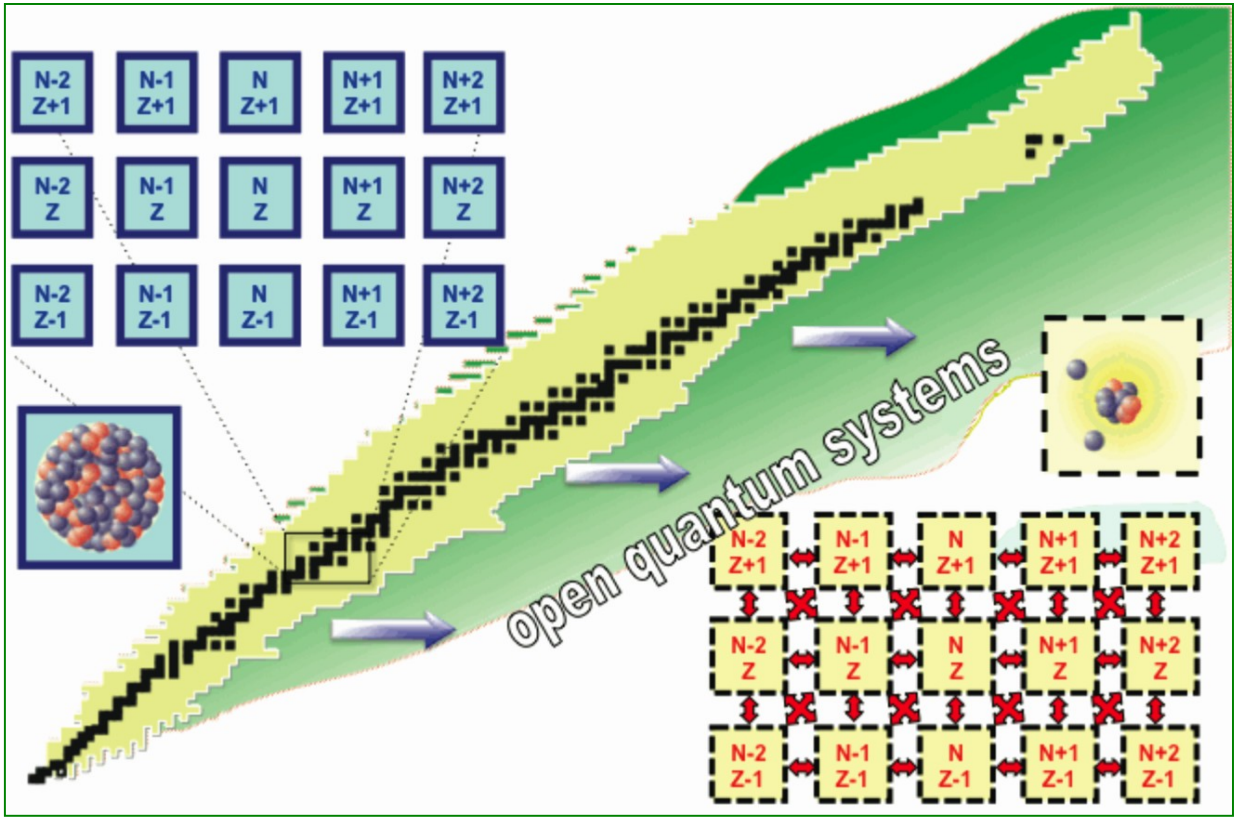
Reaction cross sections of drip-line nuclei with the Gamow Shell Model

Nicolas Michel (IMP)

Witek Nazarewicz, Kevin Fossez (MSU)
Jimmy Rotureau (Lund University)
Alexis Mercenne (Yale)
Marek Płoszajczak, José Pablo Linares (GANIL)
Yannen Jaganathen (IFJ PAN Krakòw)
R. M. Id. Betan (CONICET)

Furong Xu, Jianguo Li (PKU)
Simin Wang (Fudan University)
Yuanzhuo Ma (South Normal University)
Baishan Hu (TRIUMF)
Wei Zuo, Zuxing Yang, Penghui Chen (IMP)
Guoxiang Dong, Xiaobao Wang (Huzhou University)

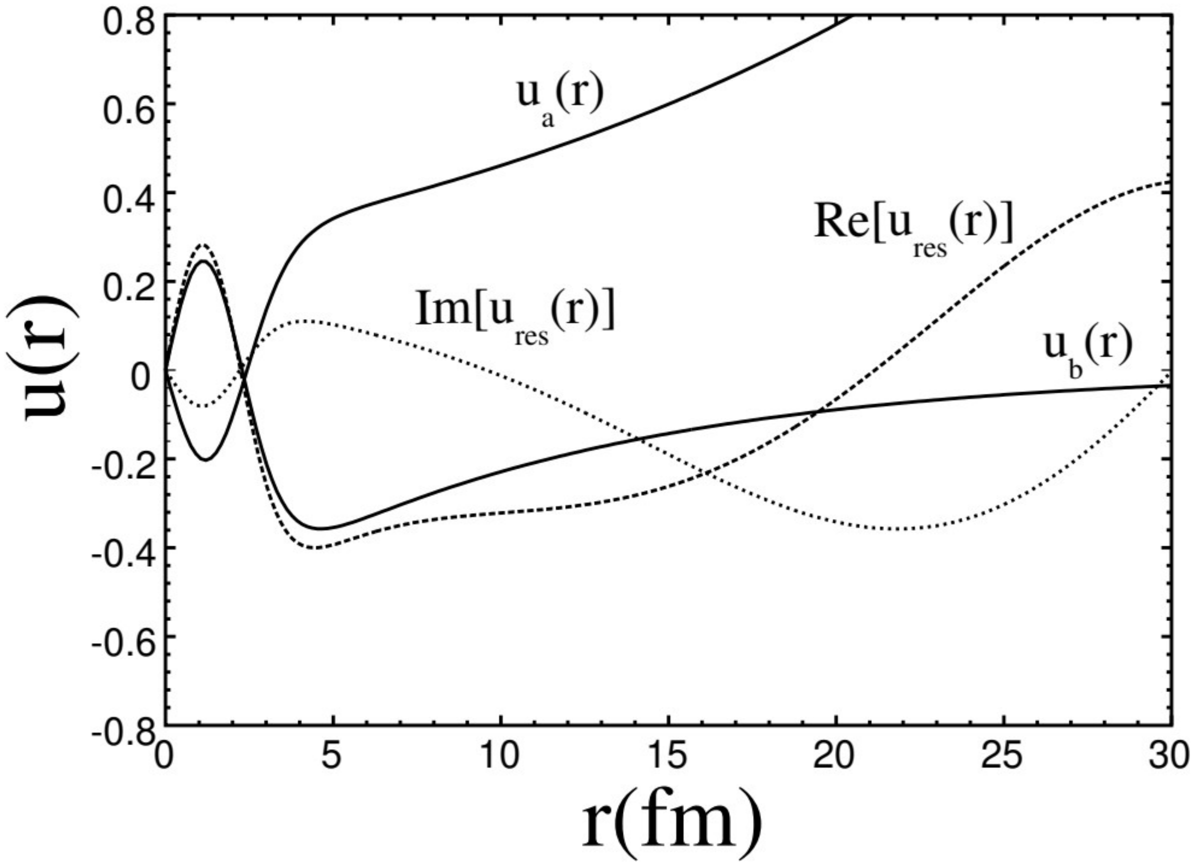
Scientific context



<u>Experimental interest</u>
Study of nuclei far from the valley of stability
Many efforts made to study drip-line nuclei

