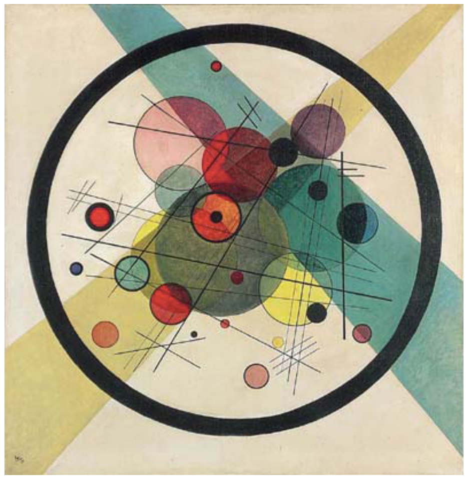
Andrea Vitturi

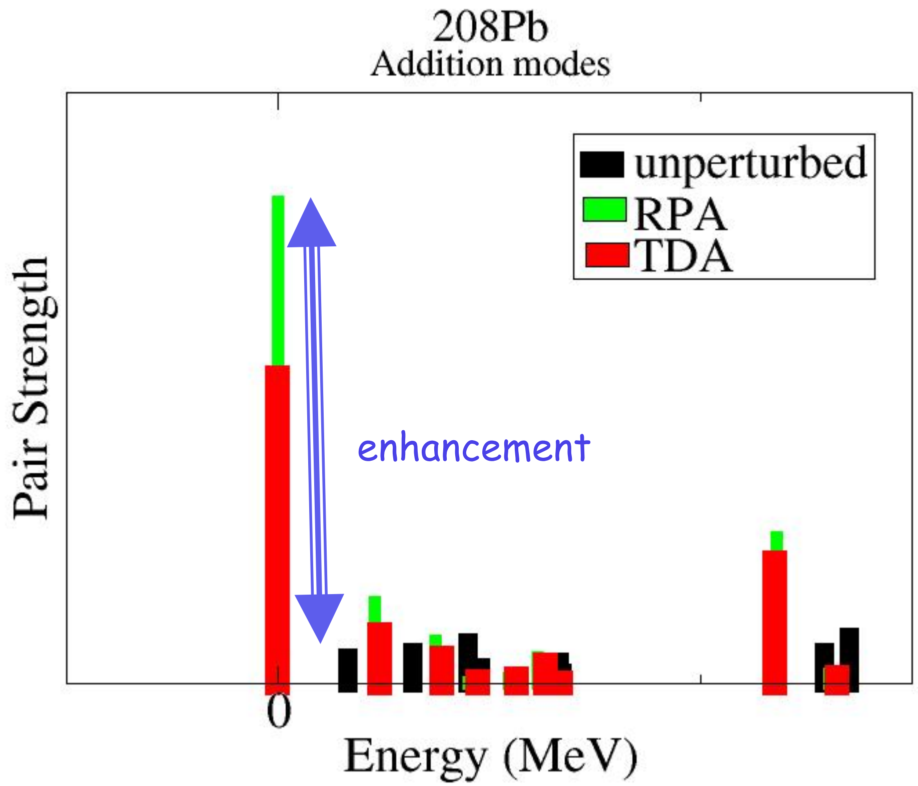
Two-particle transfer as a signature of shape phase transition and/or shape coexistence

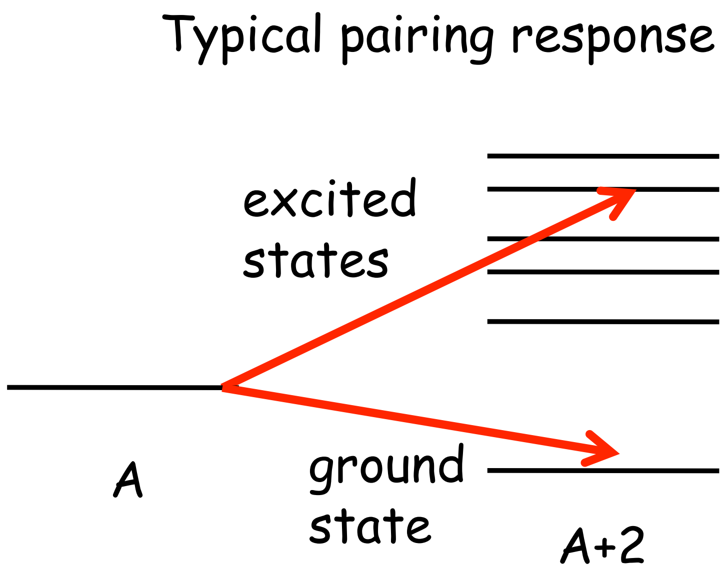


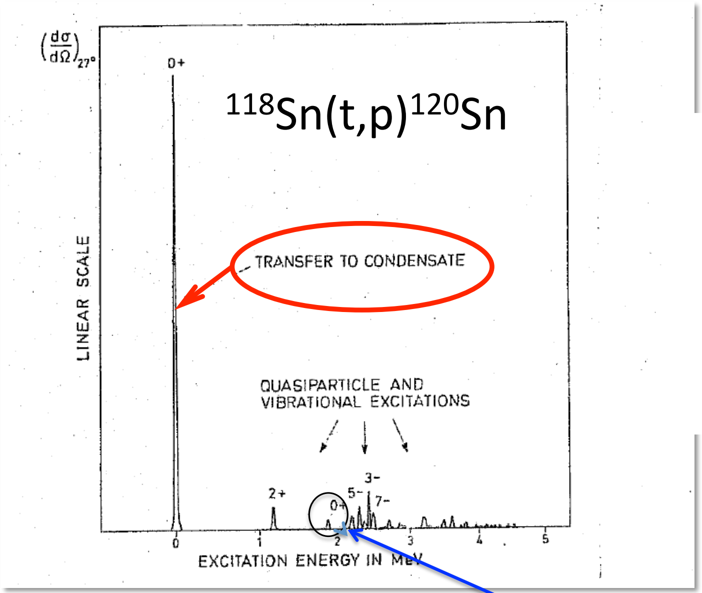
Two-particle transfer reaction are the traditional tools to study "dynamically" the effect of the pairing interaction, with special focus on the occurrence of collectivity in the ground state and the enhancement of transfer probabilities with respect to unperturbed non-collective states. The basic naïve idea is that the two-particle transfer cross section is proportional to the

Pair strength

$|P^{+}|^2 = |\sum_j [a^{+}_j a^{+}_j]_{00}|^2$ (or similar for pair removal)





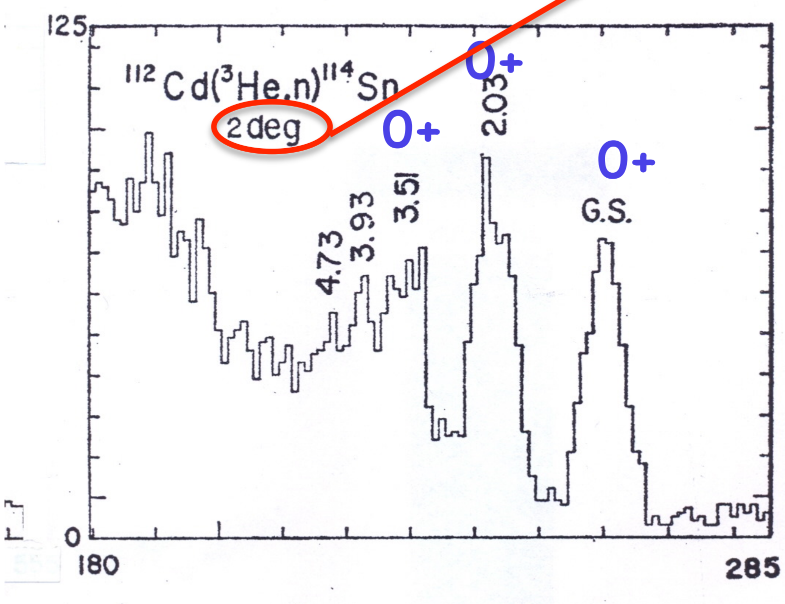


J.H.Bjerregaard *et al.* NPA 110 **1** (1968)

excited 0+ state

In most cases practically all pairing strength goes to the ground state .....

At forward direction strong selectivity favoring L=0



.....but in some cases there are strong excited 0+ states (pairing vibrations)

Basic problem with two-particle transfer reactions: the proper reaction mechanism to extract a quantitative estimate of the pairing enhancement. In fact the reaction mechanism associated with pair transfer is rather complicated and the possibility of extracting spectroscopic information on the pairing field is not obvious.
All different approaches try to reduce the actual complexity of the problem, that is at least a four-body problem (two cores plus the two transferred particles) to a more tractable framework.

Two lines of approach are most popular, simplified by:

A, Successive single-particle transfer,
based on the dominance of the mean field

B. Cluster transfer,
based on the dominance of the pairing interaction

A

Sequential two-step process: each step transfers one particle

Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A+1) nuclei, due to the correlations in initial and final wave functions

Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions. Expansion to second-order in the transfer potential

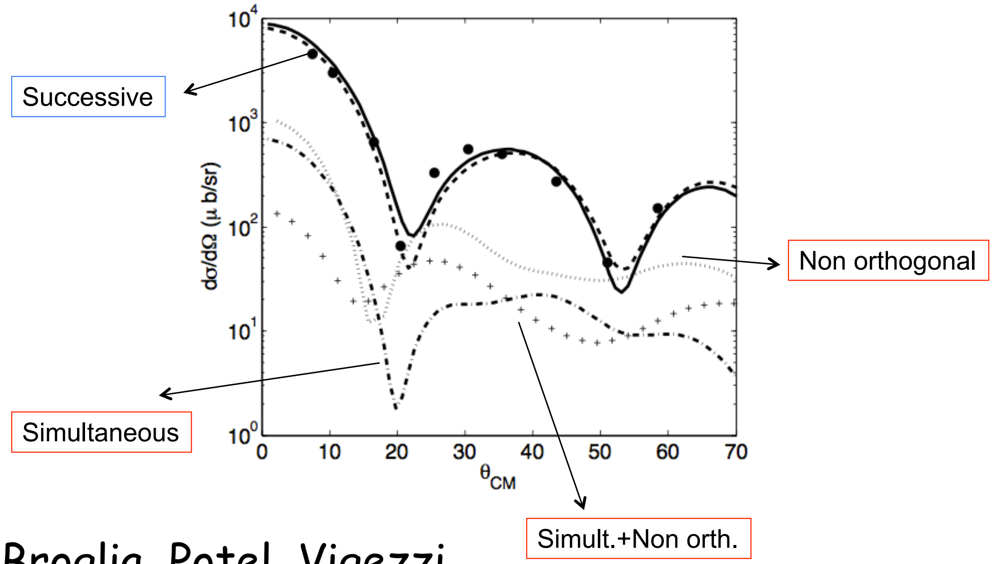
Simultaneus + Sequential + not-orthogonality

