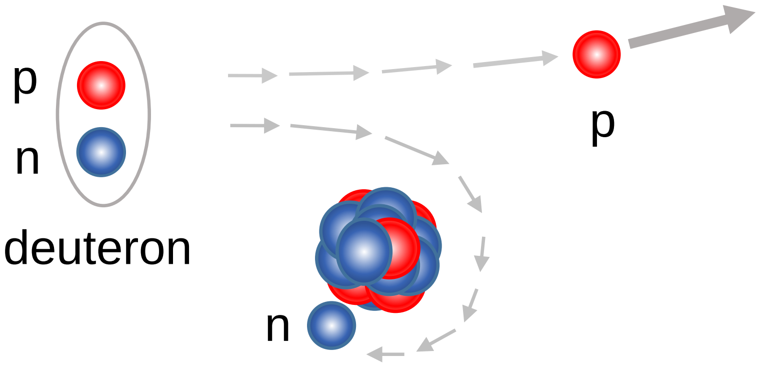
Three-nucleon force in (d,p) reactions

N.K. Timofeyuk



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The starting point for direct reactions theories for $A(a,b)B$ ($\alpha \to \beta$):

$$\left(T + \sum_{i<j} V_{ij} - E\right)\Psi = 0$$

After projecting total many-body wave function into a specific exit channel and introducing auxiliary two-body potentials U_α and U_β and after some manipulations we get

Transition amplitude for $\alpha \to \beta$ reaction

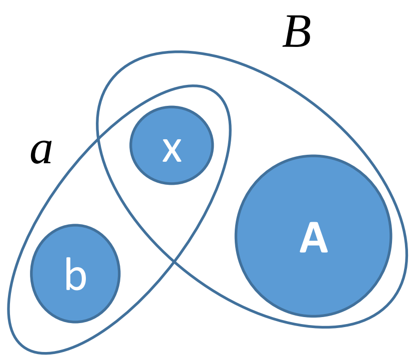
Distorted wave Born approximation

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_\beta - U_\beta(\boldsymbol{r}_\beta) \middle| \chi_\alpha^{(+)} \psi_a \psi_A \right\rangle$$

$$+ \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| \left(V_\beta - U_\beta(\boldsymbol{r}_\beta)\right) \tilde{G}_\alpha^{(+)} \left(V_\alpha - U_\alpha(\boldsymbol{r}_\alpha)\right) \middle| \Psi_\alpha^{(+)} \right\rangle$$

N. Austern, *Direct nuclear reaction theories* (1970)
G. R. Satchler, *Direct reaction theories* (1983)

$$V_\beta = V_{bB} = \sum_{i \in b} \sum_{j \in B} v_{ij} = \sum_{i \in b} \left\{ \sum_{j \in x} + \sum_{j \in A} \right\} v_{ij} = V_{bx} + V_{bA} .$$



1939: Primakoff and Holstein, PR55, 1218 (1939)

Can field interactions be substituted by two-body interactions?

Conclusion:

These investigations indicate that the replacement of field interactions by two-body action-at-a-distance potentials is a poor approximation in nuclear problems. The error made is at least of the order of v_n /c, if one compares the magnitudes, term by term, of two- and three-body potentials. Furthermore, the number of terms in the m-body interaction of an n-body nucleus: n!/m!(n-m)!,—is, in general, many times larger than the number of two-body interaction terms: n!/2!(n-2)!=n(n-1)/2, but, a direct estimate of the magnitude of the total m-body interaction

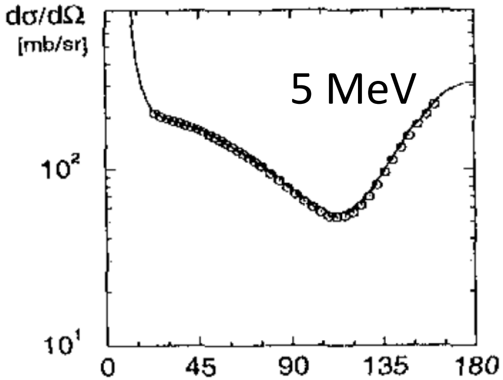
((number of m-body terms) × (average magnitude of each))

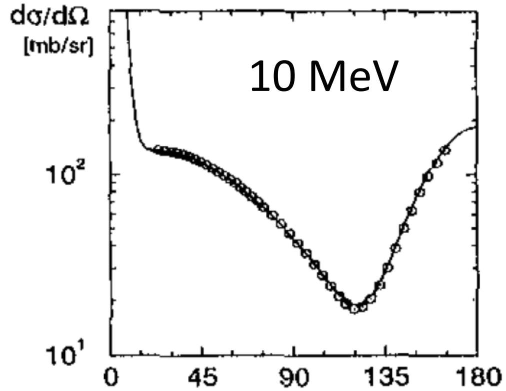
is complicated by the fact that the m-body interactions are, at least in part, of an exchange and spin-dependent character. ***It seems therefore, that a satisfactory description of nuclei, other than the deuteron, can be obtained only by an explicit consideration from the very beginning of the role played by the field.***

1957: Fujita and Miyazawa, PTP17, 360 (1957).

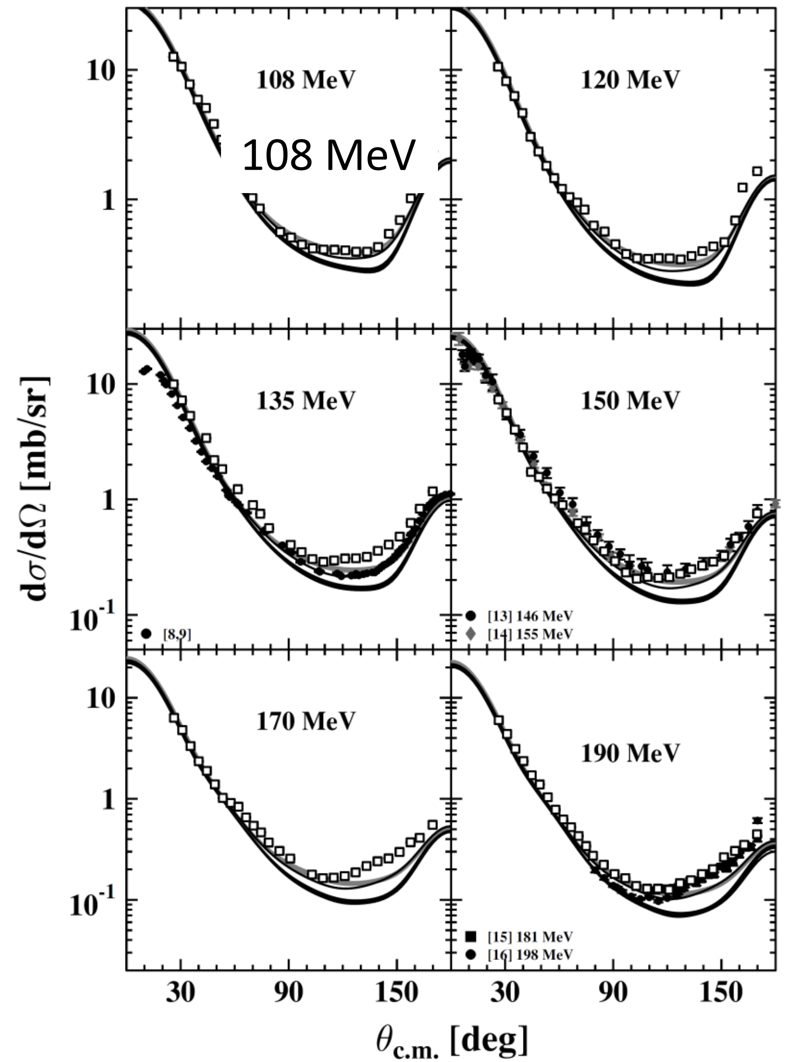
The origin of the 3N force is attributed to virtual excitations of a Delta-resonance when three nucleons interact via a pion exchange.