N. Michel, W. Nazarewicz, J. Okołowicz, M. Płoszajczak,
J. Phys. G: Nucl. Part. Phys., **37** 064042 (2010)

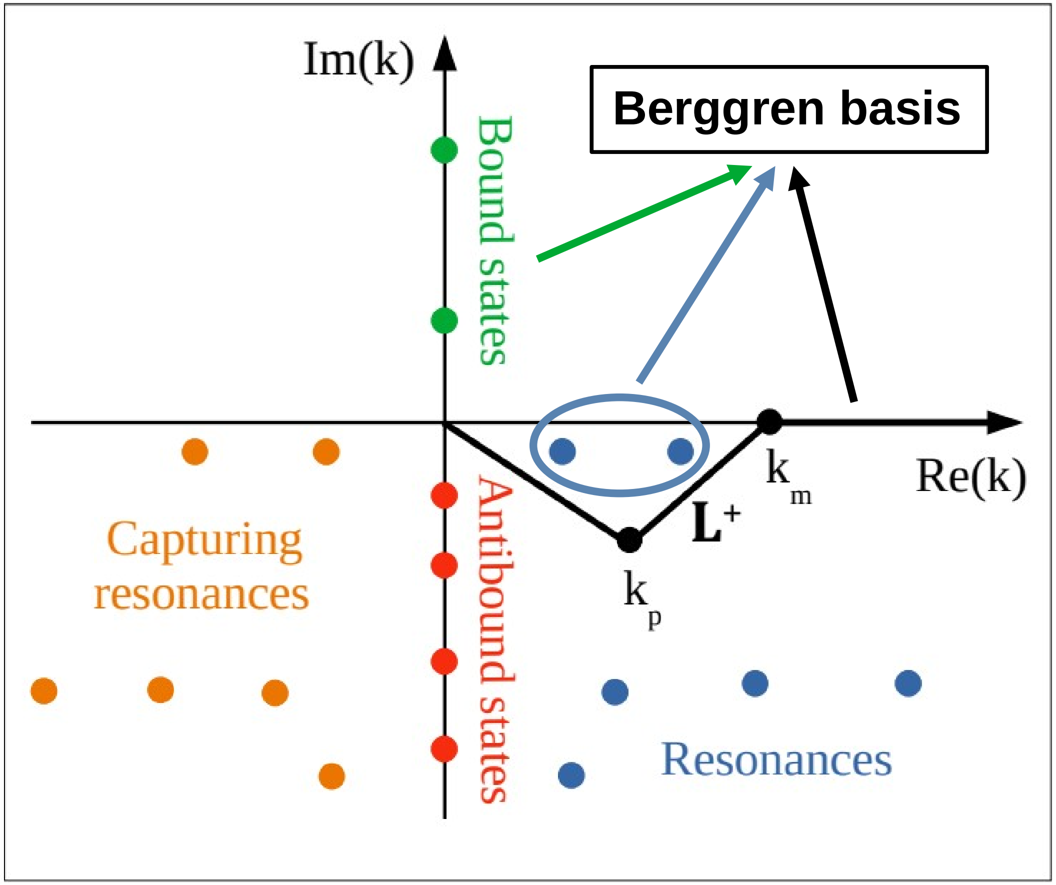
Gamow states



Bound and resonance states: same object
Same qualitative behavior inside the nucleus
Asymptote is different for bound, antibound, resonance states

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

The Berggren basis

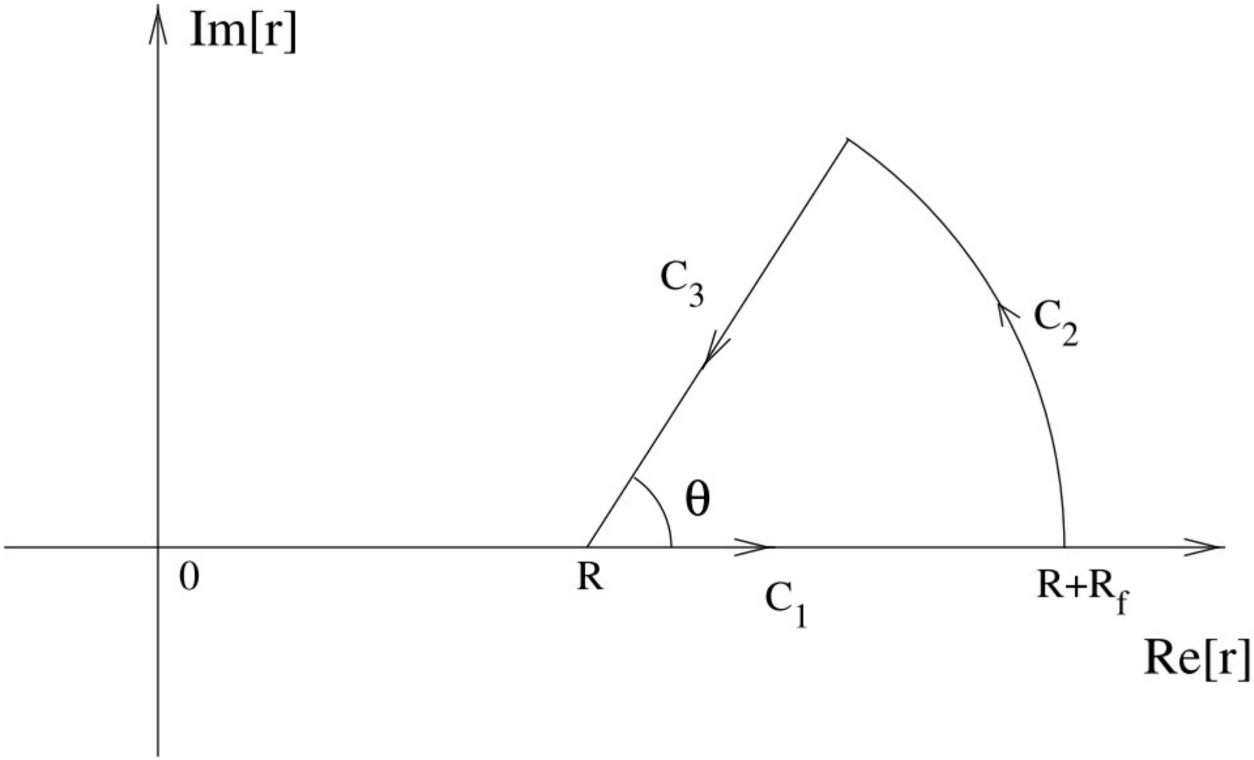


Berggren basis : bound, resonance and scattering states

Efficient discretization of the L^+ contour with Gauss-Legendre quadrature

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

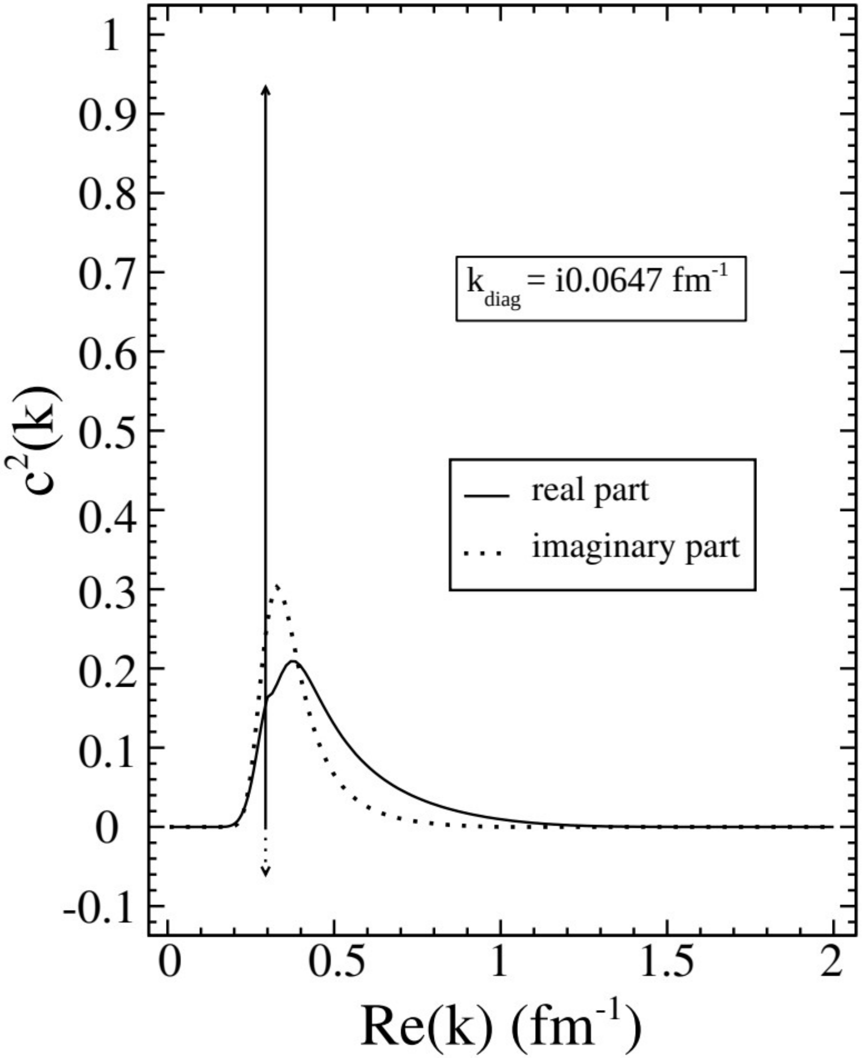
Complex scaling



Divergence of unbound states on the real axis
Resonance states: localized in the complex plane
Complex scaling method to calculate matrix elements
Bound and resonance states: normalized
Scattering states: normalized with a Dirac delta