(first-order) (second-order)

this is not the cluster contribution

these two terms may approximately cancel each other

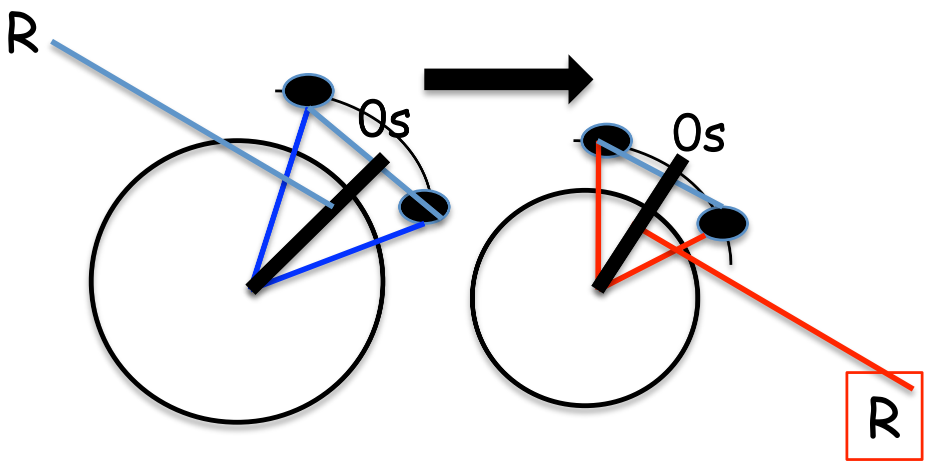
$^{122}Sn(p,t)^{120}Sn$ E_{lab} = 26 MeV



Barranco, Broglia, Potel, Vigezzi

B

Cluster-transfer model (suggested by the close radial correlation of the pairs and obviously dominant in the case of extremely large pairing force with respect to mean field)

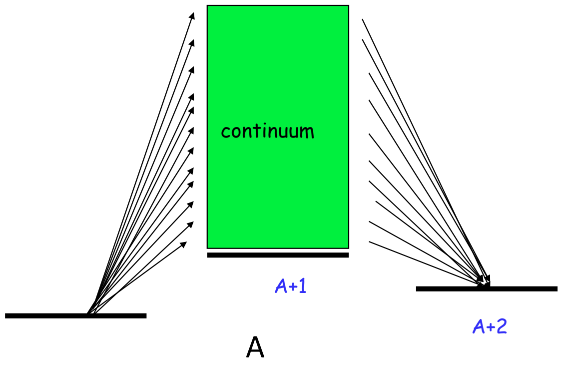


Initial and final cluster wave functions are obtained by taking the overlap between the two-particle wave functions and a 0s wave function for the relative motion

These overlaps also get enhanced by the coherent contributions of the different components generated by the pairing interaction. Note, however, that the final enhancement may be quantitatively different from the one obtained within the sequential transfer model.

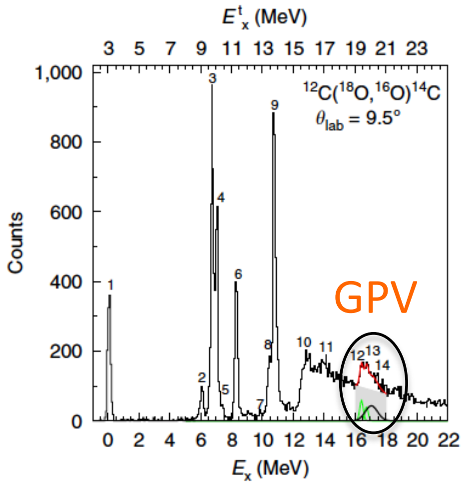
Aside from the basic problem of the reactions mechanism, a number of issues are, in my opinion, at present very interesting (each deserving at least a full seminar):

1. two-particle transfer as a tool for the study of the pairing at the drip lines and of the role of continuum states



e.g. $^{6}He(p,t)^{4}He$ via unbound ^{5}He

2. the search for high-lying pairing vibrations (Giant Pairing Vibration)



Numen (LNS)

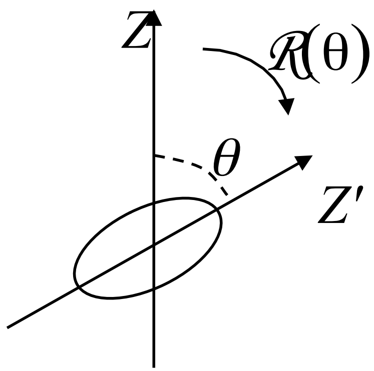
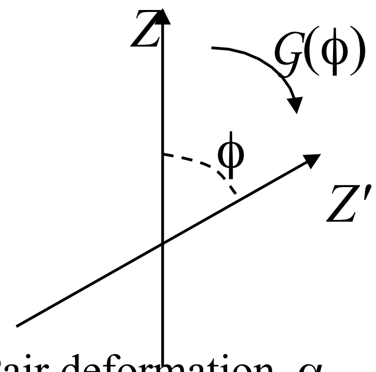
3. Interplay of T=0 and T=1 pair transfer

BUT WE MOVE NOW TO THE PROBLEM OF PHASE TRANSITIONS

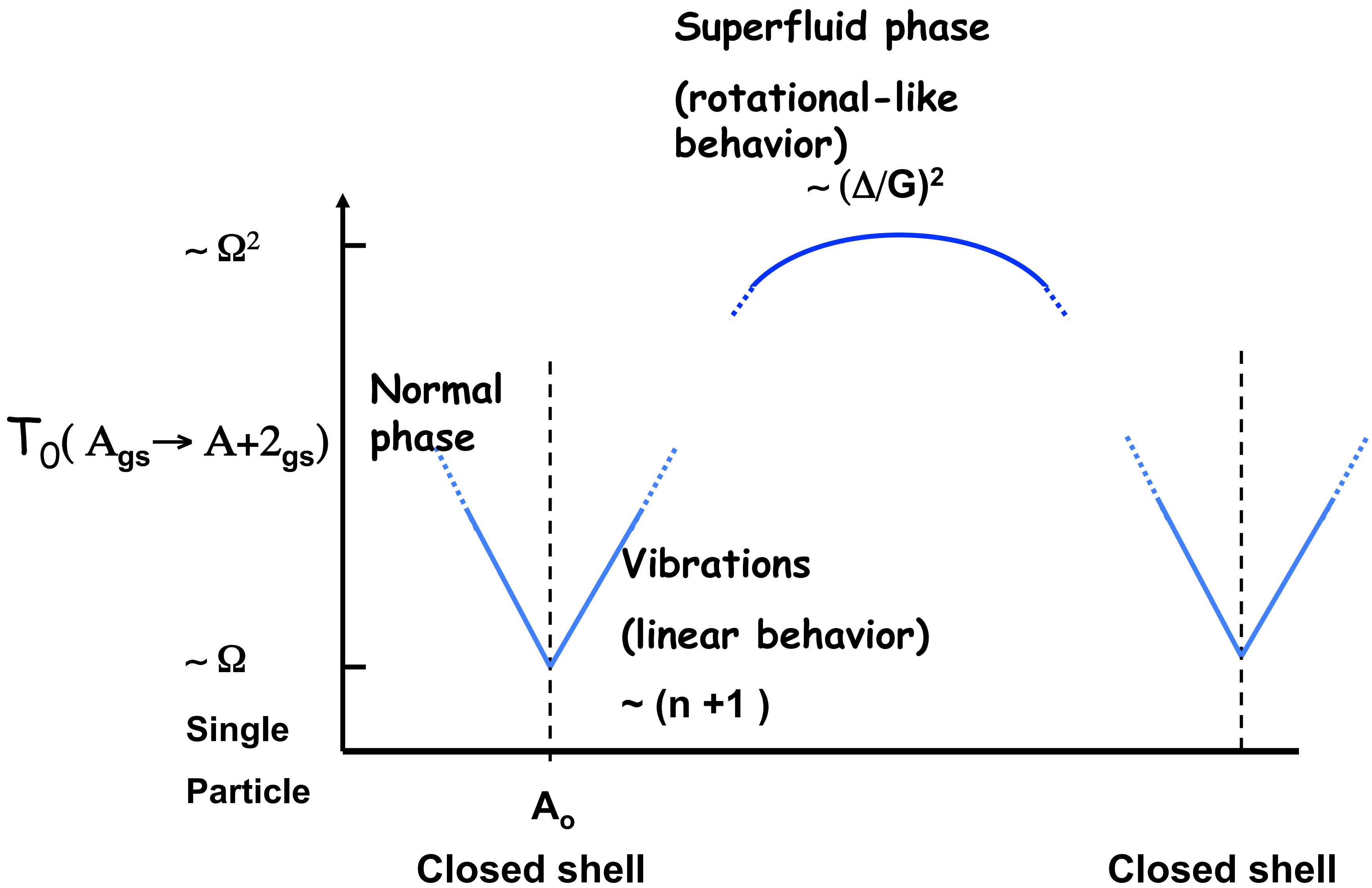
The pairing response is characterized by the pairing phase (normal or superfluid) and by the shape phase (e.g. spherical or deformed). Therefore it will be a clear signature of phase transitions (in addition to the standard signatures, as E_4/E_2, B(E2), etc) in both the

shape degree of freedom

pairing degree of freedom

Shape Transitions	Pairing Transitions
$\mathcal{R}(\theta)=\exp(-iI\theta)$	$\mathcal{G}(\phi)=\exp(-i\mathcal{N}\phi)$
Angular Momentum, I	Particle Number, $\mathcal{N}$
	
β, γ, Euler angles θ	Pair deformation, α Gauge angle, ϕ
Violation of spherical symmetry	Violation of particle number
Physical space	Abstract "gauge" space

Phase transition from "normal" to "superfluid" phases: characteristic behavior of the pair transfer matrix element

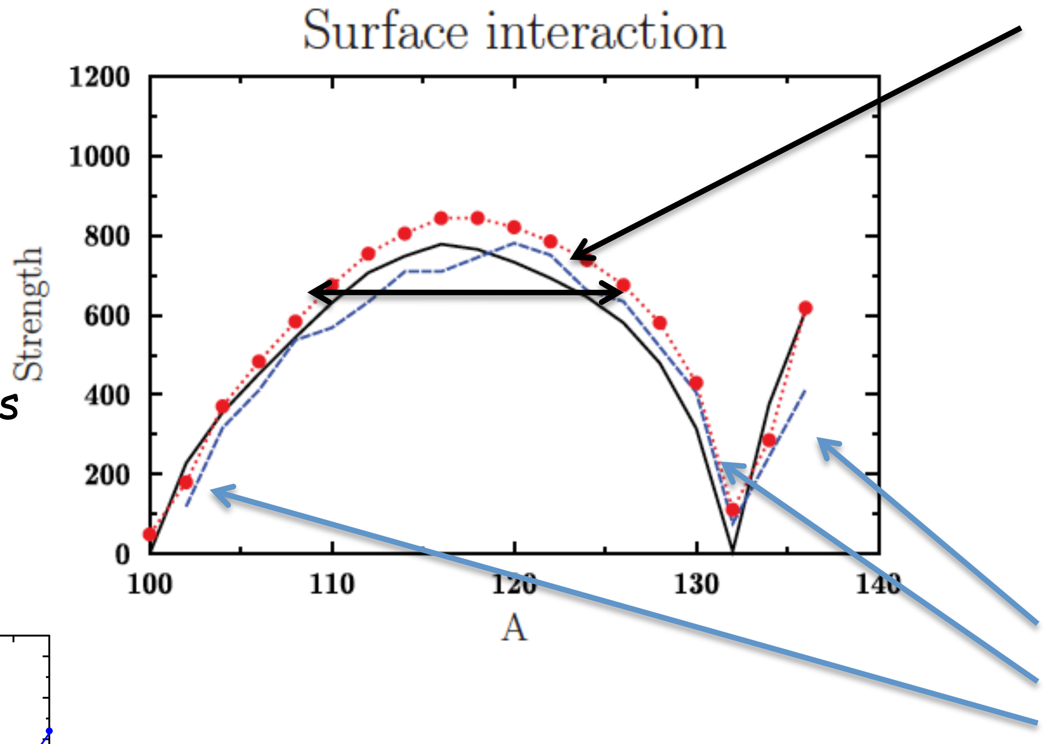


OBS: Similar phase transitions as a function of temperature or angular momentum

Grasso
Lacroix
Vitturi

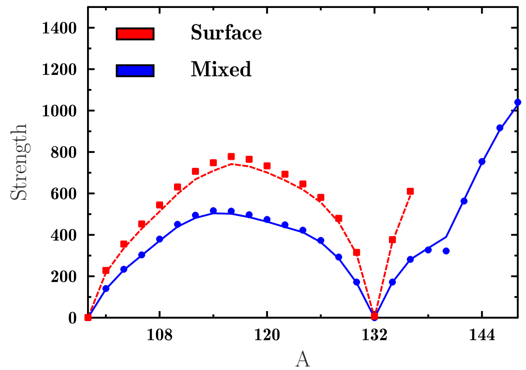
A calculation with Skyrme-HFB for Sn isotopes
with a zero-range density-dependent pairing interaction