Role of 3NF in p-d scattering:





A.Kievsky, NPA689, 361c (2001)

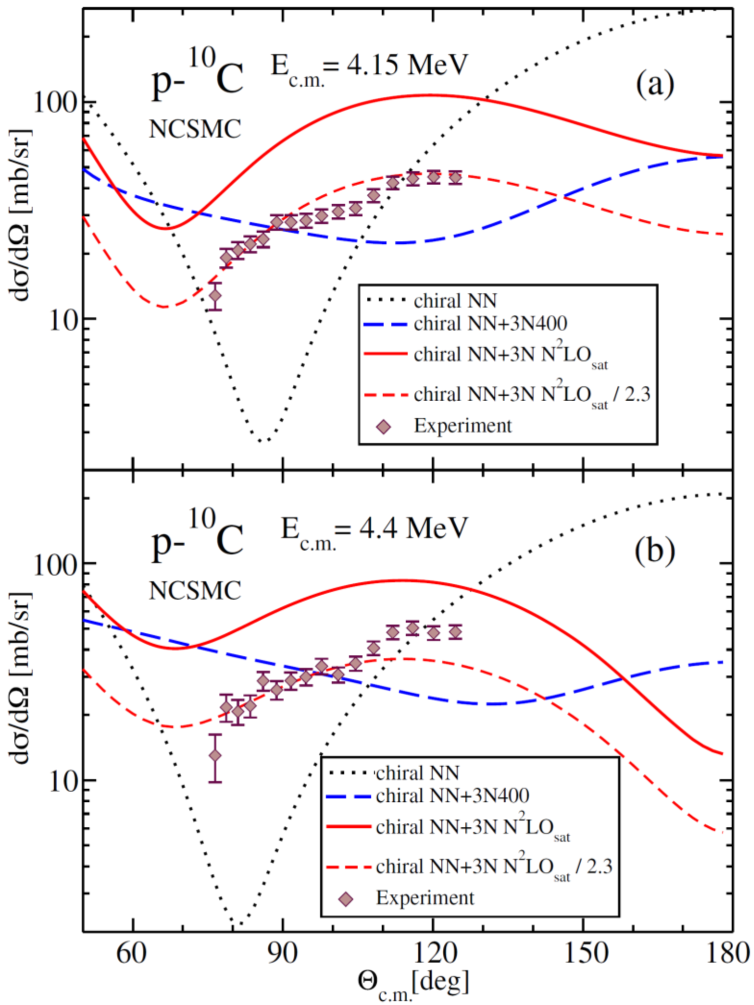


K.Ermisch et al, PRC71, 064004 (2005)

No influence at small energies.

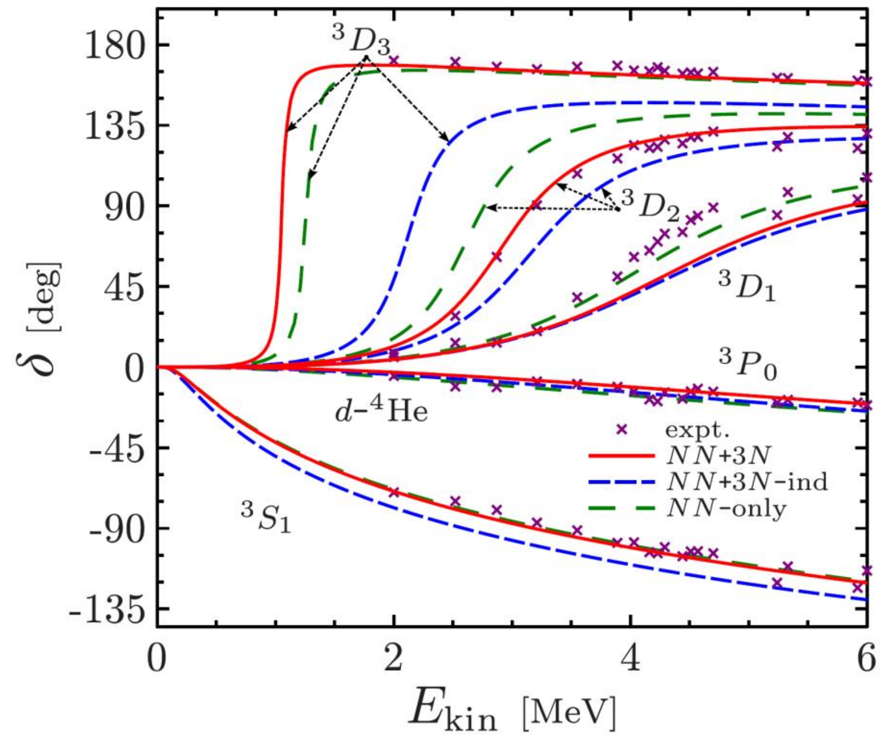
3NF contribution becomes noticeable at E > 100 MeV.

3N force in ab-initio scattering calculations for A > 4



A. Kumar et al, Phys.Rev.Lett. 118, 262502 (2017)

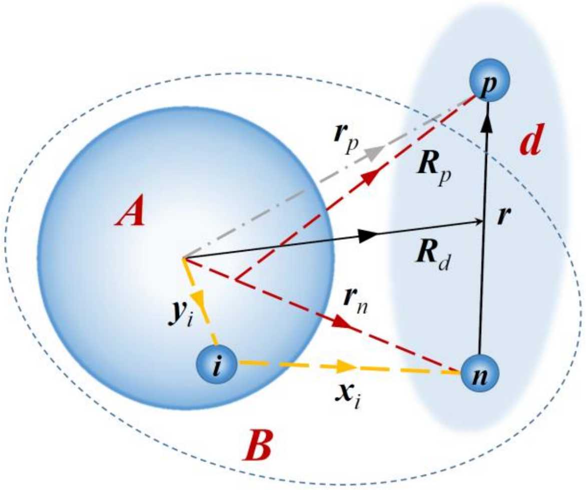
alpha - d scattering



G. Hupin, S. Quaglioni, P. Navratil, Phys.Rev.Lett. 114, 212502 (2015)

What changes does 3N force bring?

$$\left(T+\sum_{i<j} V_{ij}+\sum_{i<j<k} V_{ijk}-E\right)\Psi=0$$



Distorted wave theories: DWBA and ADWA

$$T_{(d,p)}=T_{(d,p)}^{2N}+T_{(d,p)}^{3N}$$

$$T_{(d,p)}^{2N}=\langle\Psi_{M_B}^{J_B}\psi_{M_p}^{J_p}\chi_{\boldsymbol{k}_p}(\boldsymbol{R}_p)|V_{np}(\boldsymbol{r})|\Psi_{M_A}^{J_A}\psi_{M_d}^{J_d}\chi_{\boldsymbol{k}_d}(\boldsymbol{R}_d)\rangle$$

phenomenological, 3N should be there

Hamiltonian with 3N contribution includes new term:

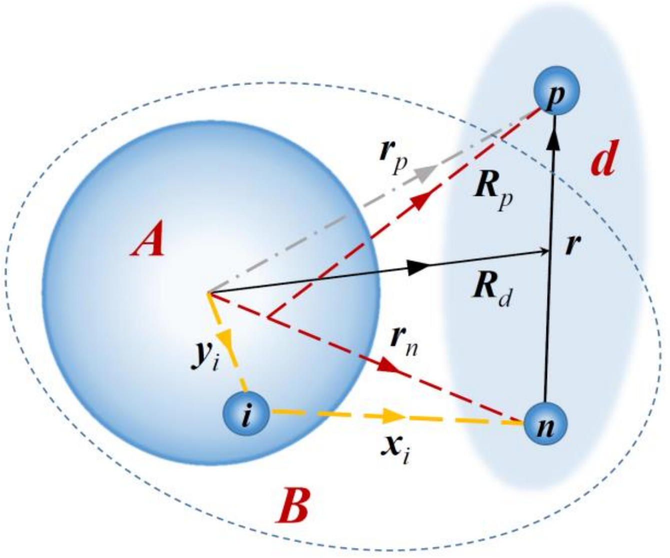
$$T_{(d,p)}^{3N}=\langle\Psi_{M_B}^{J_B}\psi_{M_p}^{J_p}\chi_{\boldsymbol{k}_p}(\boldsymbol{R}_p)|\sum_{i\in A} W_{ipn}|\Psi_{M_A}^{J_A}\psi_{M_d}^{J_d}\chi_{\boldsymbol{k}_d}(\boldsymbol{R}_d)\rangle$$

Difference between 2N and 3N contributions:

2N term contains overlap integral

$$\langle \psi_B | \psi_A \rangle = I(\boldsymbol{r}_n)$$

3N term contains different matrix element:



$$\left\langle \psi_B \left| \sum_{i \in A} W_{inp} \right| \psi_A \right\rangle = F(\boldsymbol{r}_n, \boldsymbol{R}_p)$$

It is not proportional to the overlap integral and the corresponding cross section cannot be factorized via a single spectroscopic factor!