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

One-body potential



Expansion of bound Woods-Saxon $1p_{3/2}$ proton state
Basis of unbound states: no bound basis state
Large component from basis resonance state
Scattering components smaller, but necessary

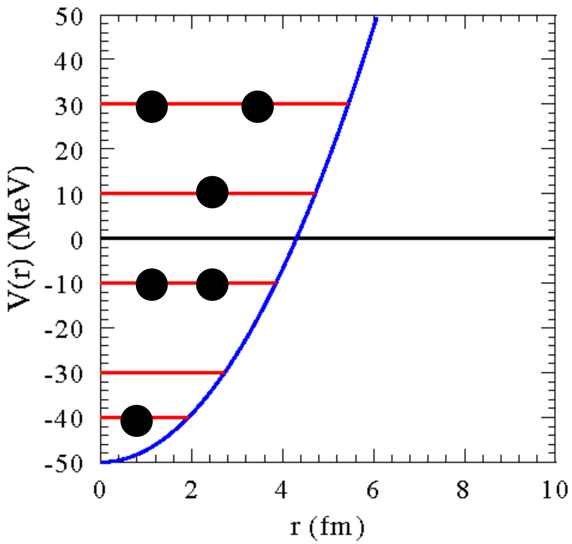
Components have large real and imaginary parts
Imaginary parts cancel in observables

N. Michel, M. Płoszajczak,
The Gamow Shell Model: the unified theory of nuclear structure and reactions,
Lecture Notes in Physics, Springer International Publishing (2021)

Gamow Shell Model (GSM)

Standard shell model

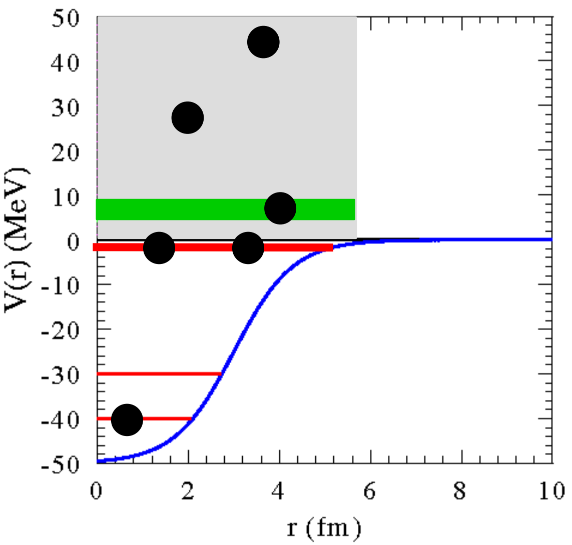
Closed quantum system description



Localized states

Gamow Shell Model

Open quantum system description



Localized states
Weakly bound/resonant states
Scattering states

Diagonalization of GSM matrices

<u>GSM Hamiltonian matrix</u>

Resonant eigenstates
representing resonances
hidden among scattering eigenstates

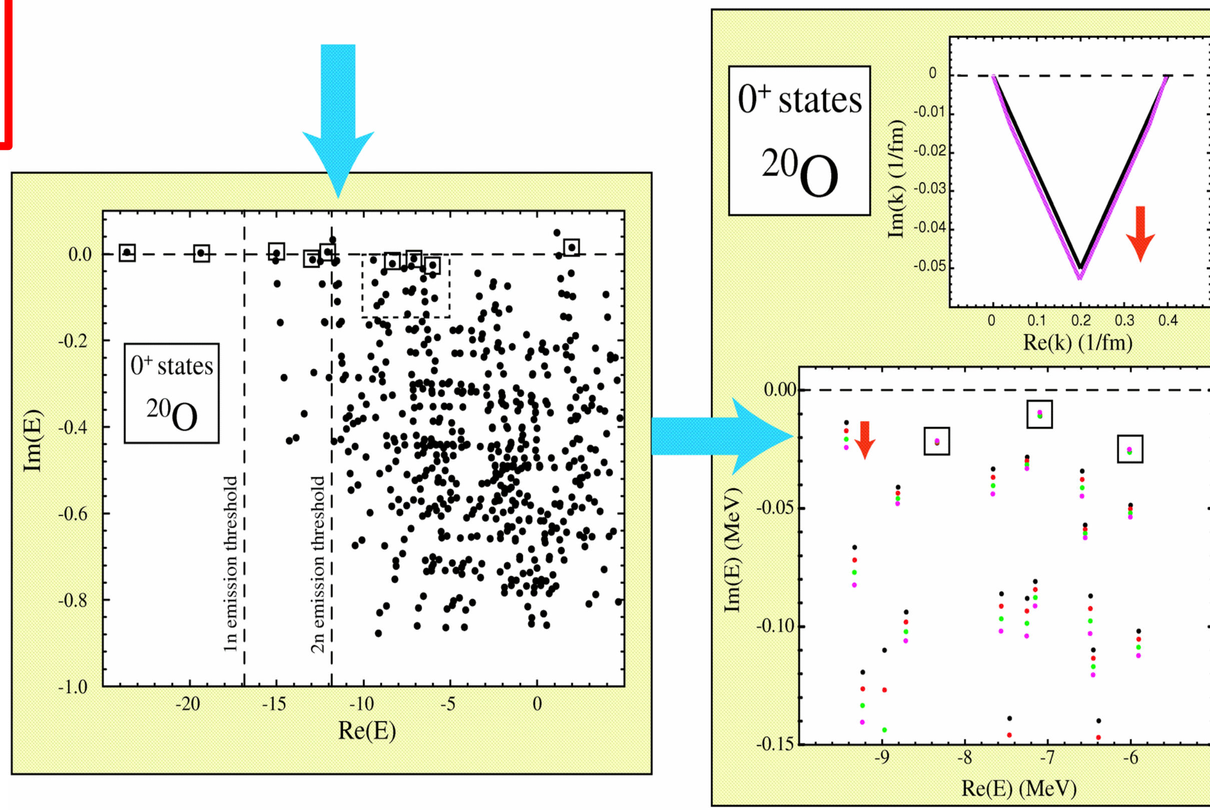
<u>The overlap method</u>

1) Diagonalization in pole space

2) Diagonalization in full space

3) Identification of physical states

Stability of the 'physical' states



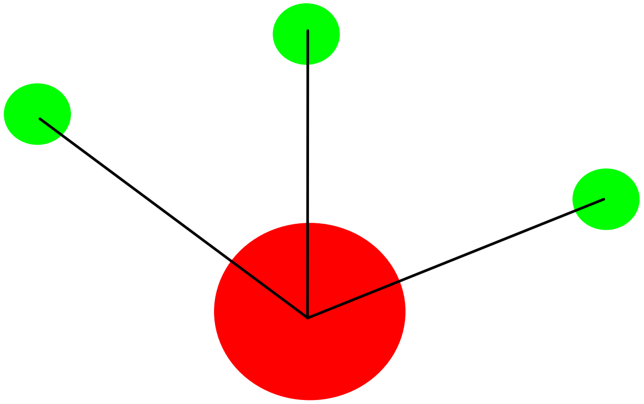
N. Michel, W. Nazarewicz, M. Płoszajczak, J. Okołowicz,
Phys. Rev. C, **67** 054311 (2003)

Cluster Orbital Shell Model (COSM)

Problematic

3A degrees of freedom (particles coordinates)

3(A-1) physically (translational invariance) → spurious states



$$\vec{r} = \vec{r}_{lab} - \vec{R}_{core}$$

Standard shell model

Calculation in a major shell (core + valence nucleons)

Lawson method (no-core shell model)

Harmonic oscillator basis only

Cluster orbital shell model (COSM)