Pairing
Rotational
smooth
behavior



Different
approximations
and/or improvements

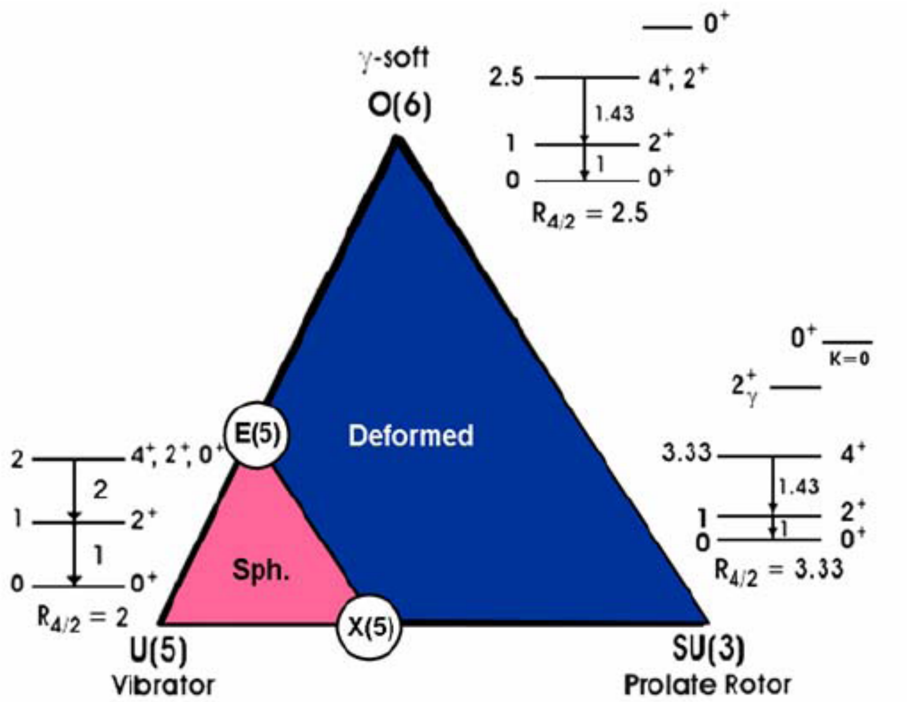
Pairing
vibrational
behavior



Different
forms of the
pairing
interaction

In a similar way pair-transfer probabilities show characteristic behaviors in correspondence of shape phase transitions

For simplicity we move within the framework of the Interacting Boson Model, but the results are similar within other microscopic models



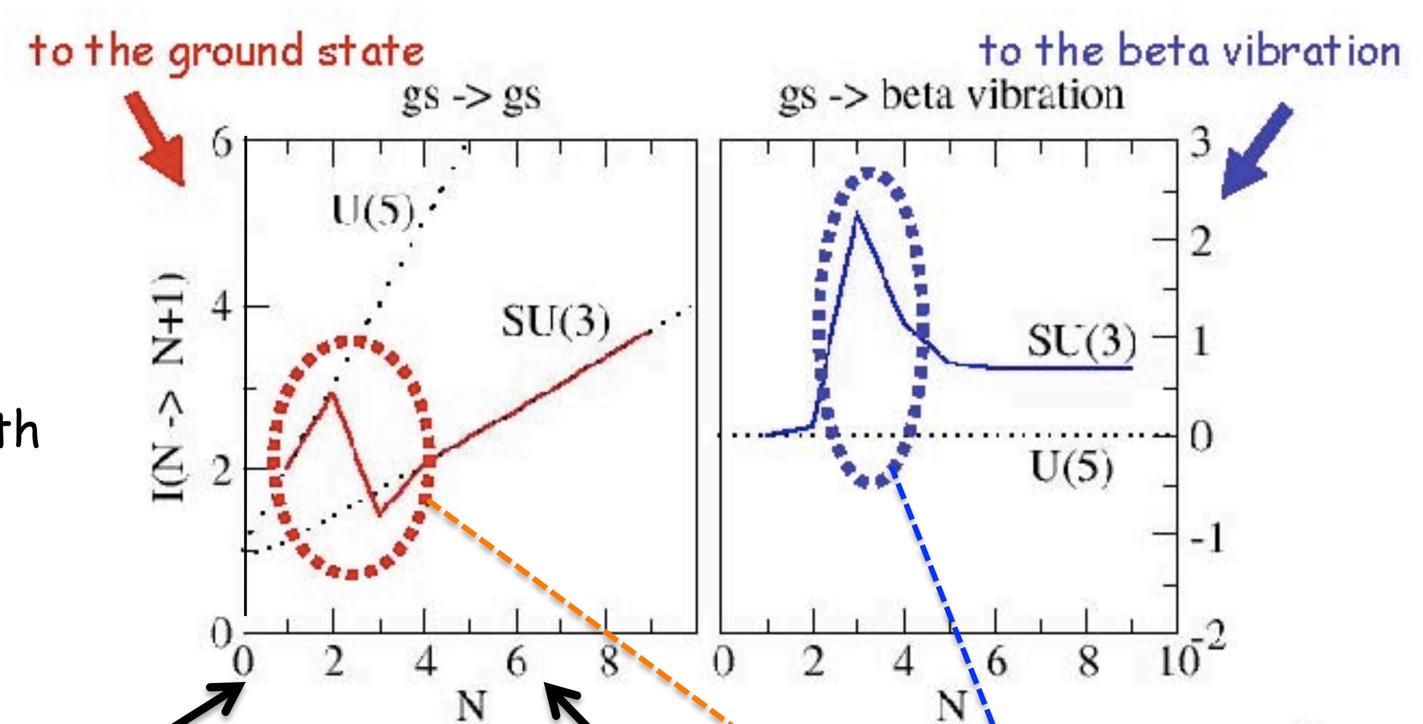
The IBM does not explicitly use the fermion degrees of freedom. From mapping procedure the "form" of the two-particle addition operator is simply assumed as s^+, neglecting higher-order terms, as s^+s^+s or $[d^+d^+]_0s$ or $[d^+s^+d]_0$ etc ……

OBS: See OAI mapping

Schematic case: Spherical shape up to N=3, axial deformation from N onwards
OBS : N number of <u>pairs</u>

Pairing strength = $|\langle N+1|s+|N\rangle|^2$

pairing strength



spherical valence pairs deformed valence pairs

There is a clear signal at the phase transition

In more details

Example: L=0 pair transfer in a phase transition from spherical to axial deformation (from U(5) to SU(3) in algebraic language)

Variational Hamiltonian

$$H(\alpha) = (1-\alpha)\, H_{U(5)} + \alpha\, H_{SU(3)}$$

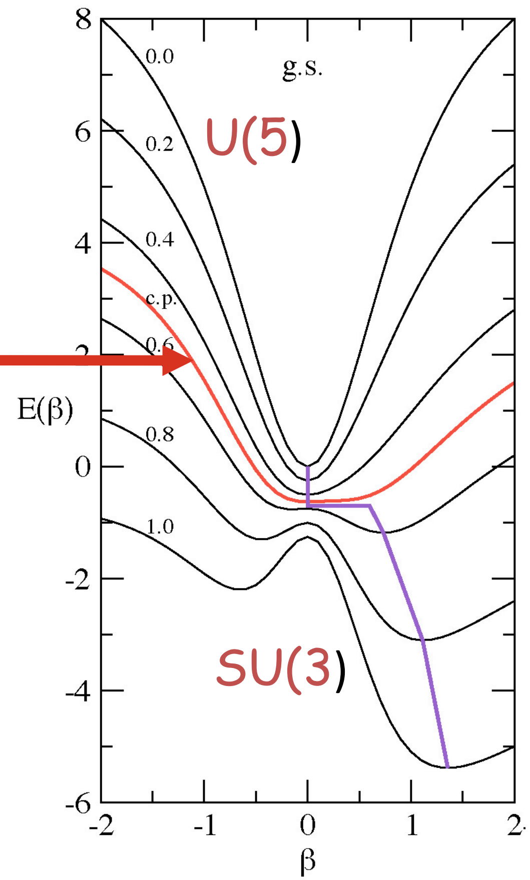
varying the parameter α from zero (sphericity) to one (axial deformation) and empirically connecting α with the mass number N along an isotope chain

critical point

(first-order phase transition)