Assumptions are needed to proceed with evaluating 3N contributions:

- 3N force is contact

$$W_{ijk} = I_3[(\tau_i \cdot \tau_j)\delta(r_{ik})\delta(r_{jk}) + (\tau_i \cdot \tau_k)\delta(r_{ij})\delta(r_{jk}) + (\tau_k \cdot \tau_j)\delta(r_{ik})\delta(r_{ji})]$$

- Nucleus A is a double-closed shell
- A and B are described by Hartree-Fock model
- Single-particle wave functions in A and B are the same
- The difference in centre-of-mass positions in A and B is neglected

With these assumptions, available DW codes can be used in which overlap function should be modified:

3N force strength

Deuteron w.f. at $r_{np} = 0$

$$I_{lj}^{\rm mod}(r) = I_{lj}(r)\left[1 + \frac{I_3\psi_d(0)}{D_0}\left(\rho_A(r) - \tfrac{1}{2}\rho_A^{(\rm n)}(r)\right)\right]$$

Overlap function (HF s.p.w.f.)

Density of nucleus A

Neutron density of nucleus A

$$D_0 = \int d\boldsymbol{r}\, V_{np}(\boldsymbol{r})\varphi_d(\boldsymbol{r})$$

D_0 is independent of NN model

$\psi_d(0)$ is model-dependent:

NN Model	Ref	$\psi_d(0)/Y_{00}(\hat{\boldsymbol{r}})$
Reid soft core	[17]	0
Argonne V18	[18]	0.079
CD-Bonn	[19]	0.30
χEFT N4LO: 0.8 fm	[20]	-0.22
χEFT N4LO: 0.9 fm	[20]	-0.11
χEFT N4LO: 1.0 fm	[20]	-0.026
χEFT N4LO: 1.1 fm	[20]	0.062
χEFT N4LO: 1.2 fm	[20]	0.14
χEFT N2LO: 1.0 fm	[21]	0.282

Finite-range effects in Plane-Wave Born Approximation

$$T_{(d,p)}^{3N} = \langle \Psi_{M_B}^{J_B} \psi_{M_p}^{J_p} \chi_{\boldsymbol{k}_p}(\boldsymbol{R}_p) | \sum_{i \in A} W_{ipn} | \Psi_{M_A}^{J_A} \psi_{M_d}^{J_d} \chi_{\boldsymbol{k}_d}(\boldsymbol{R}_d) \rangle$$

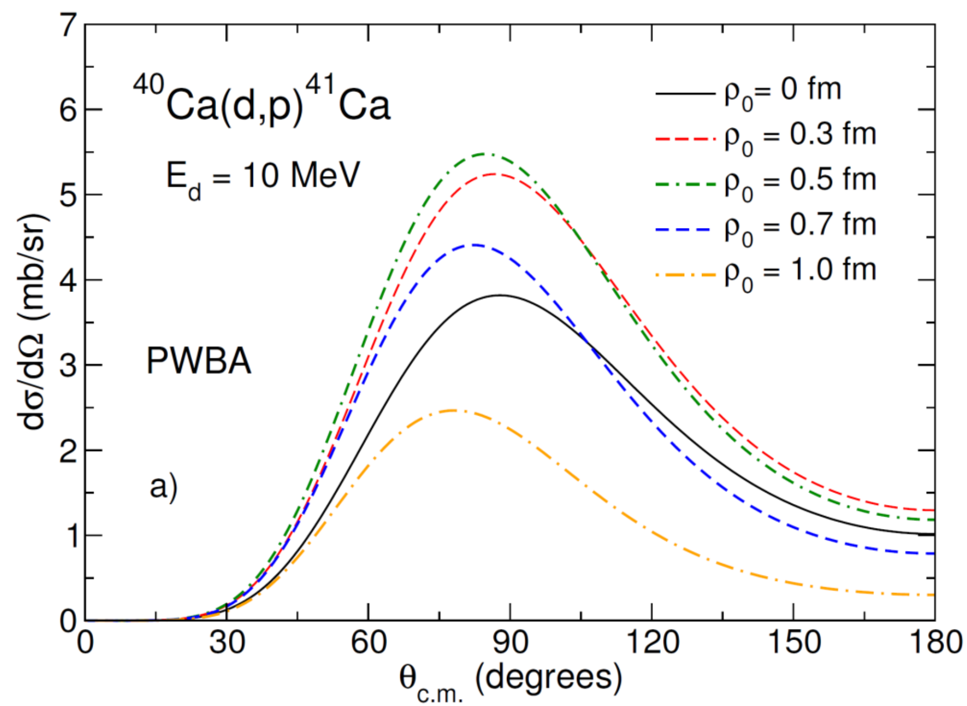
using plane waves

$$W_{ijk} = W_0 e^{-\frac{1}{3}\frac{r_{ij}^2 + r_{jk}^2 + r_{ki}^2}{\rho_0^2}}$$