Relative core coordinates → no center of mass excitation

Center of mass handled by a recoil term in the Hamiltonian

+: Formal use identical to laboratory coordinates

–: Inferior to Jacobi coordinates

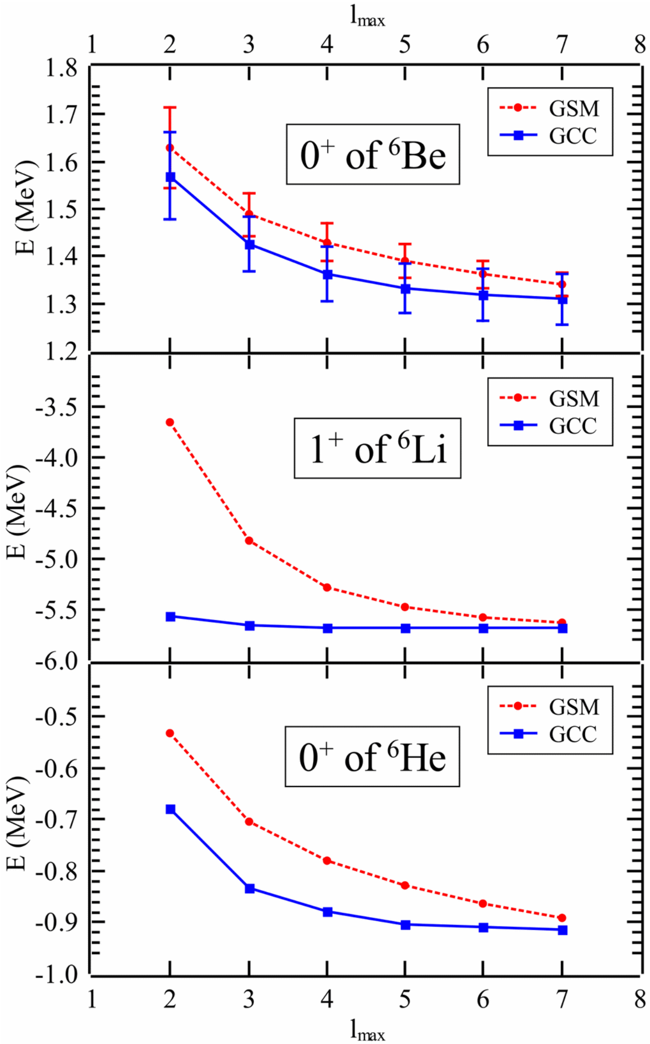
Pauli principle approximately treated with a Pauli operator on the core

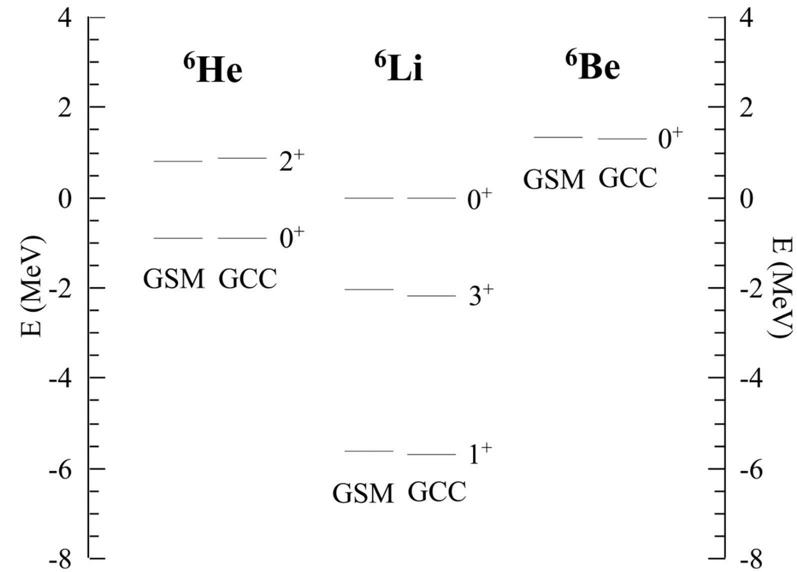
Practical use of COSM

Definition of Hamiltonian directly in COSM frame

Calculations with COSM and Jacobi coordinate models very close

Comparison of A=6 nuclei using COSM and Jacobi coordinates



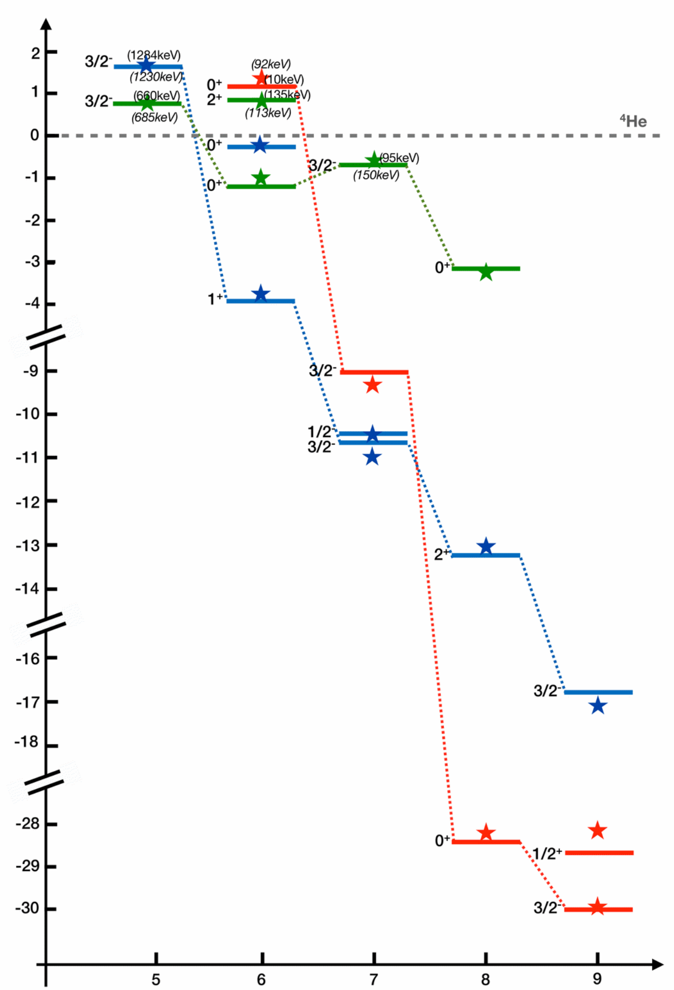


Valence nucleons above a ^{4}He core
psd Berggren space, rest HO ($l_{max} \le 7$)
Minnesota interaction

Core fitted to ^{5}He single-particle states
Standard parameters of Minnesota interaction
GCC : Gamow hyperspherical harmonics
Coupled-channel equations using Jacobi coordinates
GSM-COSM and GCC results very close

S. M. Wang, N. Michel, W. Nazarewicz, F. R. Xu, Phys. Rev. C **96**, 044307 (2017)

Energetics of light nuclei with an effective interaction



Y. Jaganathen, R. M. Id Betan, N. Michel, W. Nazarewicz, M. Płoszajczak, Phys. Rev. C **96**, 054316 (2017)

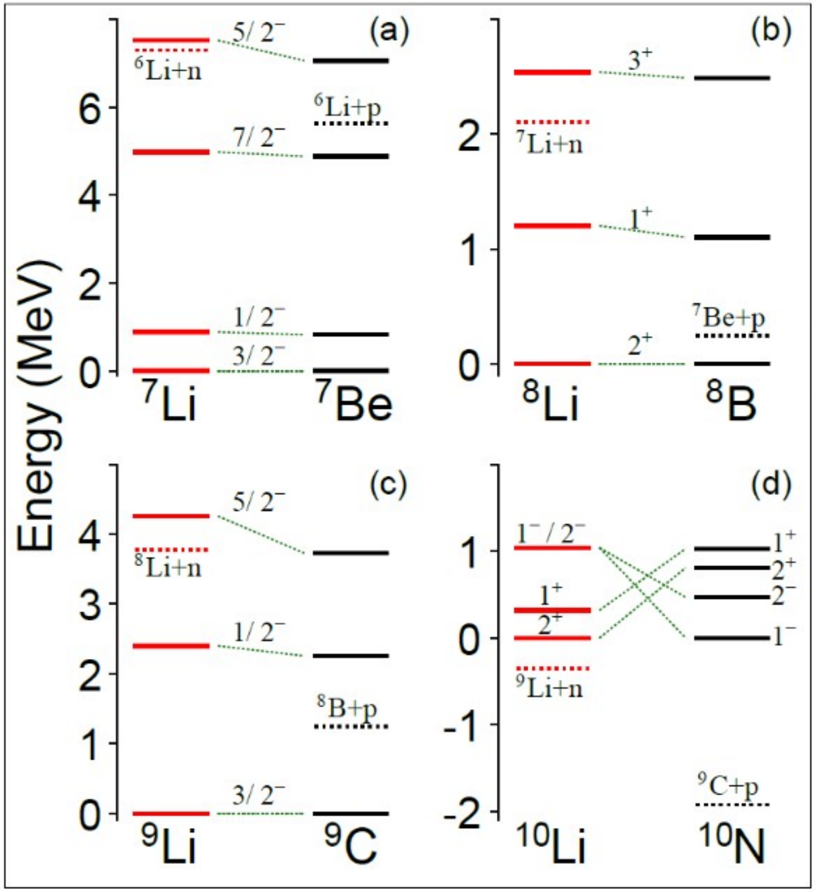
Parameter		Value
central	triplet-odd	-1.54 ± 25.53
	triplet-even	**-4.71 ± 0.76**
	singlet-odd	**-32.78 ± 7.41**
	singlet-even	**-5.79 ± 0.42**
spin-orbit	triplet-odd	-138.31 ± 819.00
tensor	triplet-odd	-12.61 ± 69.96
	triplet-even	**-16.25 ± 4.28**