$\alpha_{crit} \approx 16/34 \approx 0.5$

Energy surfaces
$E(\beta,\gamma=0)$

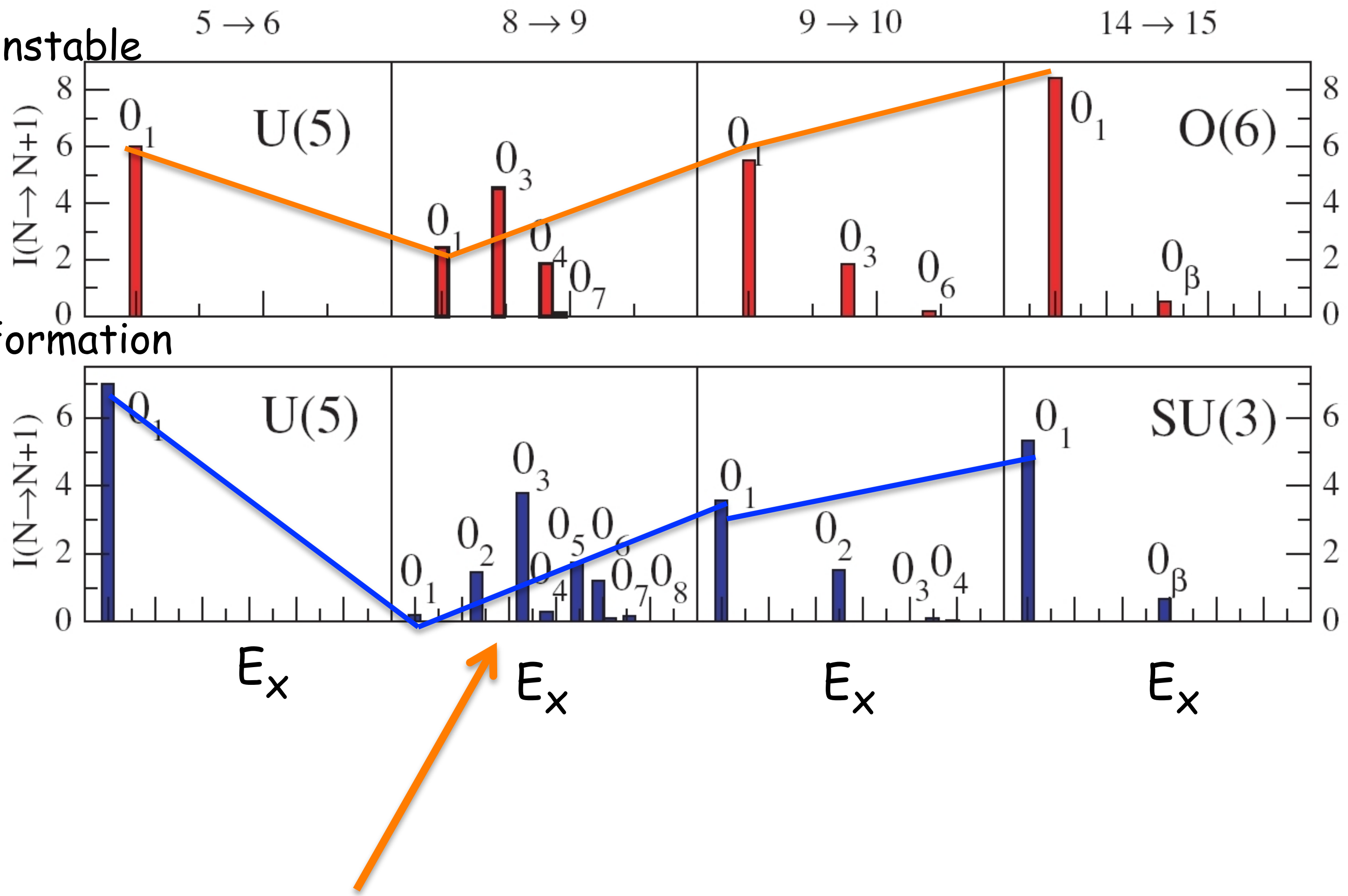


Obs: fragmentation of the pairing strength in correspondence to phase transitions along an isotope chain (in this case chosen to take place at N=8)

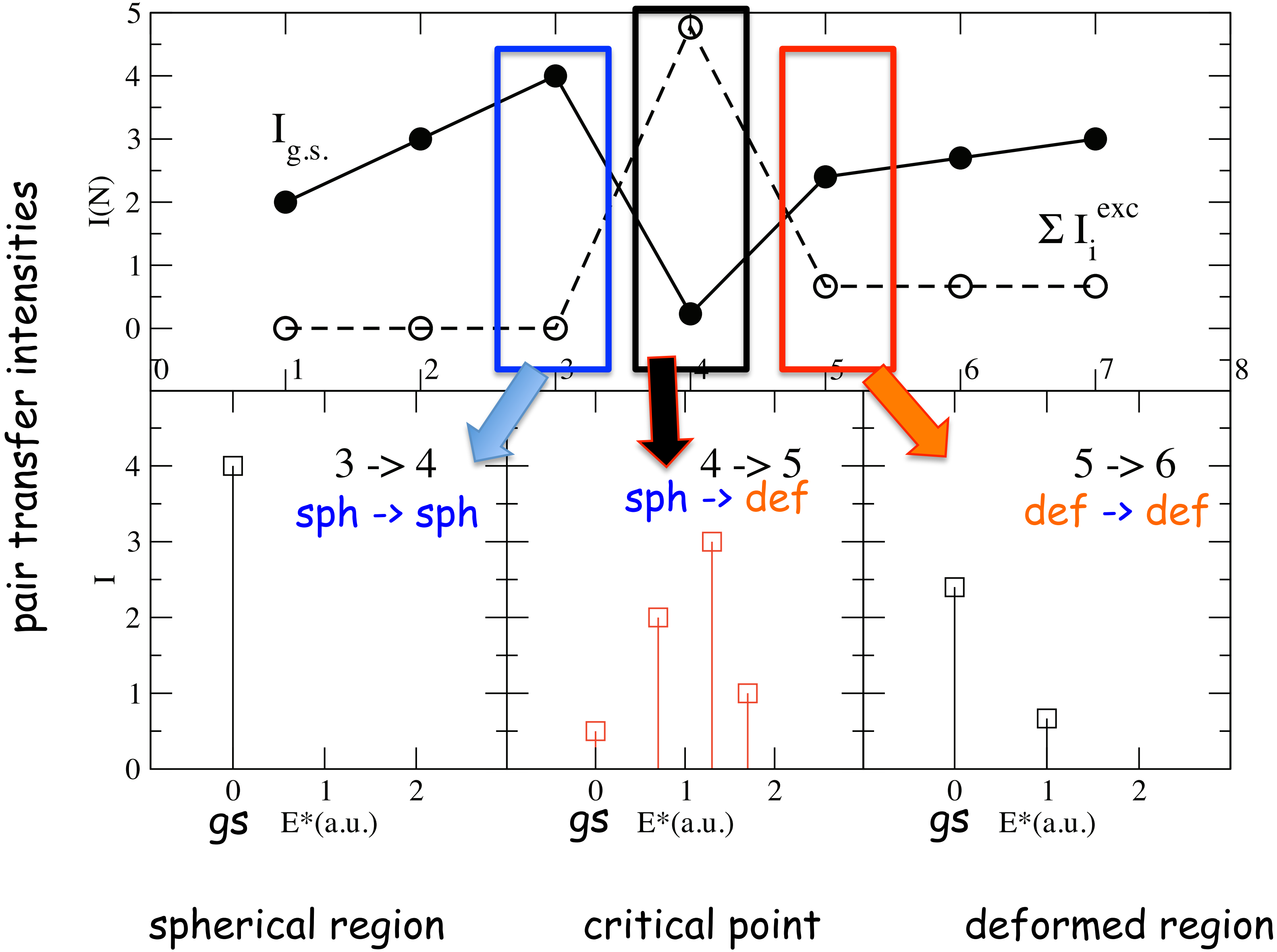
Increasing number of valence pairs →

Spherical to gamma-unstable
U(5) -> O(6)
2nd order

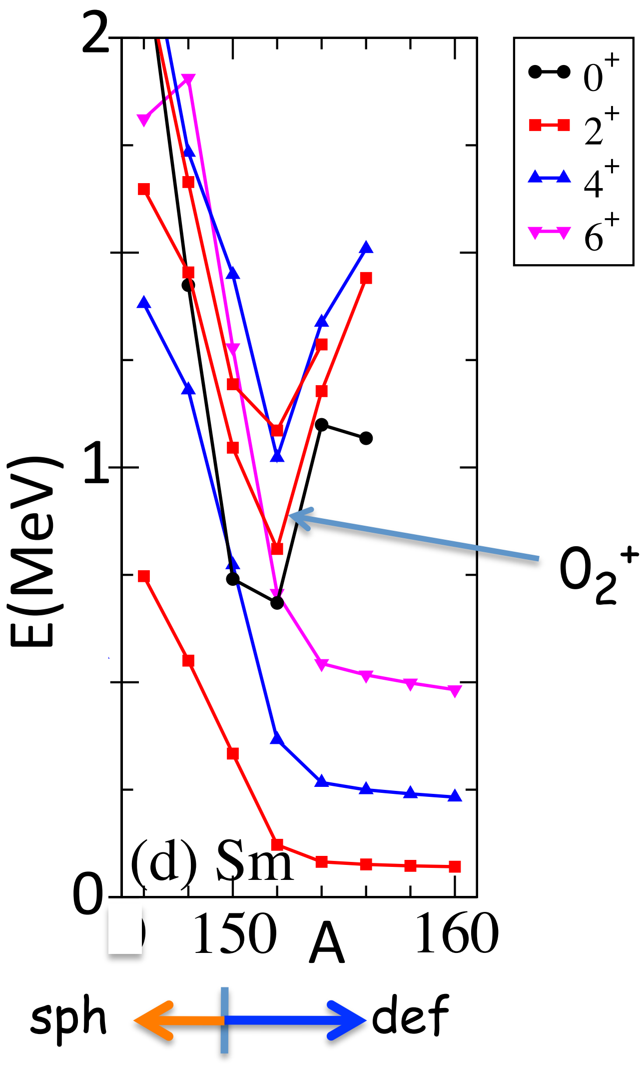
Spherical to axial deformation
U(5) -> SU(3)
1st order



fragmentation of the pairing strength


pair transfer intensities
I(N)
$I_{g.s.}$
ΣI_i^{exc}
3 -> 4
sph -> sph
4 -> 5
sph -> def
5 -> 6
def -> def
I
gs
E*(a.u.)
gs
E*(a.u.)
gs
E*(a.u.)
spherical region
critical point
deformed region

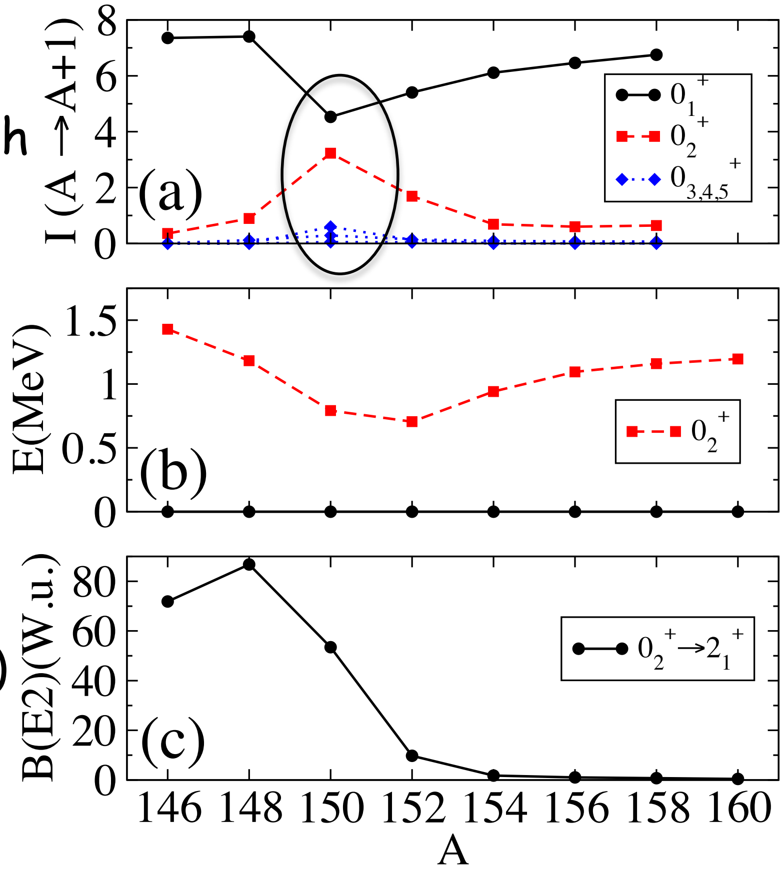
A real case: Samarium isotopes
General IBM Hamiltonian with parameters fitted to each isotope