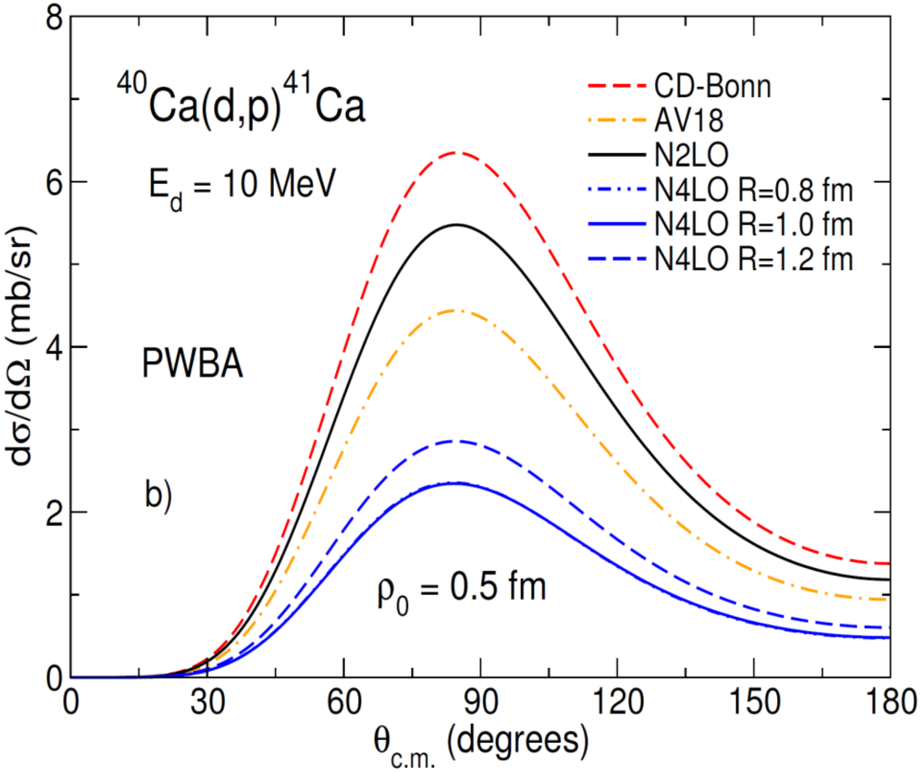
Fixed volume integral

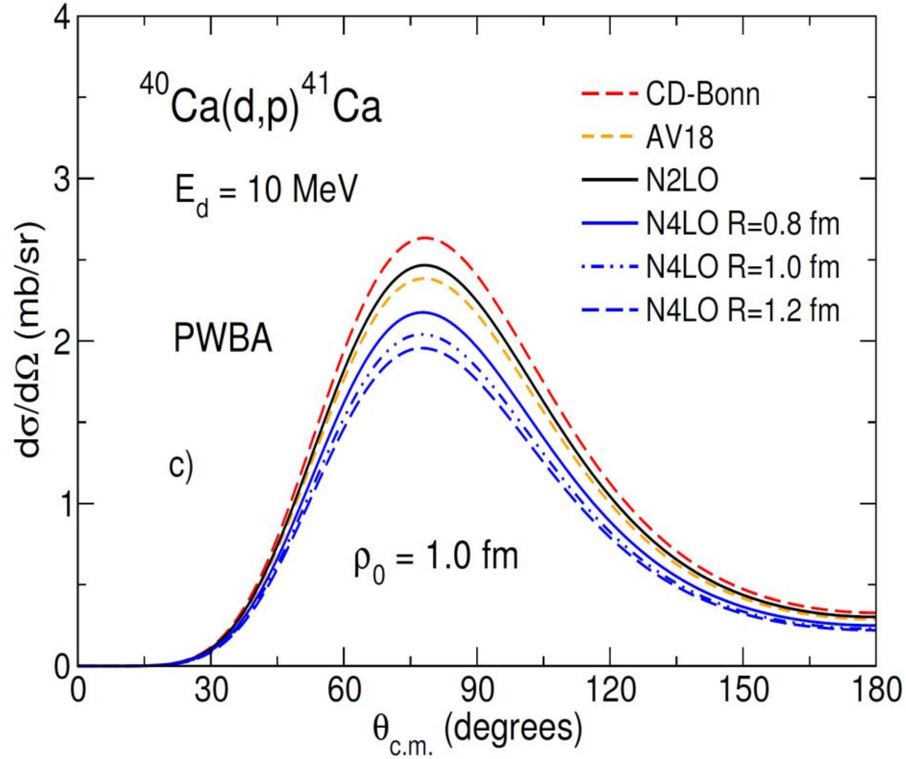
$$I_3 = \int d\boldsymbol{r}_{12} d\boldsymbol{r}_{13} W(\rho_{ijk})$$

Deuteron wave function:
χEFT at N2LO



Sensitivity to the deuteron wave function for a fixed 3N force





N. K. Timofeyuk, Phys. Rev. C 97, 054601 (2018)

Second source of 3N effects in (d,p) reactions : *deuteron breakup*

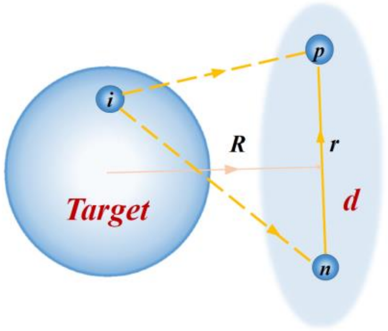
In the Adiabatic Distorted Wave Approximation **(ADWA)** the d-A distorted wave function is found from equation:

$$\left(T_{dA} + \langle \varphi_1 | U_{nA} + U_{pA} | \varphi_d \rangle - E_d\right)\chi_{dA}^{(+)}(\boldsymbol{R}) = 0$$

$$\varphi_1 = \frac{V_{np}\varphi_d}{\langle \varphi_d | V_{np} | \varphi_d \rangle}$$

Adding 3N force modifies this equation:

$$\left(T_{dA} + \langle \varphi_1 | U_{nA} + U_{pA} | \varphi_d \rangle + \langle \varphi_A \varphi_1 | \textstyle\sum_{ijk} V_{ijk} | \varphi_A \varphi_d \rangle - E_d\right)\chi_{dA}^{(+)}(\boldsymbol{R}) = 0$$

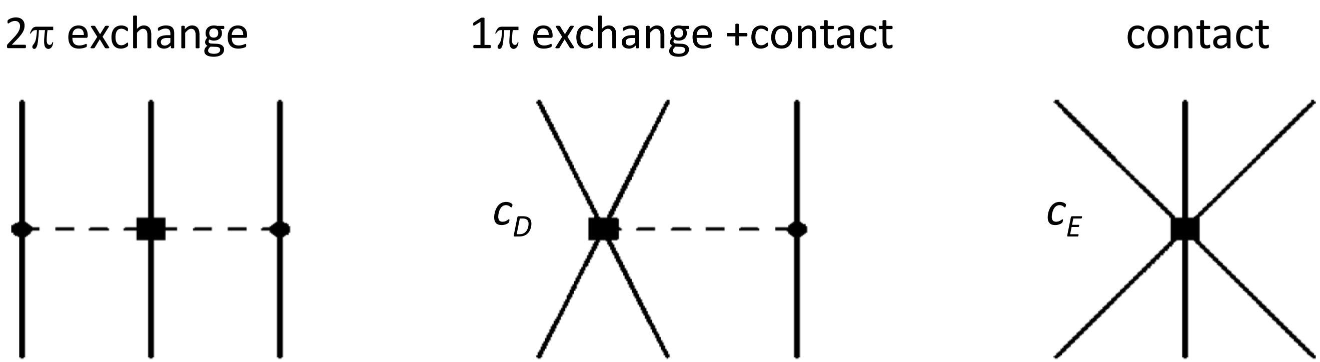


It is very important to choose 3N force that is consistent with 2N interaction that generates deuteron wave function.

N.K. Timofeyuk, M.J. Dinmore and J.S. Al-Khalili, Phys. Rev. C **102, 06461** (2020)

3N force in Chiral Effective Field Theory (χEFT).

At next-to-next-to-leading (N2LO) order

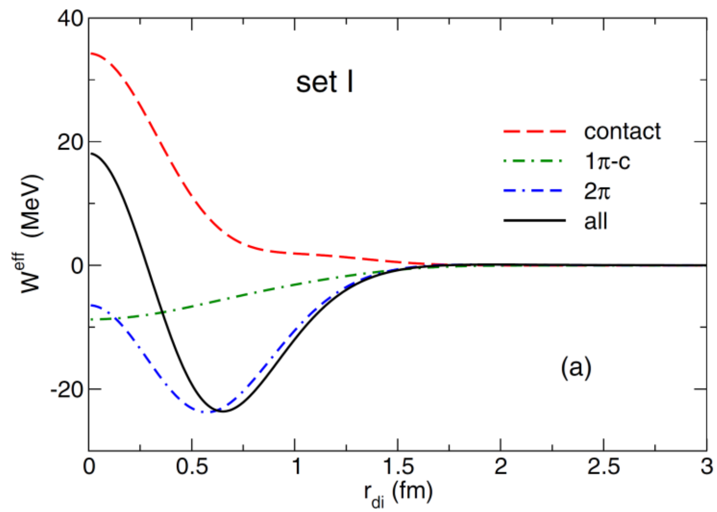


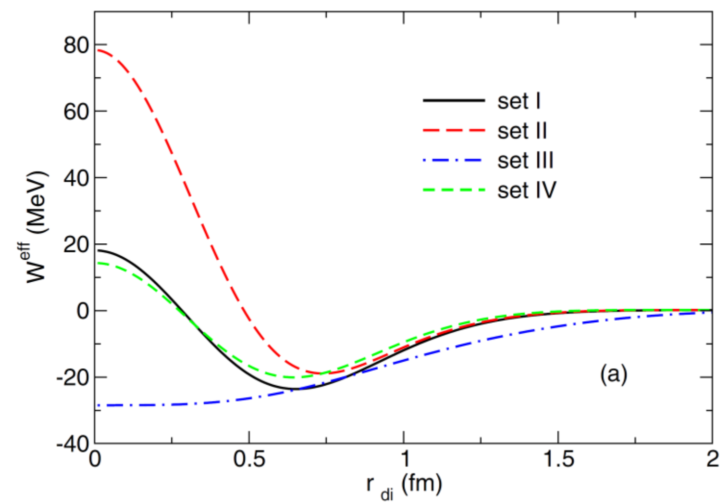
Local version of the χEFT at N2LO from *J.E.Lynn et al* [*PRL 116 062501(2016)*] has been used with 4 sets of low-energy constants. They were fitted to describe light nuclei, neutron-alpha scattering and neutron matter.