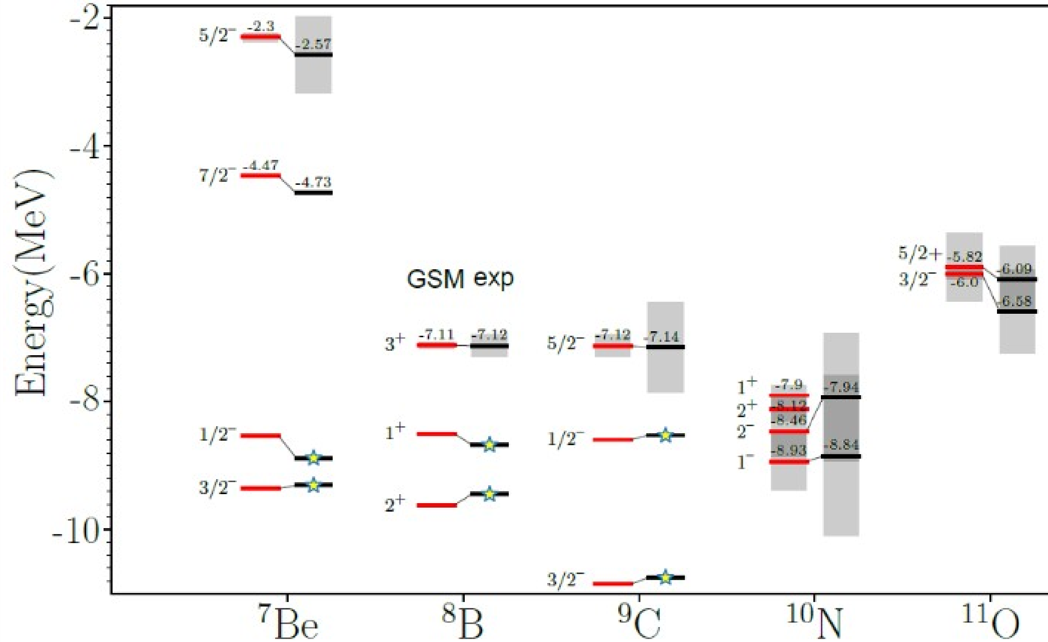
Valence nucleons above a ^{4}He core
psd Berggren space
FHT (Furutani-Horiuchi-Tamagaki) interaction

Core fitted to nucleon + ^{4}He phase shifts
He, Li, Be isotopic chains considered
Parameters fitted to experimental data
Uncertainties of model calculated

In progress:
Bayesian statistical analysis

Applications : Li isotopes and mirror partners

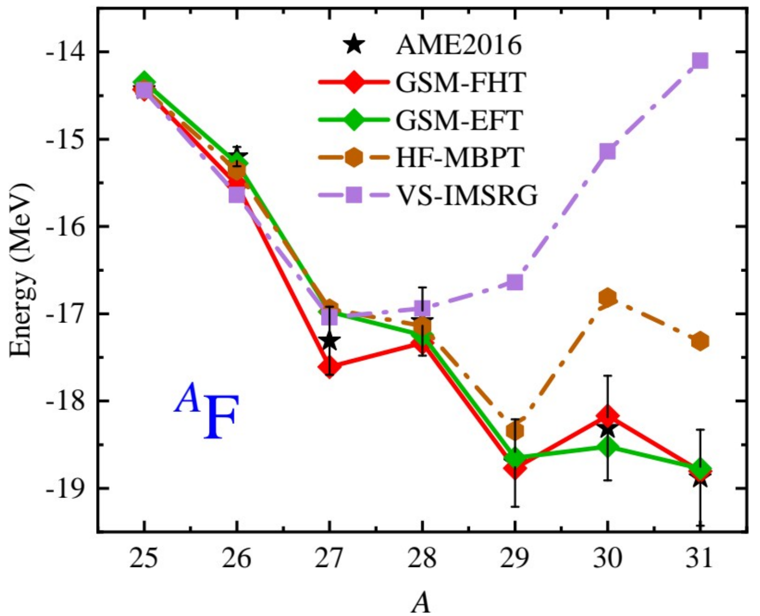


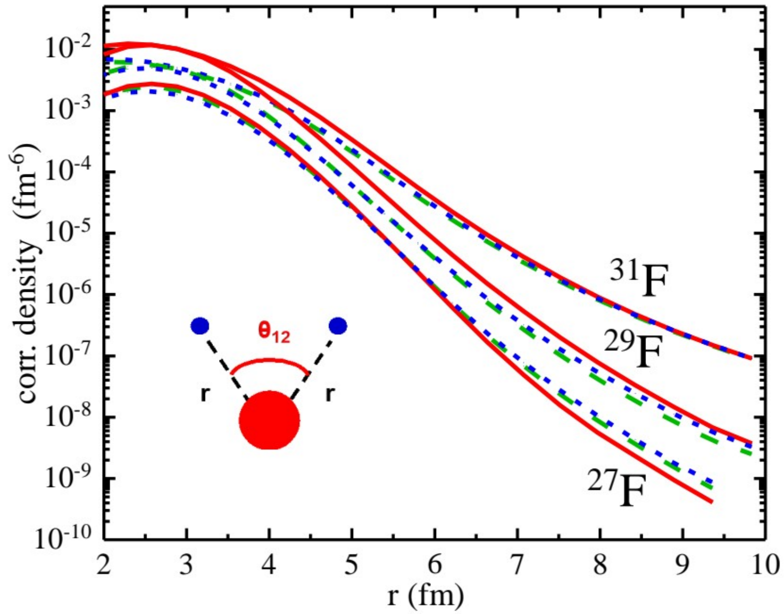


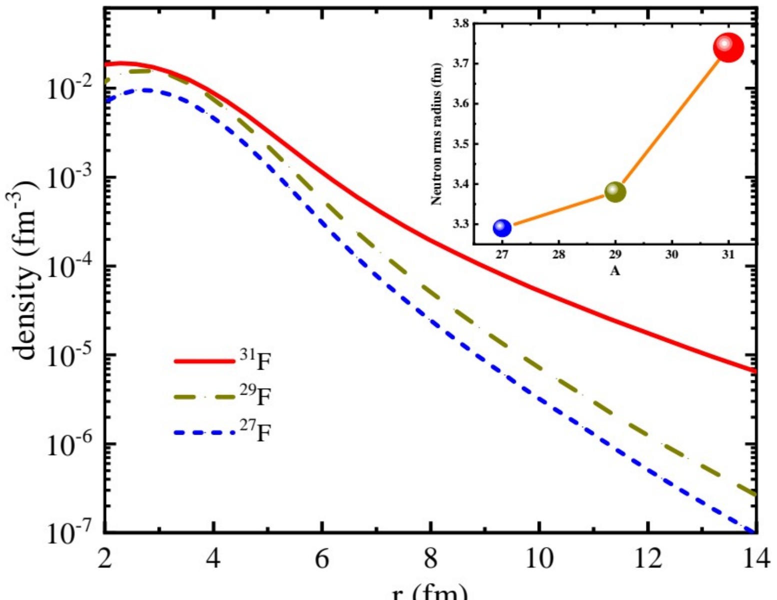
⁴He core + valence nucleons in sp Berggren space
FHT interaction renormalized using effective field theory
Mirror isotopes: proton-rich, broad resonances
strong Thomas-Ehrman shifts
Proton-rich not well known experimentally
GSM predictions consistent with recent analyses