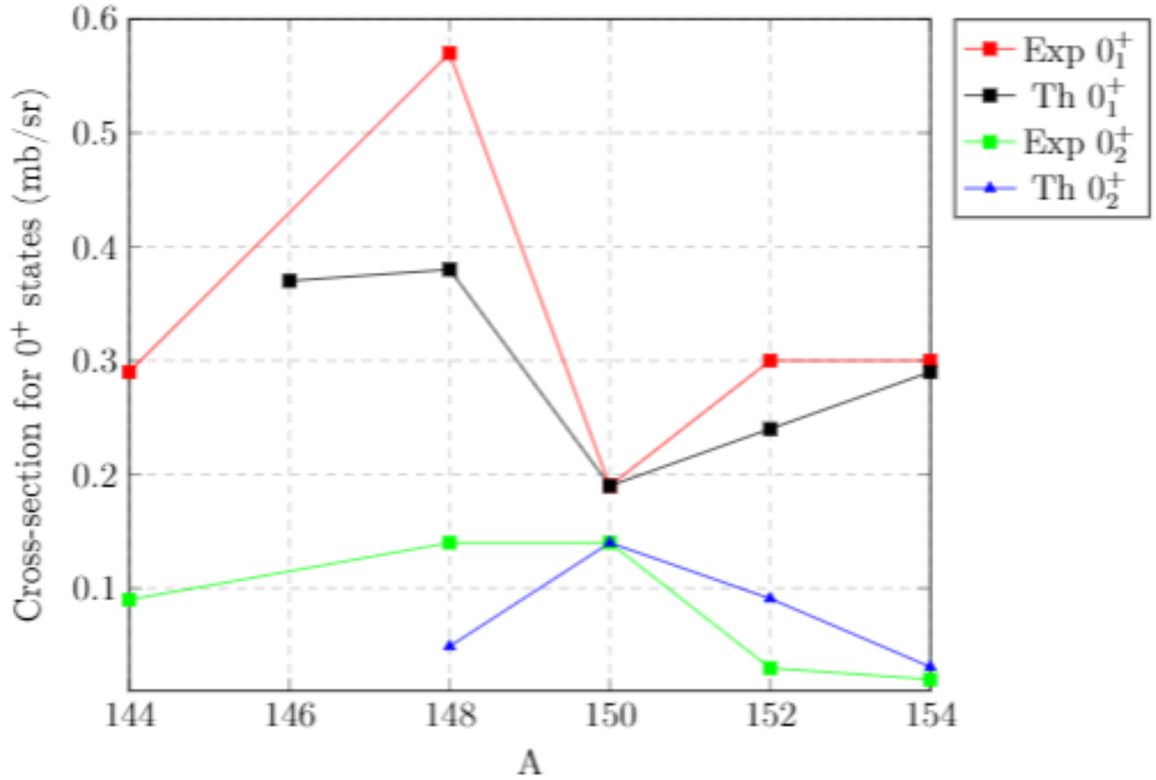
Pair strength

$E(0^+{}_2)$

$B(E2)(0^+{}_2\text{->}2^+{}_1)$



From pair strength to cross sections (within a full microscopic approach, with simultaneous and successive contributions, and with spectroscopic two-neutrons amplitudes provided by IBM structure calculations)



Second-order DWBA for ^{A}Sm(p,t) $^{A-2}$Sm

Optical potentials for proton,deuteron and triton BG,DG and XL

Cross section at $\theta_{cm=}28^{0}$

Alternative scenario: Shape coexistence.

It is a very broad phenomenon that supposes the presence of states with very different shapes or deformation, for instance vibrational-like and deformed, in a narrow excitation energy range. The existence of different configurations is associated with particle-hole (np-nh) excitations across the shell closure. Typically, vibrational-like states correspond to 0p-0h excitations while the deformed ones are associated to 2p-2h excitations.
When both families of states cross in the ground state, it experiences an abrupt change of deformation with consequences in the systematics of the two-neutron separation energy, the quadrupole moment or the B(E2: $2^+_1 \rightarrow 0^+_1$) values.

QPT and shape coexistence show therefore similar systematics and in many cases it is not simple to disentangle which one is the responsible of the rapid onset of deformation.

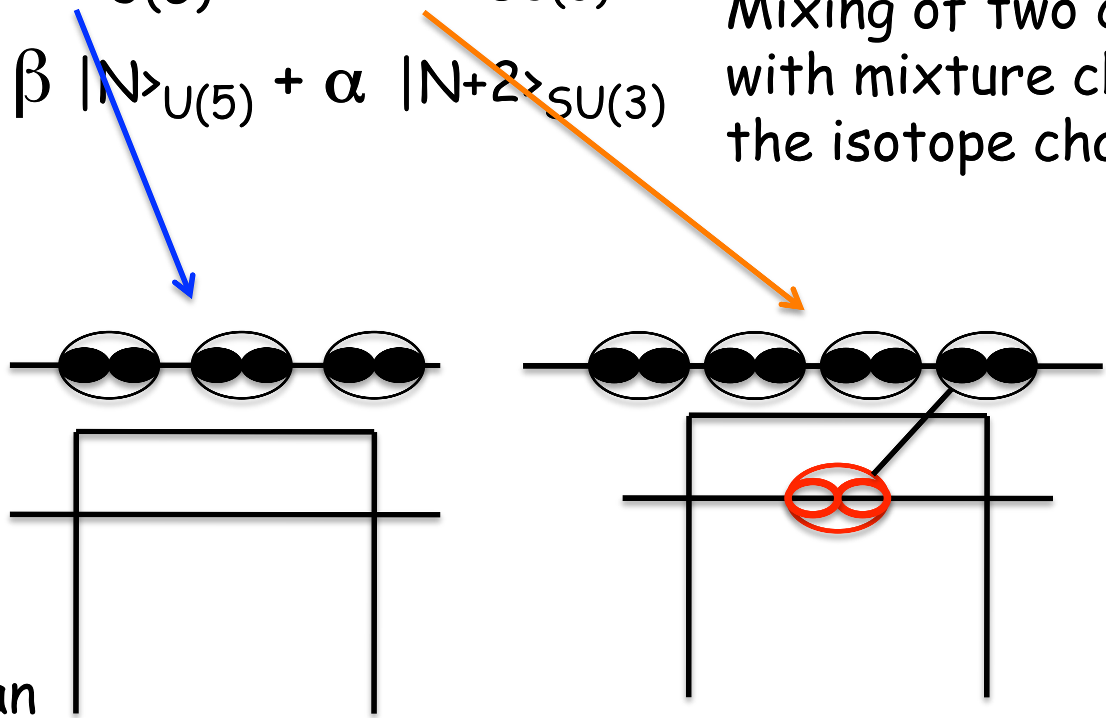
Can two-particle transfer processes help in clarifying the picture?

Schematic scenario: two-level shape co-existence, for example of a spherical and a deformed state within the same nucleus

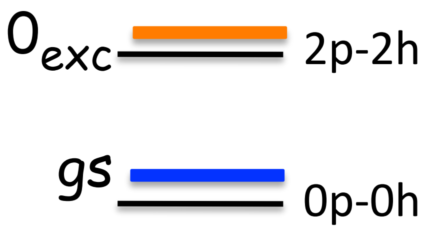
$$|0^+_{gs}, N\rangle = \alpha\, |N\rangle_{U(5)} + \beta\, |N+2\rangle_{SU(3)}$$

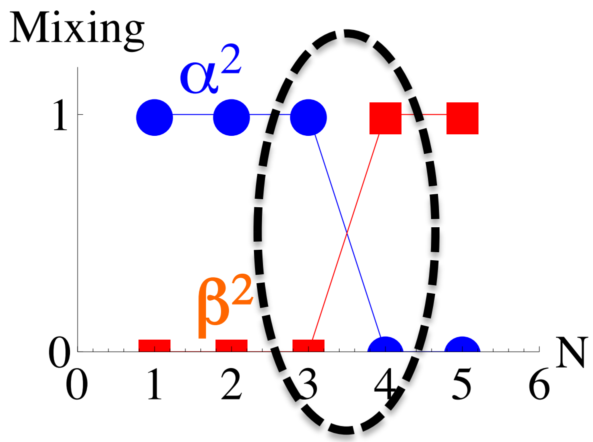
$$|0^+_{exc}, N\rangle = -\,\beta\, |N\rangle_{U(5)} + \alpha\, |N+2\rangle_{SU(3)}$$

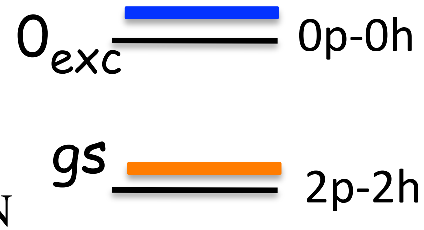
Mixing of two configurations, with mixture changing along the isotope chain



A simple model: along the isotope chain a sharp inversion of the structure

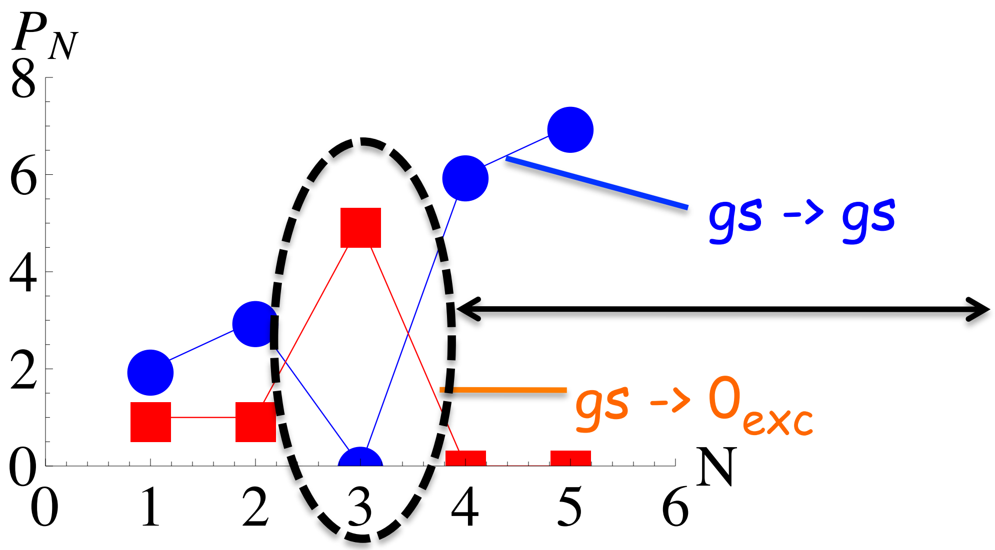






Transfer operator in now more complex: S^+ + S (one can create a particle pair or destroy a hole pair)