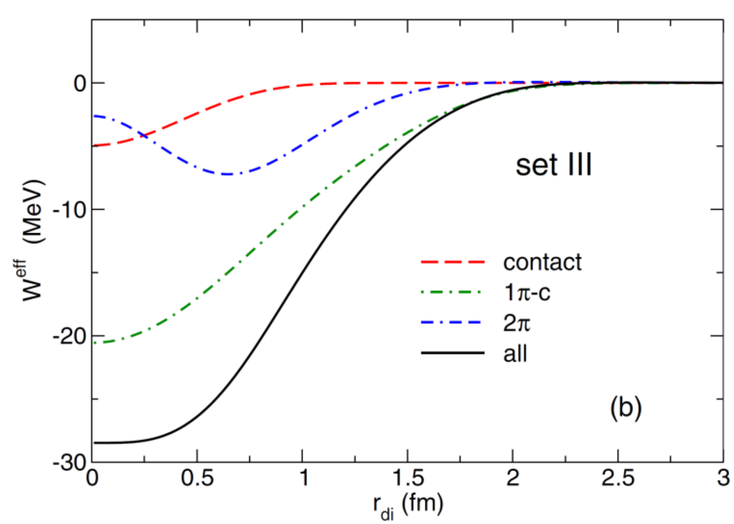
Set	Format	R_{3N}	c_E	c_D
I	$E1$	1.0	0.62	0.5
II	$E\tau$	1.0	−0.63	0.0
III	$E\tau$	1.2	0.09	3.5
IV	$E_{\mathcal{P}}$	1.0	0.59	0.0

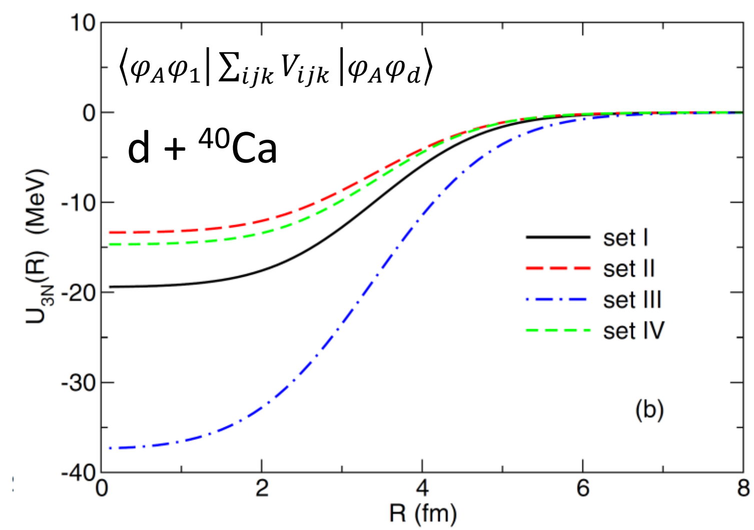
When calculating $\langle \varphi_A \varphi_1 | \sum_{ijk} V_{ijk} | \varphi_A \varphi_d \rangle$ we first integrate over *n-p* distance r:

$$W_{di}^{\text{eff}}(r_{di}) = \langle \varphi_1 | \sum_{ijk} V_{ijk} | \varphi_d \rangle$$



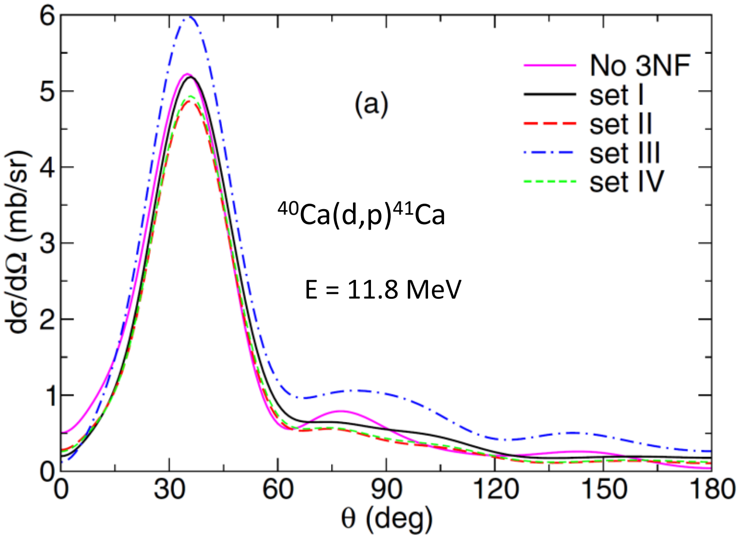


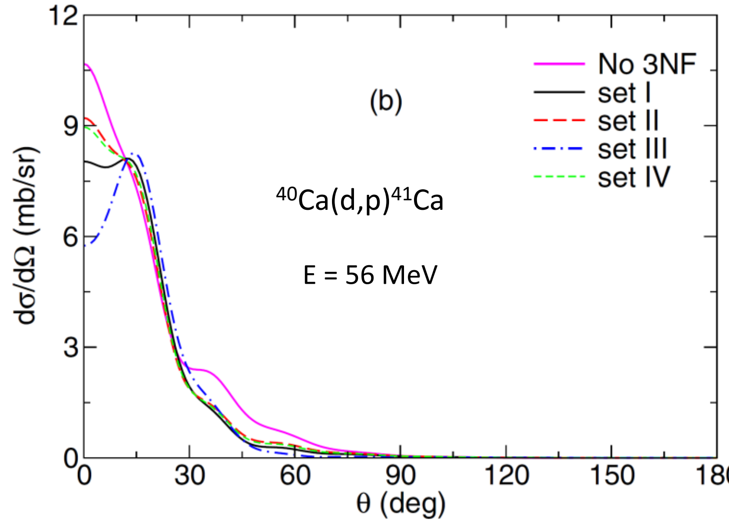


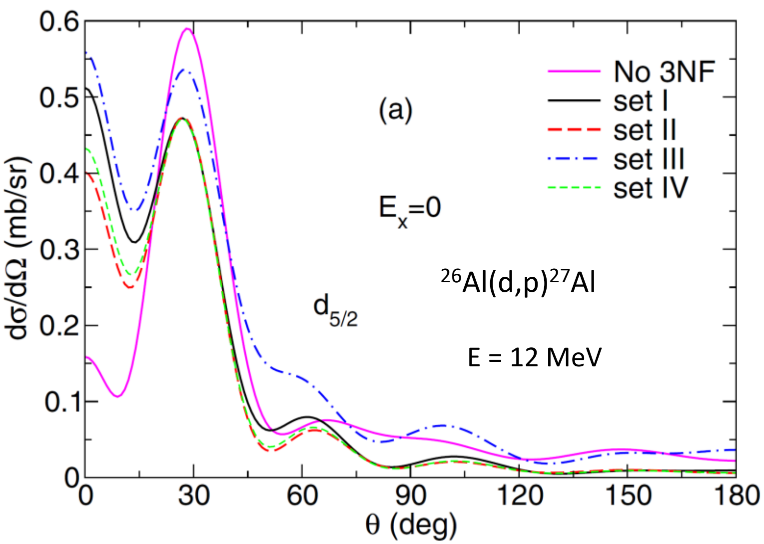


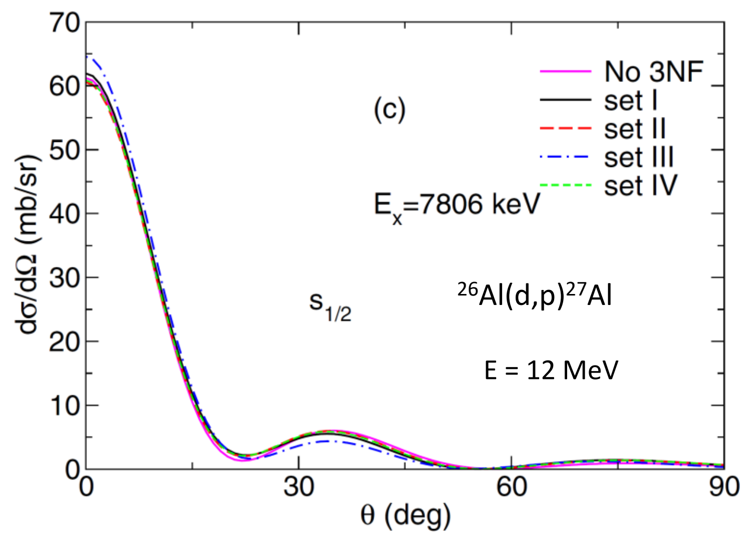
N.K. Timofeyuk, M.J. Dinmore and J.S. Al-Khalili, Phys. Rev. C **102, 06461** (2020)

