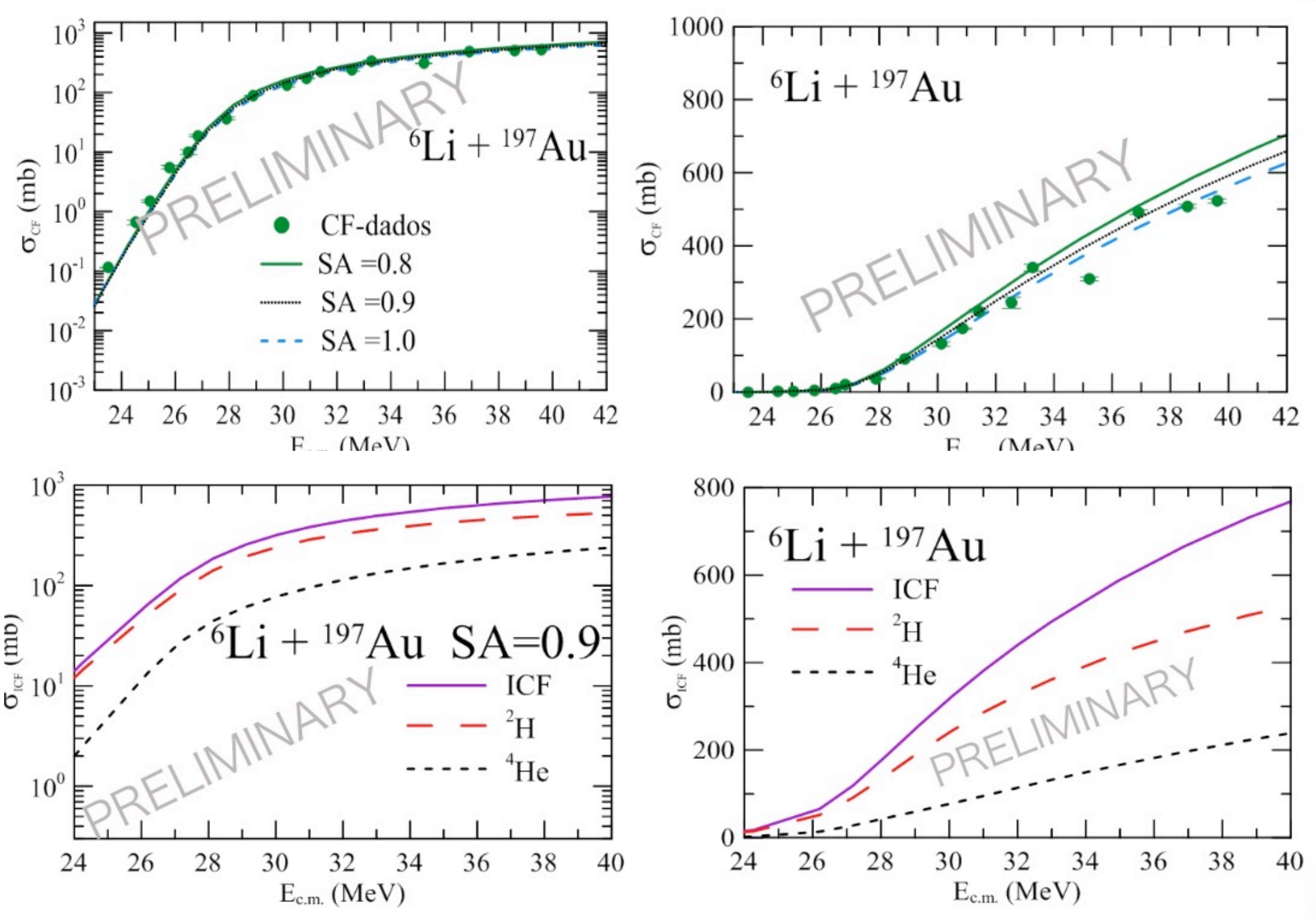
ICF$_t$ very well described

ICF$_\alpha$ overpredicted. Why? Problem of the theory, of the dat?????