X. Mao, J. Rotureau, W. Nazarewicz, N. Michel, R. M. Id Betan, Y. Jaganathen, Phys. Rev. C **102**, 024309 (2020)

Two-neutron halo of ^{31}F





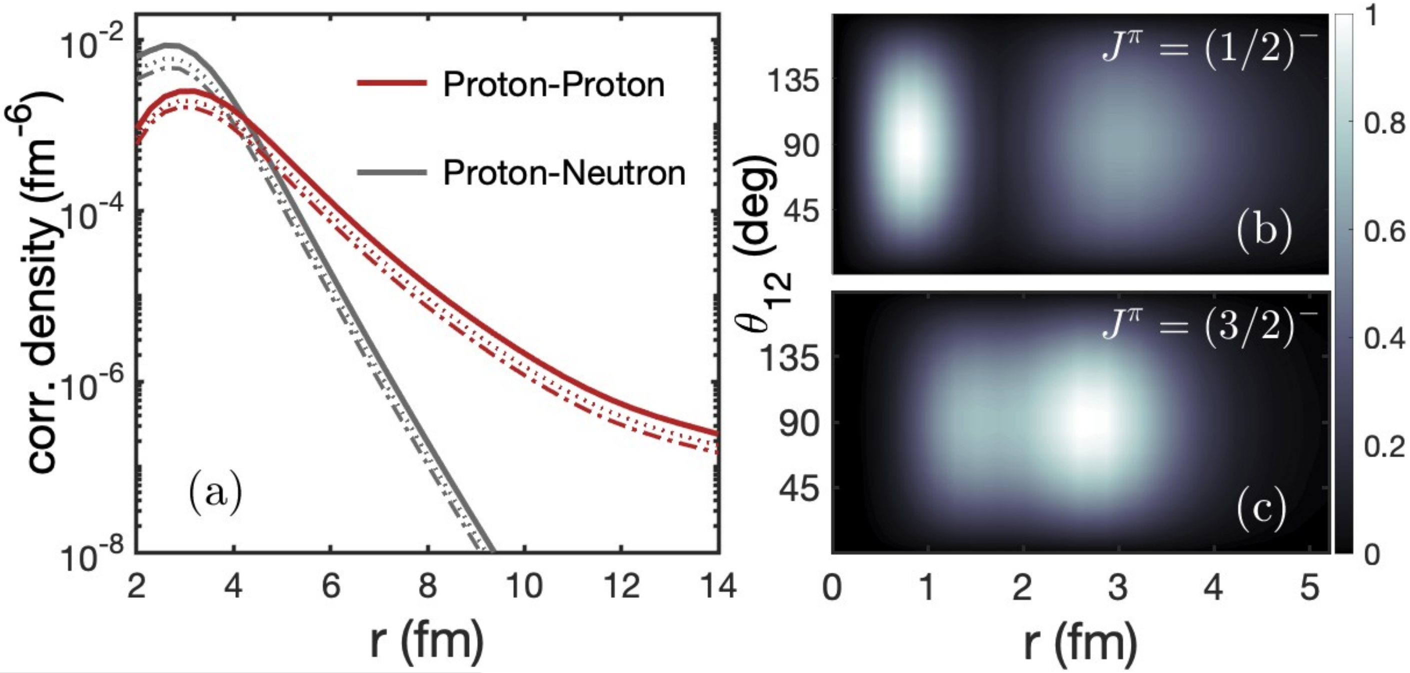


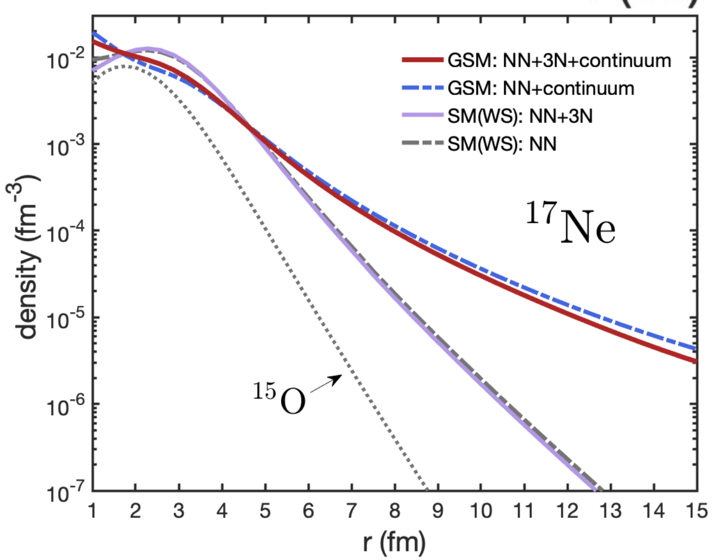
Valence nucleons above a ^{24}O core
sdpf HO + Berggren space
FHT and EFT (effective field theory) interactions

Neutron-rich fluorine isotopes not well known
Only GSM can include continuum coupling
Good description of fluorine isotopes at neutron drip-line
^{31}F predicted to be a two-neutron halo
Confirmed from GSM calculations with FHT and EFT
Clustering still in question

N. Michel, J.G. Li, F.R. Xu, W. Zuo, Phys. Rev. C **101**, 031301(R) (2020)

Two-proton halo of ^{17}Ne





Realistic chiral N^{3}LO potential: Q-box with ^{14}O core
Two-nucleon and three nucleon forces included
Neutrons: bound $0p_{1/2}$ + sd shell
Protons: $s_{1/2}$ and $d_{5/2}$ Berggren basis states

Inter-nucleon correlations + continuum coupling
^{17}Ne two-proton halo supported by GSM calculations
Three-nucleon forces: not prominent in halo formation

Y.Z. Ma, F.R. Xu, N. Michel, S. Zhang, J.G. Li, B.S. Hu,
L. Coraggio, N. Itaco, A. Gargano, Phys. Lett. B **808**, 135673 (2020)

Asymptotic normalization coefficients (ANC) with GSM

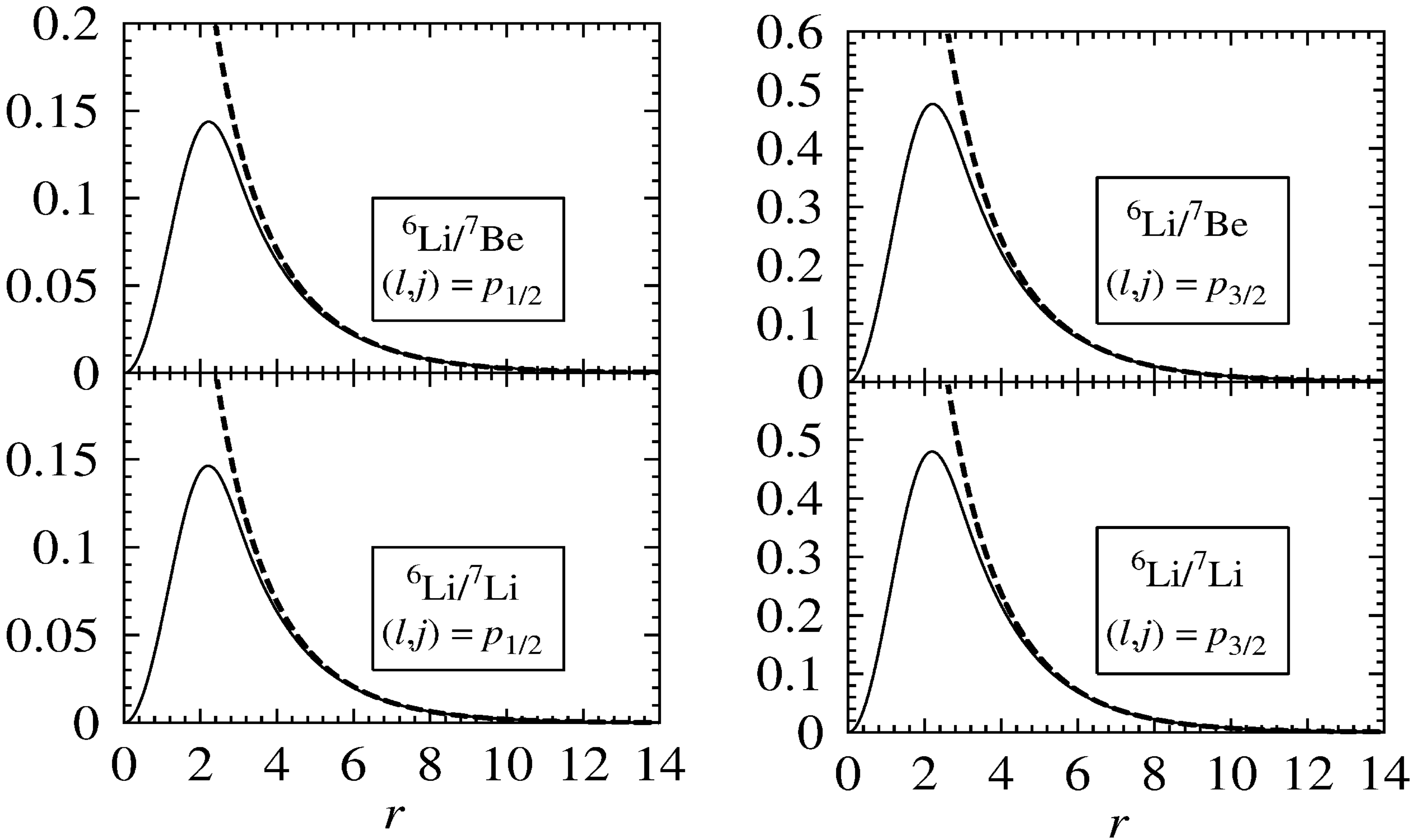
Overlap functions: nucleon form factor times A-1 core
One-body asymptotic behavior of many-body functions
Decay directly related to one-nucleon separation energy
Asymptotic decay: Hankel/Coulomb wave function times ANC