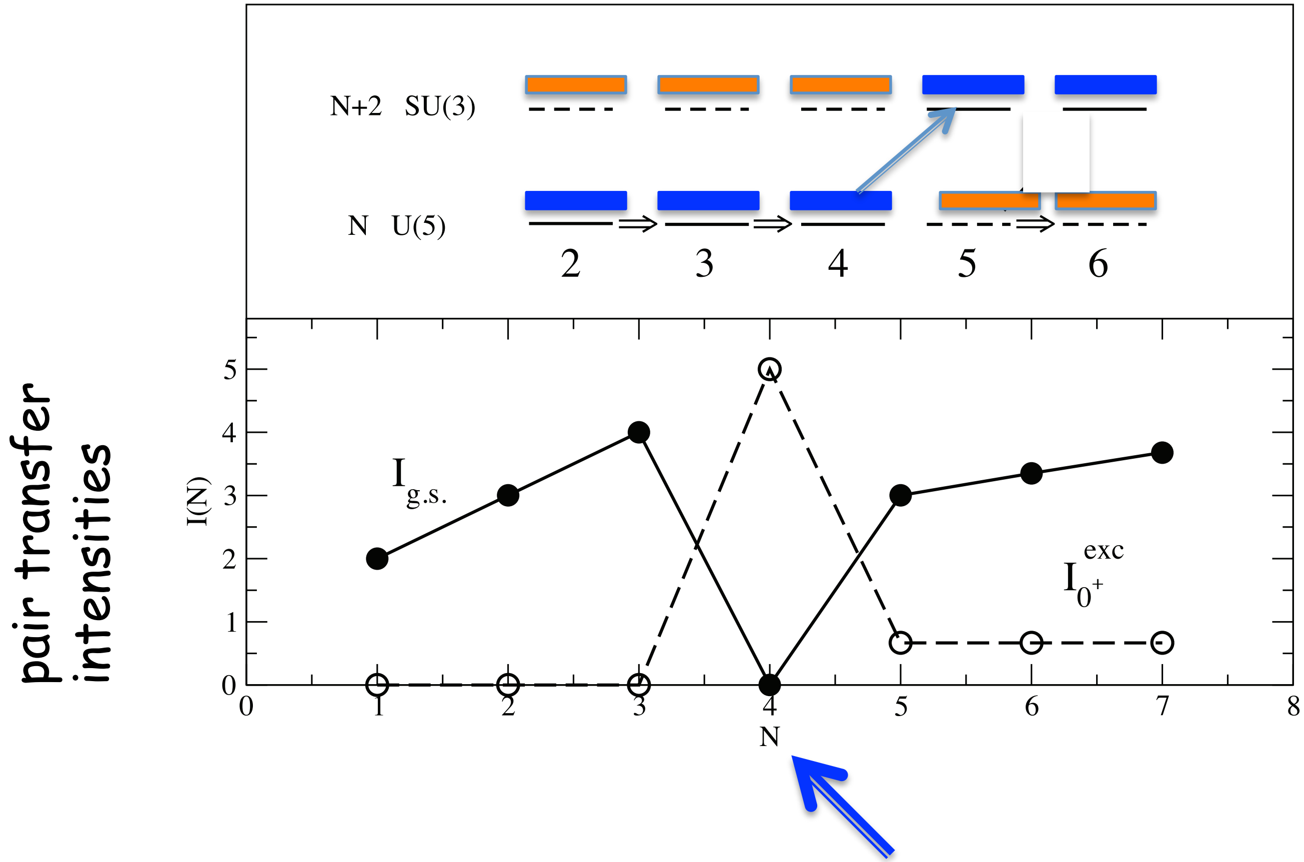
Pair creation probability
N -> N+2



There is a clear signal at the crossing point deformed

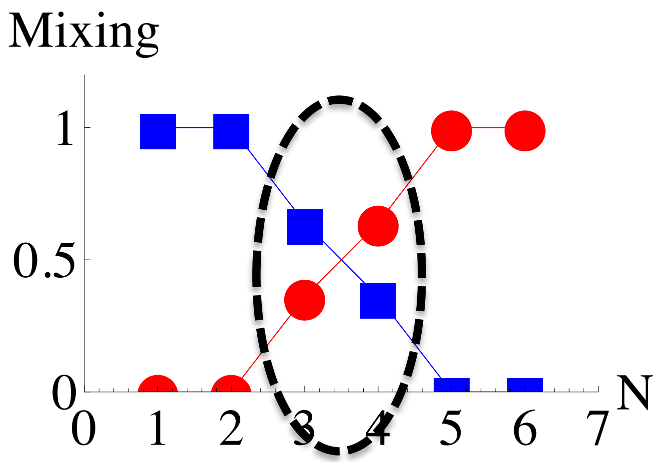
spherical

Another case: crossing between 4 and 5 pairs



As in the previous situation of the standard phase transition a clear discontinuity appears at the critical point. However, the pair strength is always practically concentrated in a single state, without the fragmentation illustrated in the case of the phase transition

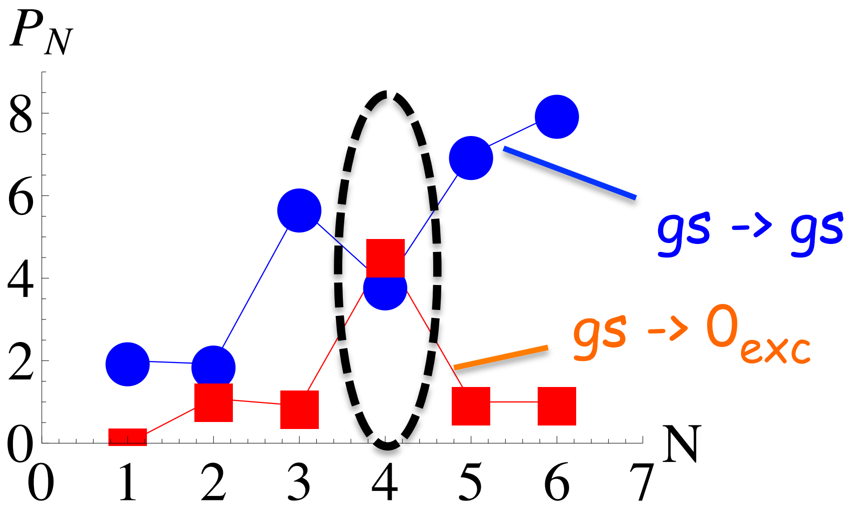
Another case: two-level shape-coexistence with a smoother transition



OBS: Cf. E0 transitions between the two 0+ states

Transfer operator for pair removal : s + s^+ (one can destroy a particle pair or create a hole pair)

Pair removal probability N <- N+2



A more detailed description: approach based on the IBM with extension to configuration mixing (IBM-CM).

The standard IBM is modified to deal also with particle-hole excitation, e.g., 2p-2h excitations. In this case the original Hilbert space based on the N valence bosons is enlarged to $[N] \oplus [N + 2]$. The $[N + 2]$ space corresponds to considering two extra bosons that come from the promotion of a pair of protons across the corresponding shell closure, generating an extra boson made of holes and another made of particles.

The considered Hamiltonian for the case of IBM-CM is

$$H = H_N + H_{N+2} + \Delta + V_{mix}$$

H_N: Hamiltonian in the normal sector

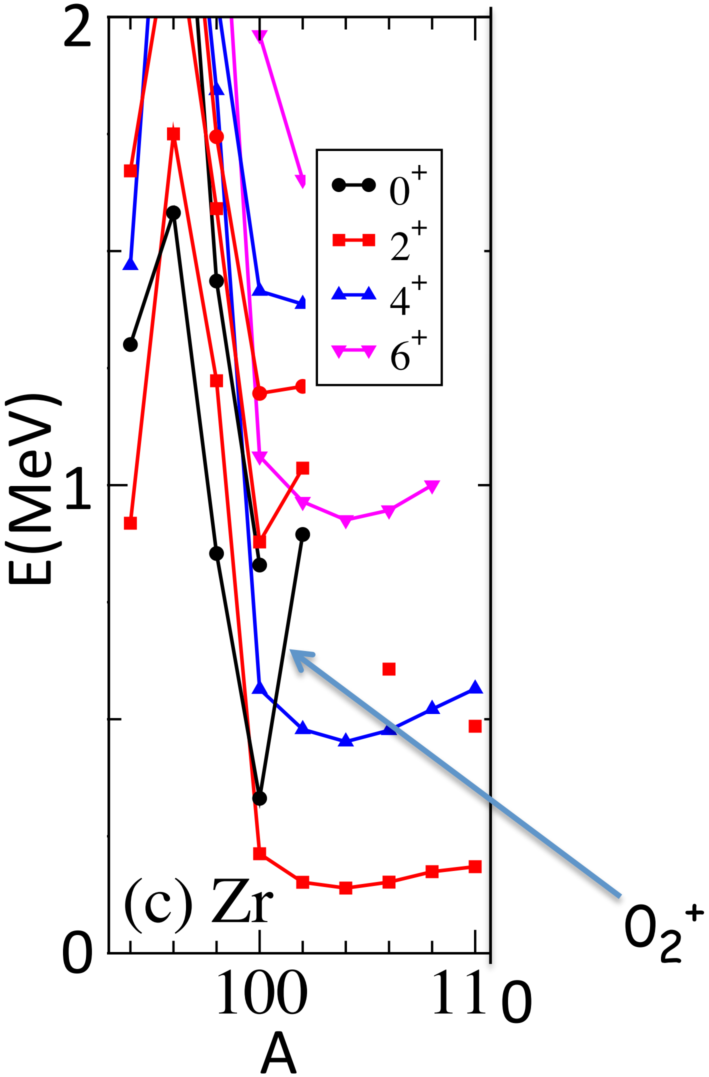
H_{N+2}: Hamiltonian in the intruder sector

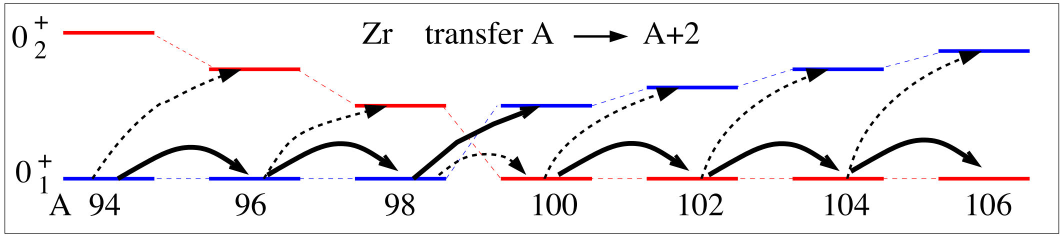
Δ: energy needed to promote a pair across the shell closure

V_{mix}: mixing between the two sectors

The parameters of the Hamiltonians are varied to fit each nucleus .