^{6}Li - preliminary results

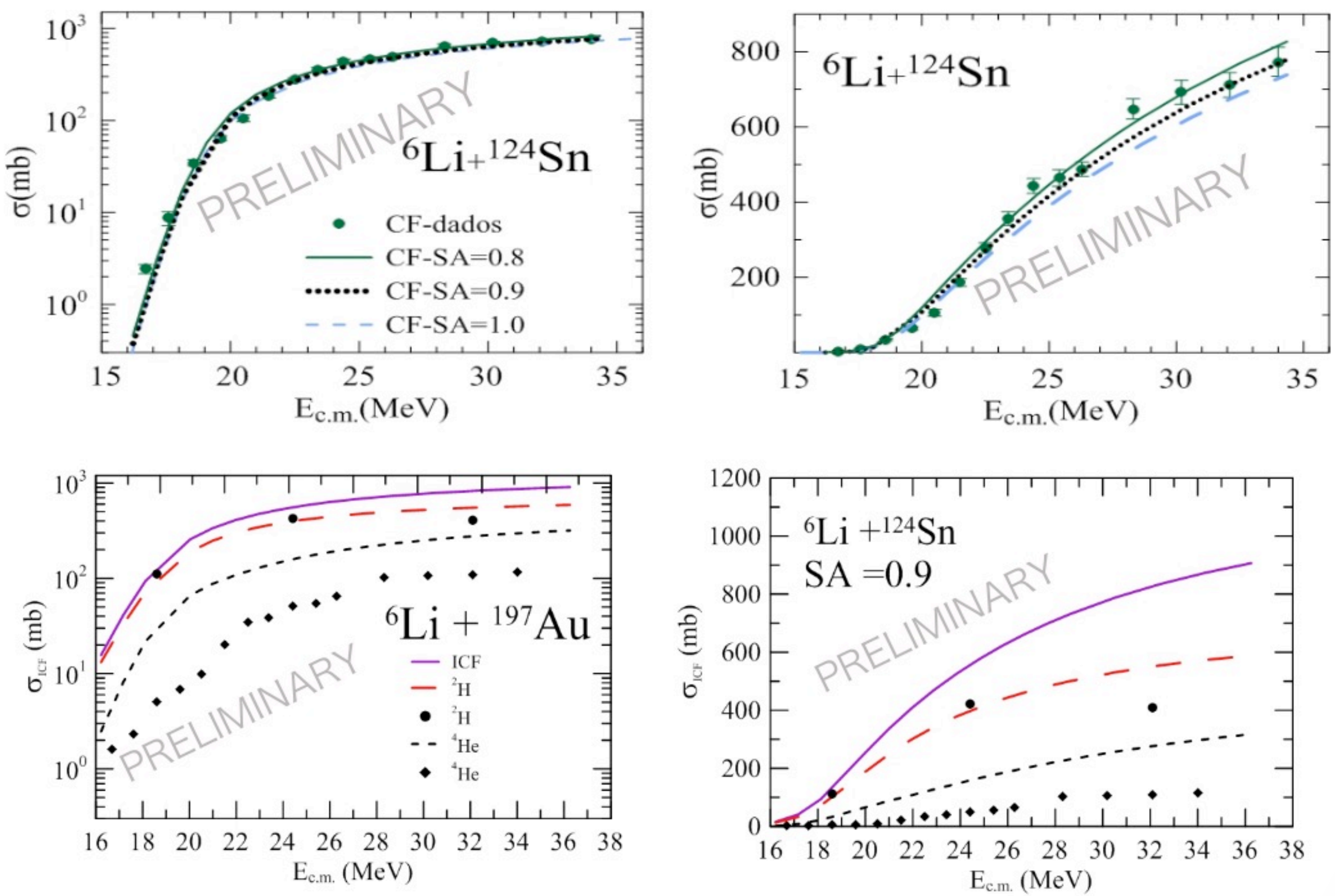
S = 0.9

^{197}Au



CF - well described   ICF – no data

^{124}Sn



CF - well described

ICF$_t$ – well dewcribed

ICF$_\alpha$ - overestimatd

Conclusions

- We have proposed a new quantum mechanical method to evaluate CF and ICF in collisions of weakly bound nuclei
- The method was applied to the ^{7}Li + heavy target system and the results were compared with the data.
- Considering that our calculations use standard interaction and have no free parameters, the agreement between theory and experiment is excellent
- Calculations of other systems are in progress

Future plans

- Study other systems (e.g. ^{6}Li on heavy targets, ^{6,7}Li on medium mass targets)
- Include spectroscopic factors* (cluster structure of g.s. is just an approximation)
- Include target excitation (important in fusion of deformed targets)*
- Include core-excitations*
- Extension to 4-body CDCC (ex: ^{9}Be collisions)*
- Include transfer channels ??????
- Include BU triggered by transfer ?????

* Requires another version of the CDCC code

Team members

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- Jonas Leonardo Ferreira (UFF)
- Mariane Rodrigues Cortes (UFF)

Thank you :D

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