Physical interest: radiative capture, transfer cross section
Cross sections proportional to the square of the ANC

Determination of the ANC in standard models: DWBA + shell model
Standard models: continuum coupling neglected
GSM: unified determination of ANCs

Applications: nuclei bearing A=6-8 nucleons
GSM calculations: ^{4}He core + phenomenological Gaussian interaction
Astrophysical interest: solar neutrinos, puzzled Li abundance, …

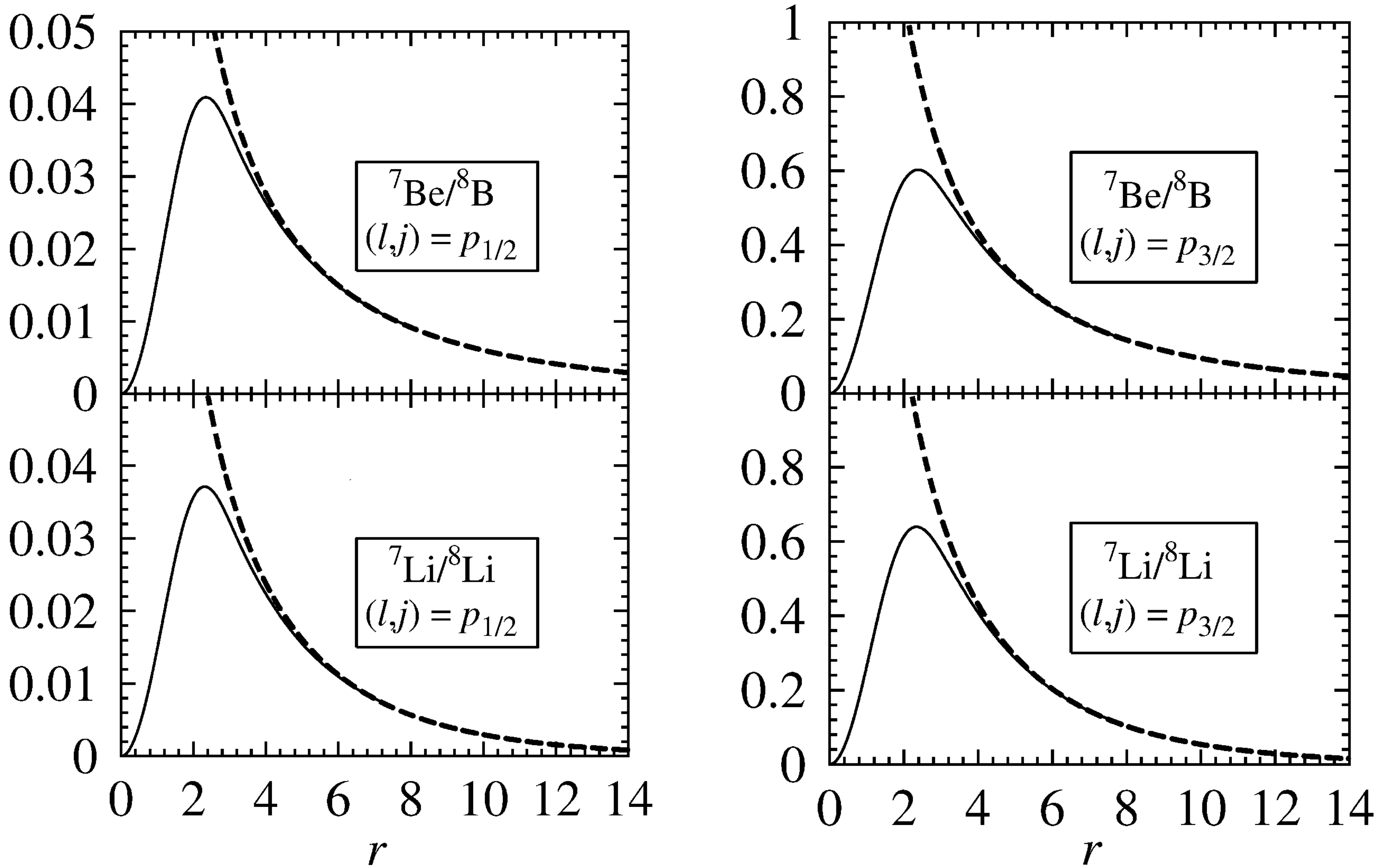
^{6}Li/^{7}Be, ^{6}Li/^{7}Li: ground states



$1^+ \rightarrow 3/2^-$: weak coupling to the continuum (well-bound states)

J. Okołowicz, N. Michel, W. Nazarewicz, M. Płoszajczak,
Phys. Rev. C **85**, 064320 (2012)

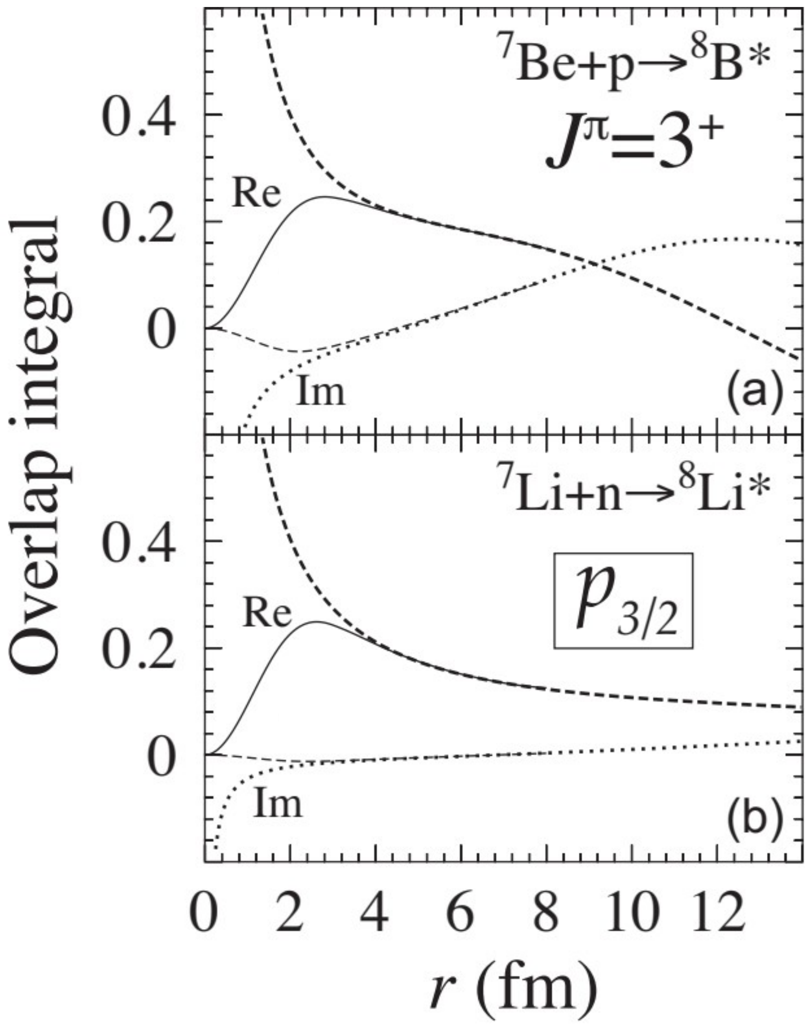
$^{7}Be/^{8}B$, $^{7}Li/^{8}Li$: ground states