(d,p) results in ADWA with global *N-A* optical potential KD03

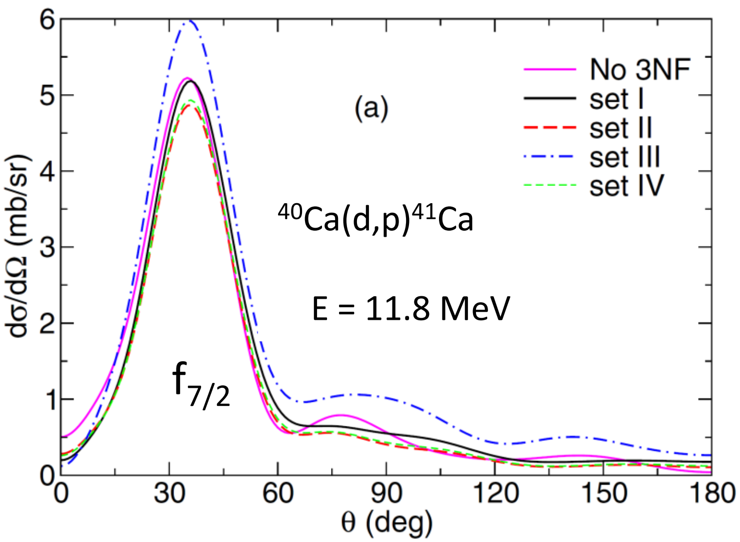


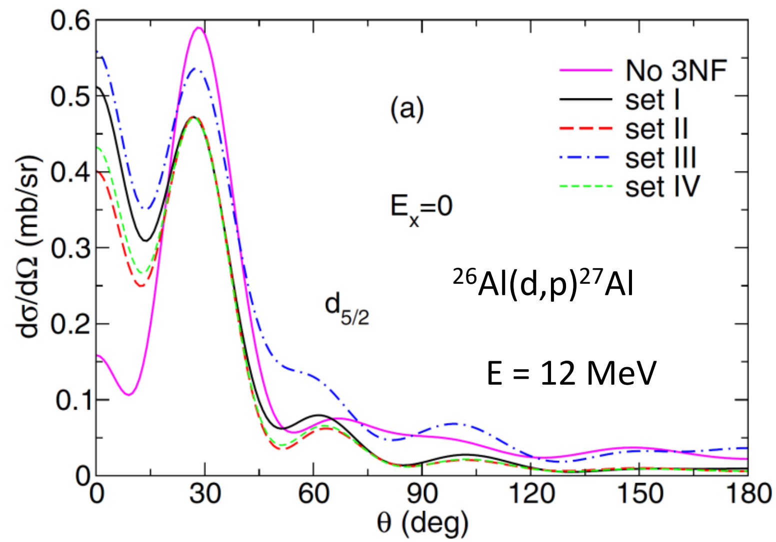




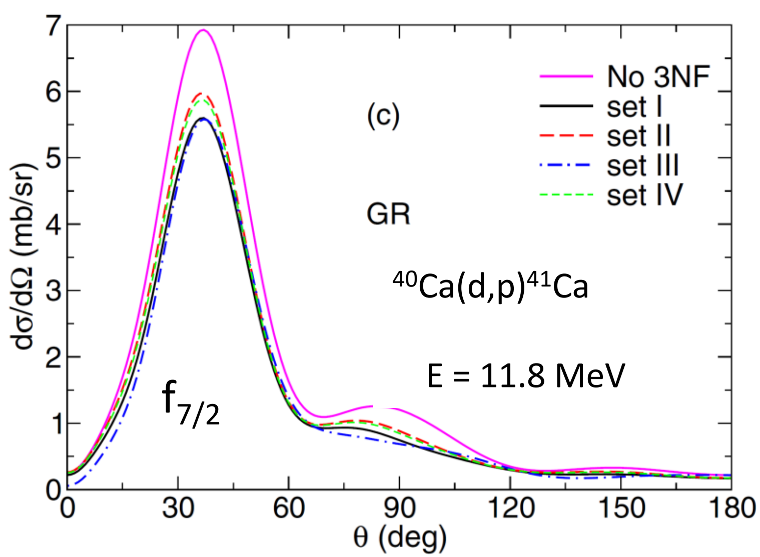


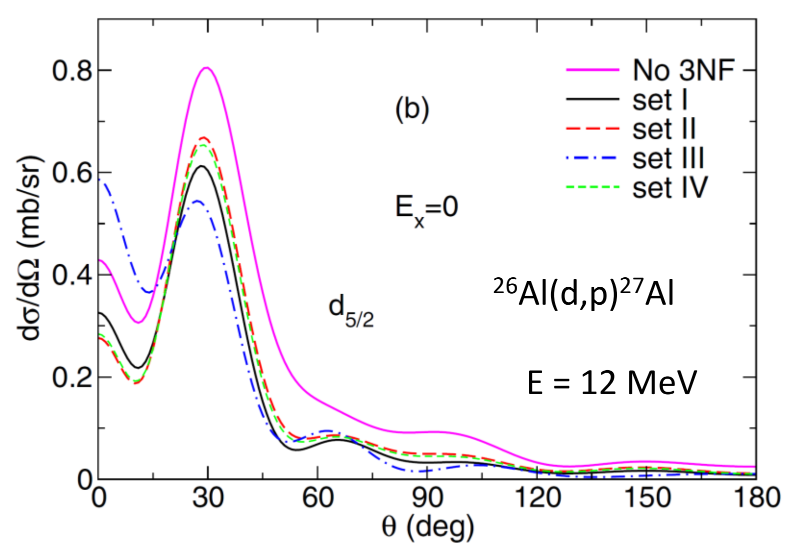
ADWA with local optical potentials: KD03



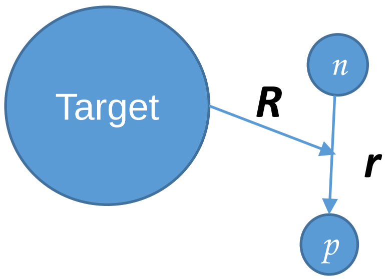


ADWA with non-local energy-independent optical potential: Giannini-Ricco





Optical potentials in the A + n + p three-body model



The total system is a many-body object

$$\Psi = \phi_{g.s.}\chi_0(\boldsymbol{r},\boldsymbol{R}) + \sum_{i\neq 0}\phi_i\chi_i(\boldsymbol{r},\boldsymbol{R})$$

$$\Psi_P = P\Psi \qquad \Psi_Q = Q\Psi$$

$$Q = \sum_{i\neq 0}|\phi_i\rangle\langle\phi_i|$$

Excluding excited states following Feshbach ideas we find that the ground-state channel function can be found from a three-body model

$$(T_3 + V_{np} + \langle\phi_{g.s.}|V_{opt}|\phi_{g.s.}\rangle - E_3)\chi_0 = 0$$

with the optical potential

$$V_{opt} = U_{nA} + U_{pA} + U_{nA}\frac{Q}{e}U_{pA} + U_{pA}\frac{Q}{e}U_{nA} + \ldots$$

$$U_{NA} = v_{NA} + v_{NA}\frac{Q}{e}U_{NA}$$

Optical potential for 3-body system has two-body and three-body terms

R.C. Johnson and N.K. Timofeyuk, PRC 89, 024605 (2014)

Neglecting multiple scattering in ADWA:

$$\left(T_{dA} + \langle \varphi_1 \varphi_A | U_{nA} + U_{pA} | \varphi_d \varphi_A \rangle - E_d\right) \chi_{dA}^{(+)}(\boldsymbol{R}) = 0$$

Averaging procedure gives

$$\langle \varphi_1 \varphi_A | U_{NA} | \varphi_d \varphi_A \rangle \approx \left\langle \varphi_A \middle| v_{NA} + v_{NA} \frac{Q}{E_{\text{eff}} + i0 - T_N - (H_A - E_A)} U_{NA} \middle| \varphi_A \right\rangle$$

where $$E_{eff} = \frac{1}{2} E_d + \frac{1}{2} \frac{\langle \varphi_d | V_{np} T_{np} | \varphi_d \rangle}{\langle \varphi_d | V_{np} | \varphi_d \rangle}$$

← half the n-p kinetic energy in deuteron ranges between 44 and 120 MeV

Comparing to the N-A optical potential:

$$\langle \varphi_A | U_{NA} | \varphi_A \rangle \approx \left\langle \varphi_A \middle| v_{NA} + v_{NA} \frac{Q}{E_N + i0 - T_N - (H_A - E_A)} U_{NA} \middle| \varphi_A \right\rangle$$

Three-body problem for (d,p) reactions should be solved with energy-independent nonlocal nucleon potentials taken at effective energy equal to half the deuteron energy plus a shift.