Zirconium isotopes: a case of shape-coexistence with crossing

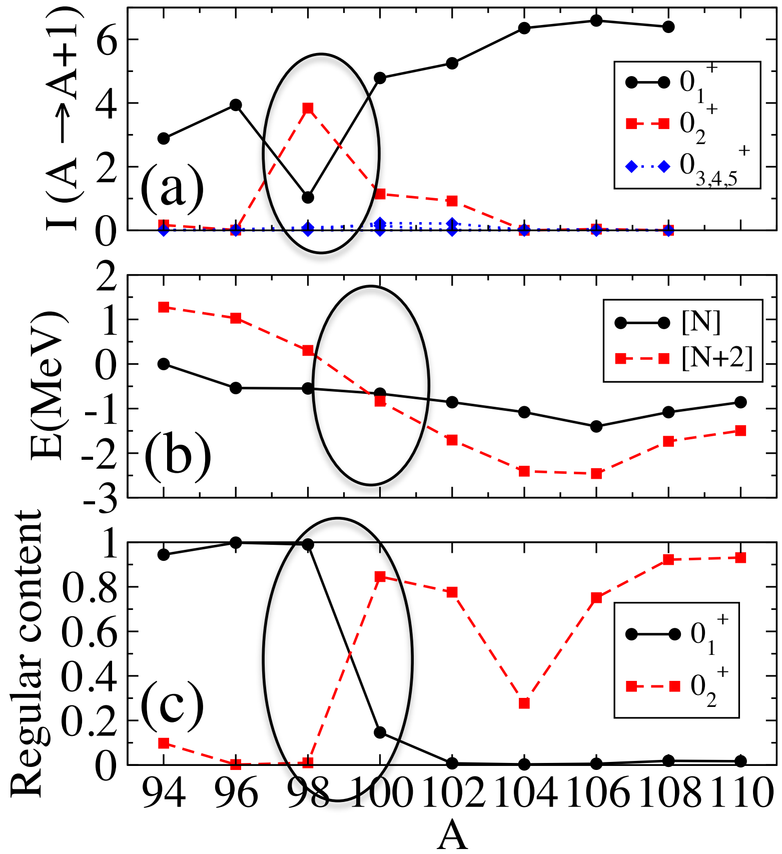




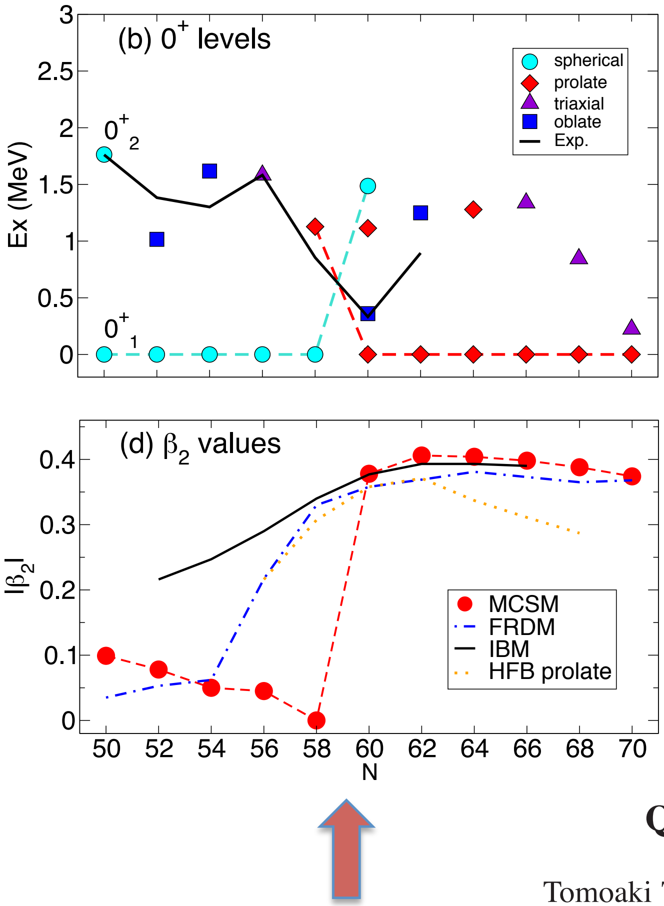
Pair strength

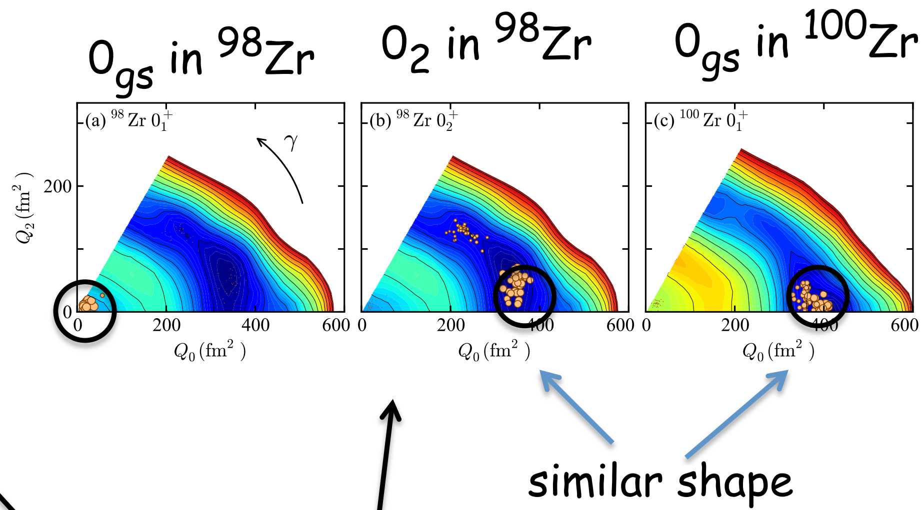
$E(0^+_1, 0^+_2)$

Regular content



The same case in more details (shape phase transition / shape coexistence in Zr isotopes) with full microscopic wave functions coming from Monte Carlo large-scale shell model calculations and "proper" reaction model





Quantum Phase Transition in the Shape of Zr isotopes

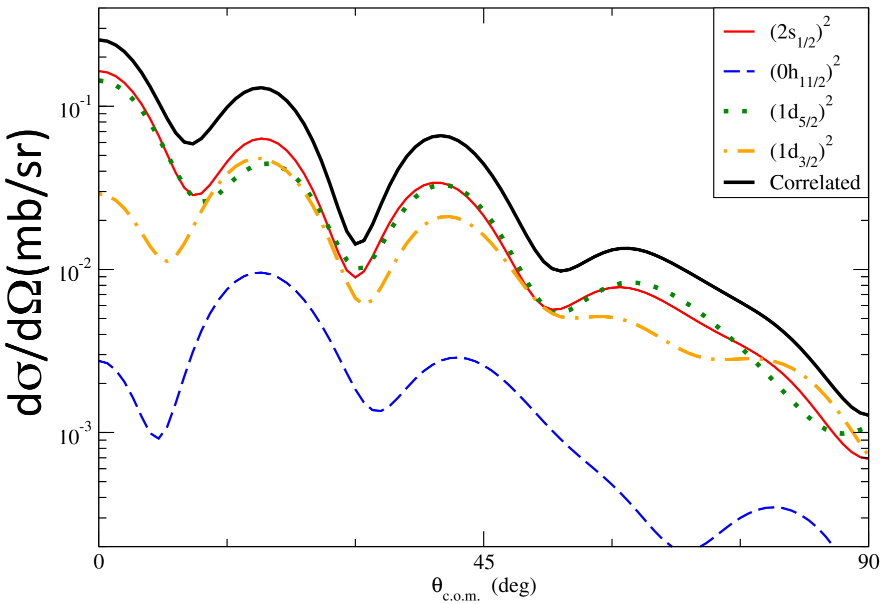
Tomoaki Togashi[1], Yusuke Tsunoda[1], Takaharu Otsuka[1,2,3,4] and Noritaka Shimizu[1]

relevant 2-particle spectroscopic amplitudes

	90>92gs	92>94gs	94>96gs	96>98gs	98>100gs	98>100 ($0+_4$)	100>102gs
d5/2	**0.74**	**0.86**	**0.86**	0.13	0.0	0.16	0.08
S1/2	0.10	0.08	0.10	**0.90**	0.0	0.16	0.05
d3/2	0.13	0.18	0.16	0.07	0.0	**0.90**	0.04
h11/2	0.22	0.20	0.19	0.08	0.0	0.14	**0.55**

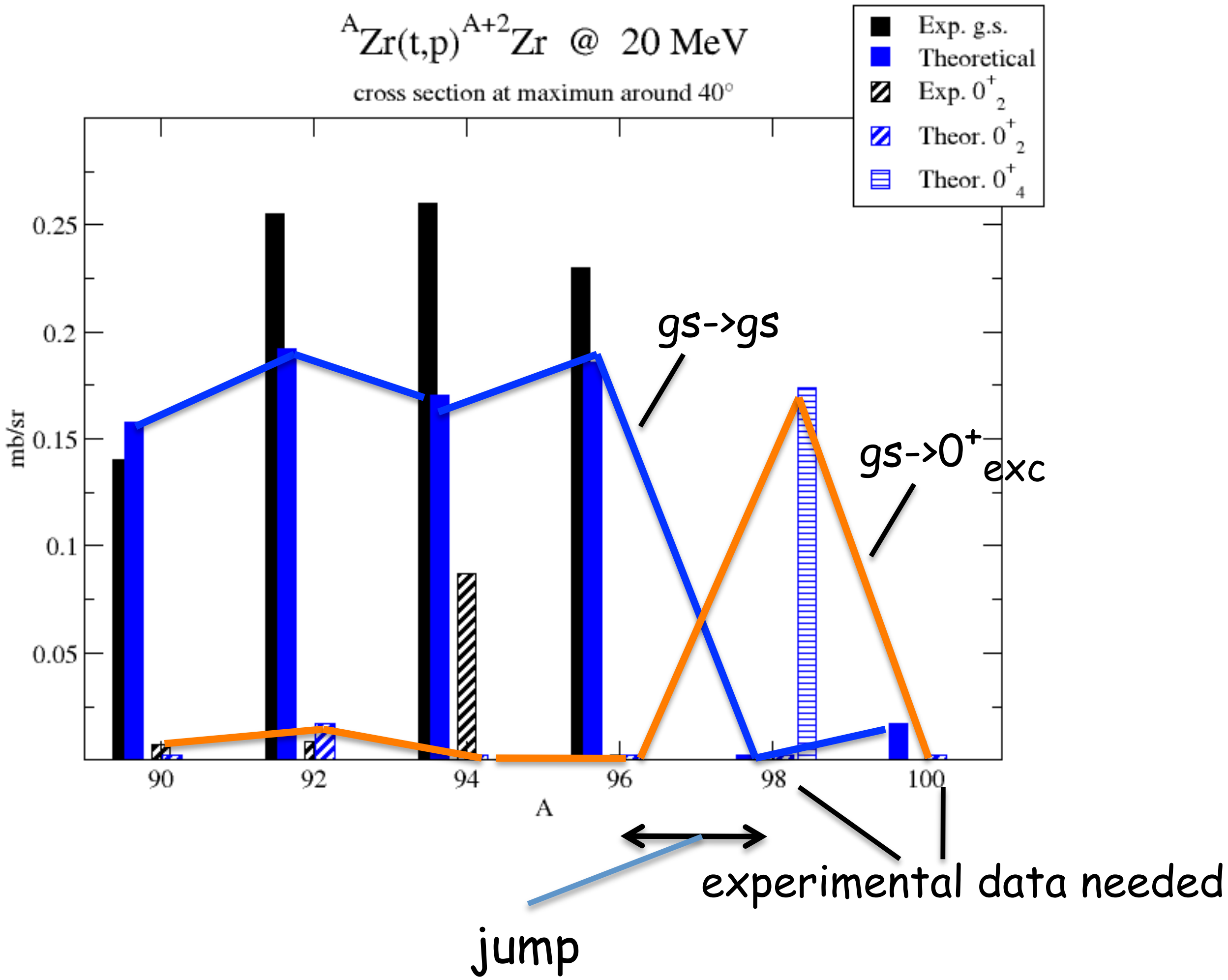
$d_{5/2}$ $s_{1/2}$ $d_{3/2}$ $h_{11/2}$

$^{96}Zr(t,p)^{98}Zr$ @ 30MeV

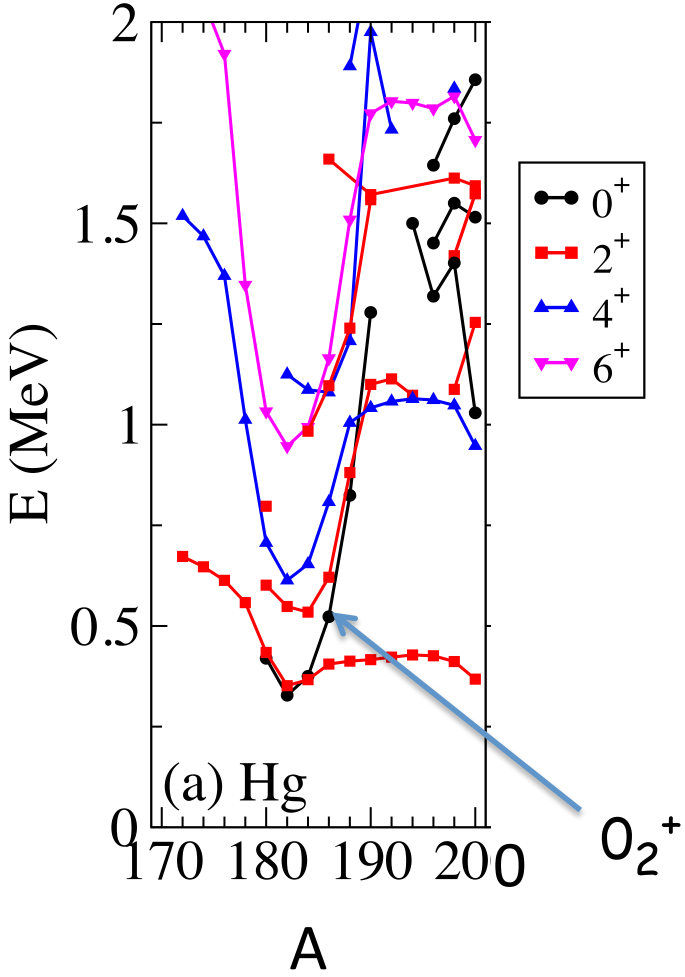


Cross sections for pure configurations

Calculation of two-particle transfer reactions using:
sequential model for the reaction mechanism
one- and two-particle spectroscopic amplitudes from the Tokyo group