The optical operator contains multiple scattering to all orders:

$$V_{opt} = \underbrace{U_{nA} + U_{pA}}_{U^{(0)}} + \underbrace{U_{nA} \frac{Q}{e} U_{pA} + U_{pA} \frac{Q}{e} U_{nA}}_{U^{(1)}} + \ldots$$

Including multiple scattering effects in the leading order within the ADWA:

$$\langle \varphi_1 \varphi_A | U^{(0)} + U^{(1)} | \varphi_d \varphi_A \rangle \approx 2 \langle \varphi_1 \varphi_A | U^{(0)} | \varphi_d \varphi_A \rangle - \sum_{N=n,p} \langle \varphi_1 \varphi_A | v_{NA} | \varphi_d \varphi_A \rangle$$

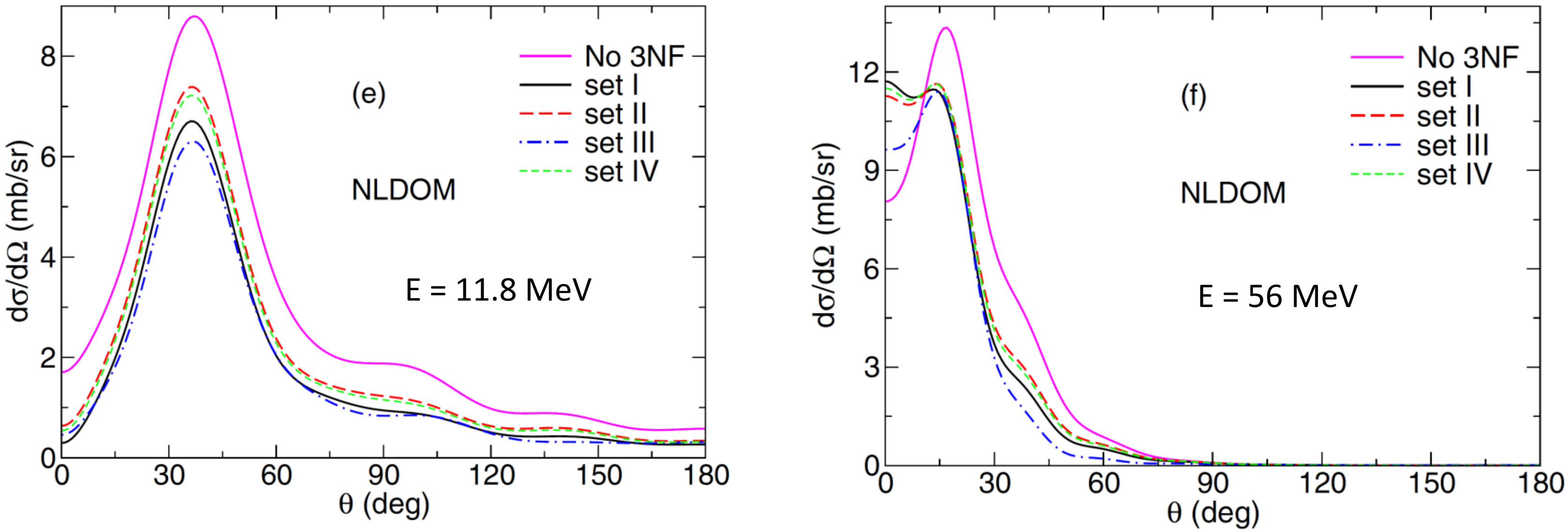
Relation to phenomenological optical potentials

M.J.Dinmore, N.K.Timofeyuk, J.S.Al-Khalili, R.C.Johnson
Phys.Rev. C 99, 064612 (2019)

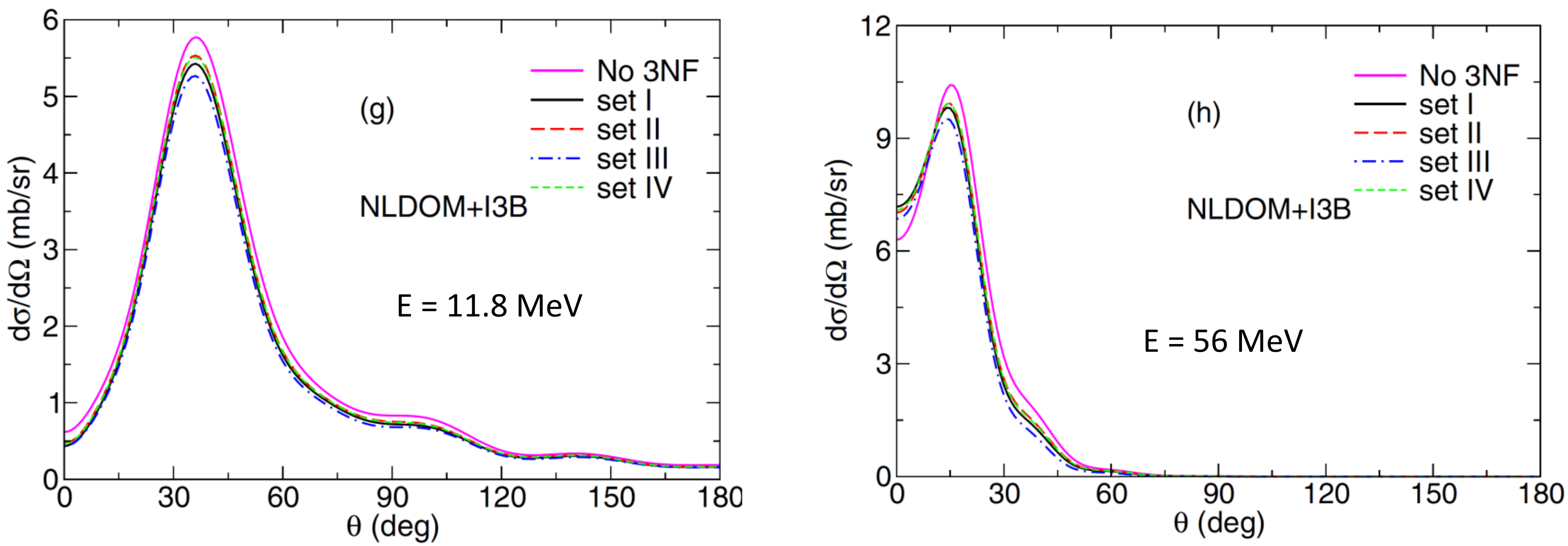
$$\langle \phi_A | U^{(0)} + U^{(1)} | \phi_A \rangle = V_{nA}^{HF} + 2\Delta V_{nA}^{\text{dyn}}(E) + V_{pA}^{HF} + 2\Delta V_{pA}^{\text{dyn}}(E)$$

Dynamical part of the phenomenological optical potential, taken at a shifted energy, should be doubled.