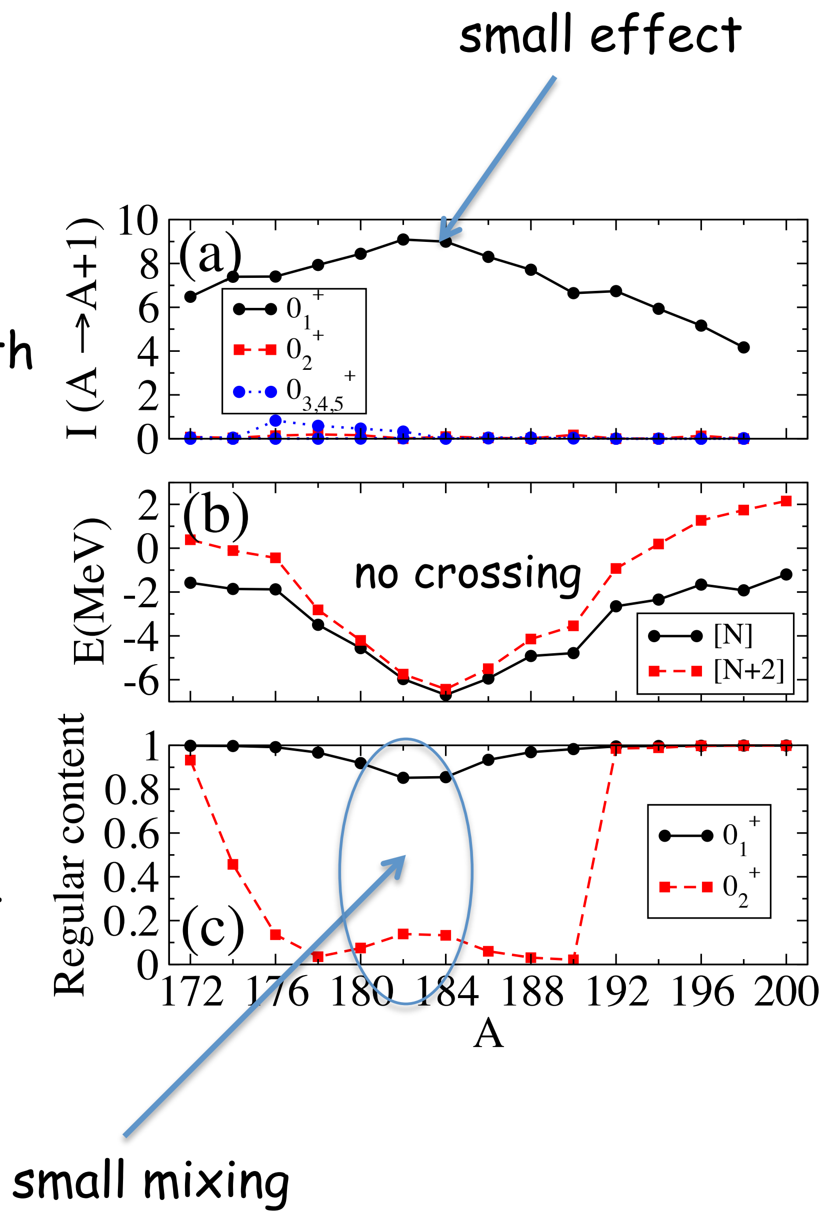
Mercury isotopes: a case of shape-coexistence with no crossing and no mixing



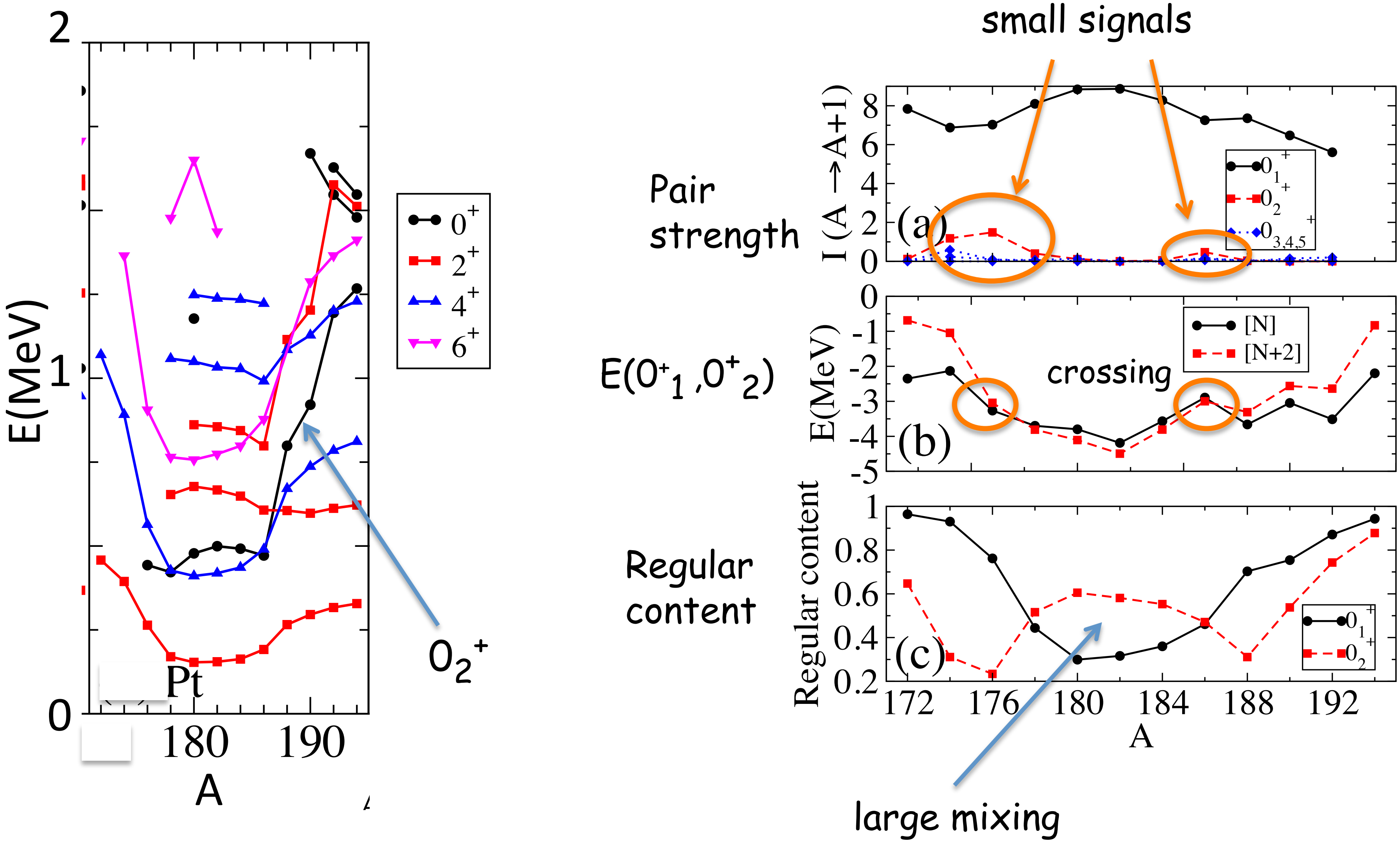
Pair strength

$E(0^+_1, 0^+_2)$

Regular content



Platinum isotopes: a case of shape-coexistence with crossing and large (but stable) mixing, and no significant signal in the pair strength



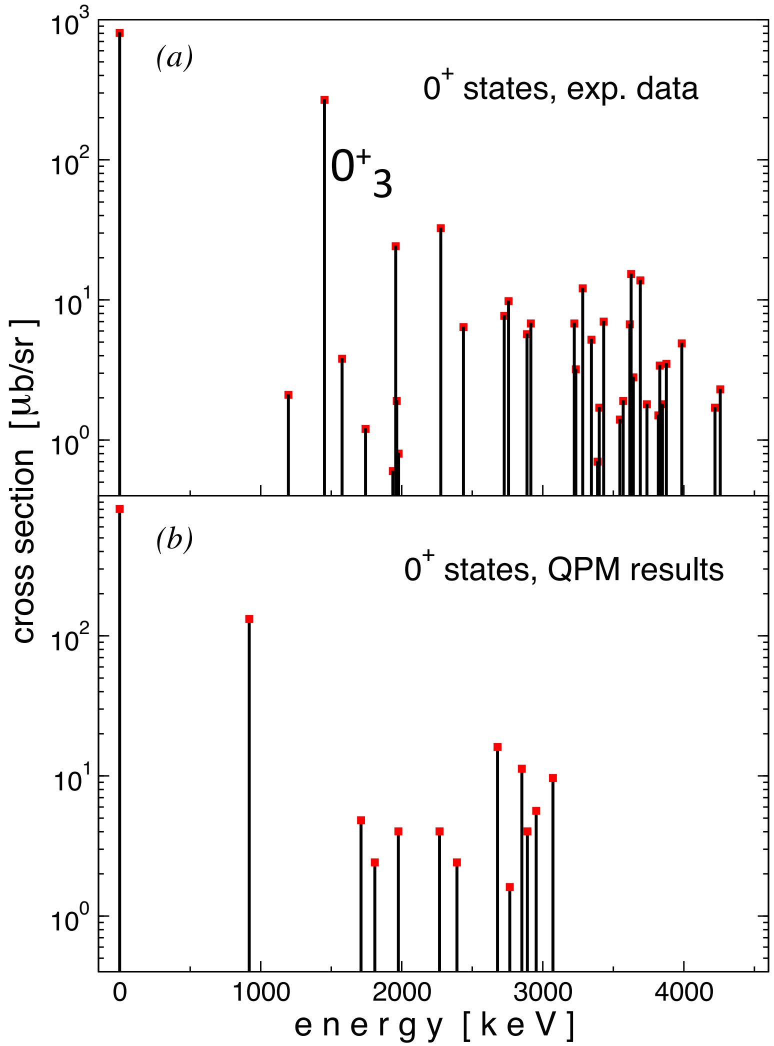
Conclusions:

Pairing response (tested in two-particle transfer reactions but also in other dynamical processes involving pairs of particles) gives strong constraints on nuclear wave functions. The effect is amplified in correspondence of critical situations associated with shape phase transitions or crossing with an intruder state, with "abnormal" population of excited 0+ states and weakening of the ground state transition. In spite of this clear signal, however, it seems difficult to clearly disentangle the shape coexistence picture from the QPT one only using the two-neutron transfer intensity.

Collaborators (on different points):
Jose' Enrique Garcia Ramos, Pepe Arias, Clara Alonso, Lorenzo Fortunato, Jose' Antonio Lay, Praveen Jodidar, Ruben Fossion, Taka Otsuka and Yusuke Tsunoda

The situation is complicated for the large number of 0+ states

$^{160}Gd(p,t)^{158}Gd$



Levon etal
Phys. Rev. C 100, 034307 (2019)