ADWA with non-local dispersive optical potential for $^{40}Ca(d,p)^{41}Ca$

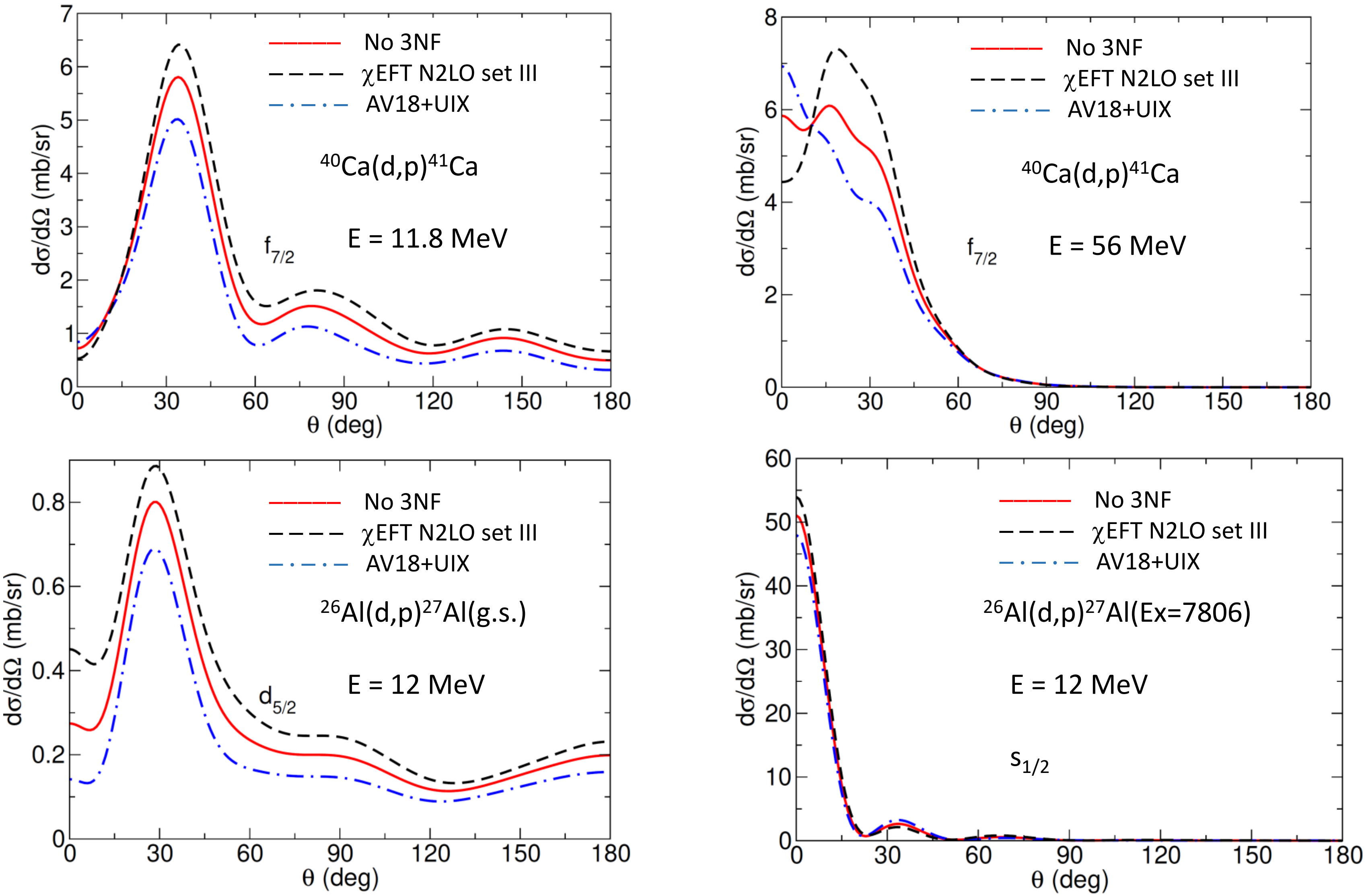


… and including induced 3-body (I3B) force



Watababe (folding) model with local optical potential KD03:

$$\left(T_{dA} + \langle \varphi_d | U_{nA} + U_{pA} | \varphi_d \rangle + \langle \varphi_A \varphi_d | \textstyle\sum_{ijk} V_{ijk} | \varphi_A \varphi_d \rangle - E_d\right) \chi_{dA}^{(+)}(\boldsymbol{R}) = 0$$

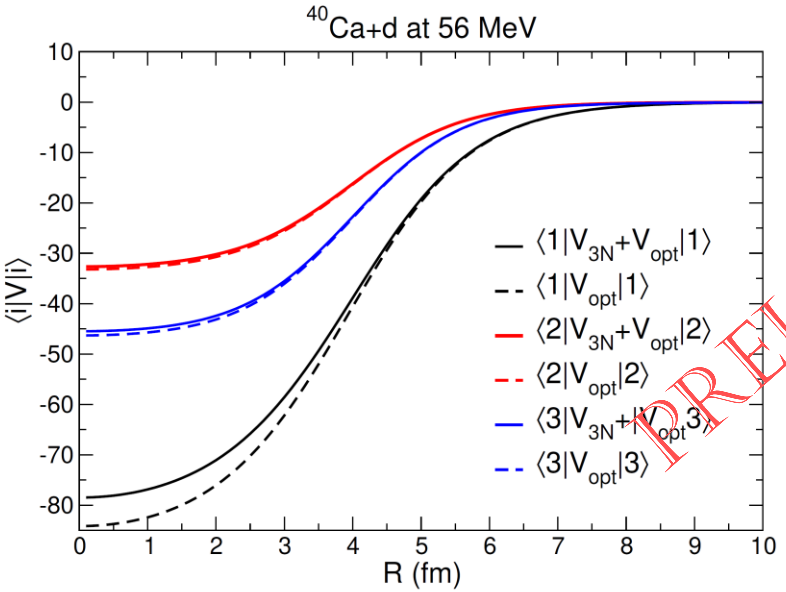


The work in progress:

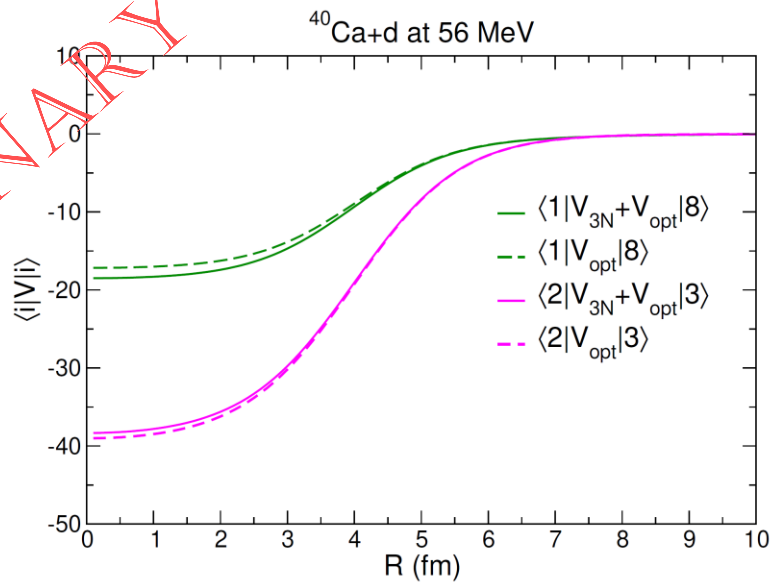
Contribution from 3N force in (d,p) reactions within CDCC, in collaboration with Mario Gomez-Ramos (Seville) and Laura Moschini (Surrey)

NN + 3N : AV18+UIX

Diagonal couplings:



Nondiagonal couplings:



Conclusions

There are two sources of contributions from 3N force in (d,p) reactions:

- Correction to (d,p) amplitude
- Correction to n+p+A interaction potential

Both contributions are very sensitive to the short-range physics and it is crucial to use 3N force that is consistent with the 2N force that determines the n-p observables

The n+p+A potential is very sensitive to the 2N+3N model.

The 3N contribution can be noticeable depending on the choice of a reacting system, deuteron energy, neutron separation energy and orbital momentum of the transferred neutron. It also depends on the choice of N-A optical potential and on induced three-body effects.