$3/2^{-} \rightarrow 2^{+}$: ^{8}B one-proton halo visible

J. Okołowicz, N. Michel, W. Nazarewicz, M. Płoszajczak,
Phys. Rev. C **85**, 064320 (2012)

$^7Be/^8B$, $^7Li/^8Li$: unbound states



$3/2^- \rightarrow 3^+$: complex overlap functions of unbound states

J. Okołowicz, N. Michel, W. Nazarewicz, M. Płoszajczak,
Phys. Rev. C **85**, 064320 (2012)

Reaction observables with GSM

Resonating Group Method (RGM) channels: products of target and projectile states
Target and projectile states from GSM
Projectiles: proton/neutron (scattering, radiative capture),
deuteron (scattering, transfer, radiative capture in progress),
^{4}He, ^{3}He, t (coded, tests in progress)

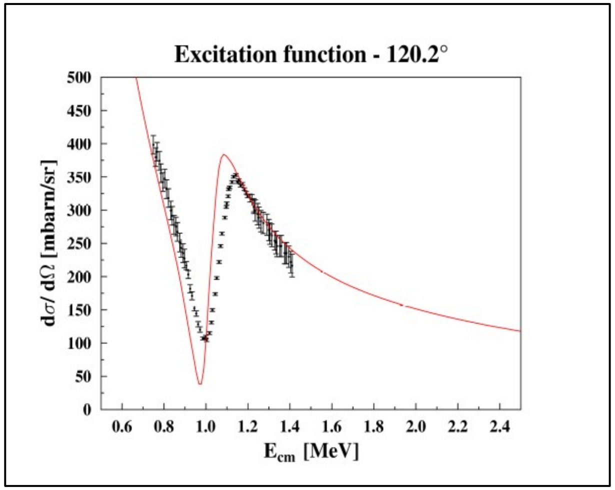
Potentials calculated from GSM: microscopic
Coupled-channel equations to solve : GSM-CC
Same Hamiltonian in GSM and RGM: unification of structure and reaction models
Fine tuning of Hamiltonian parameters

Applications

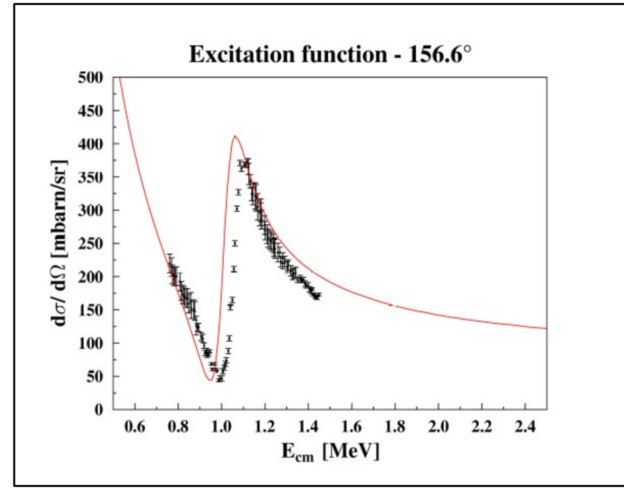
Direct and transfer reactions of low energy
Nuclei of experimental and astrophysical interest

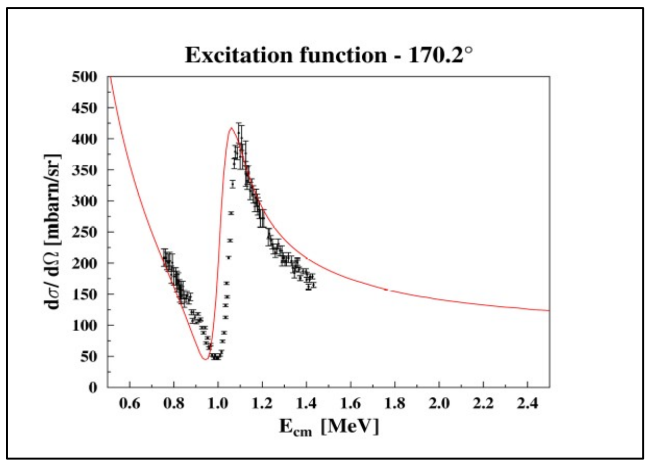
Elastic and inelastic scattering reactions
Proton and neutron radiative capture

^{18}Ne(p,p) scattering reaction



Valence protons above a ^{16}O core
psd Berggren space
Surface Gaussian interaction



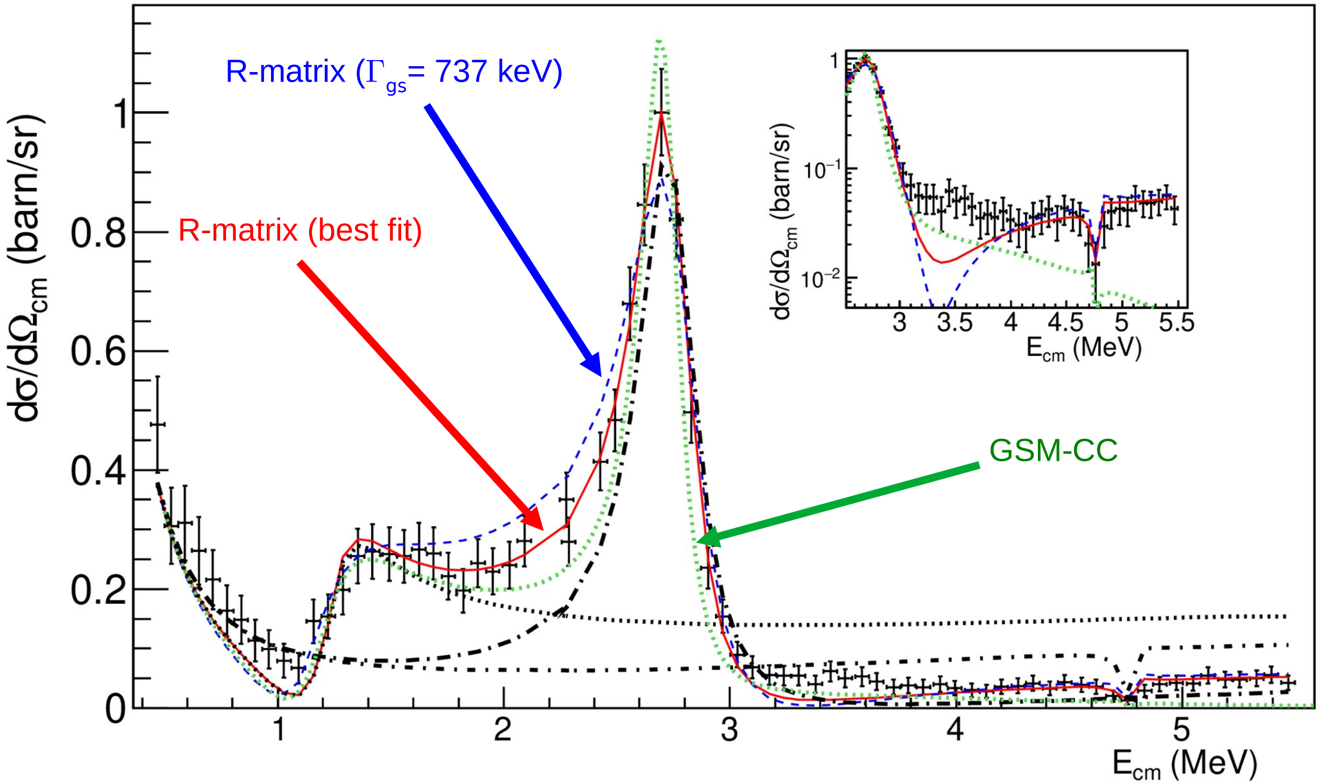


Y. Jaganathen, N. Michel, M. Płoszajczak, Phys. Rev. C **89**, 034624 (2014)

^{19}Na spectrum fitted to experimental data
^{19}Na states unbound but very narrow
Good agreement with experimental data

$^{14}O(p,p)$ scattering reaction

Θ = 180 deg



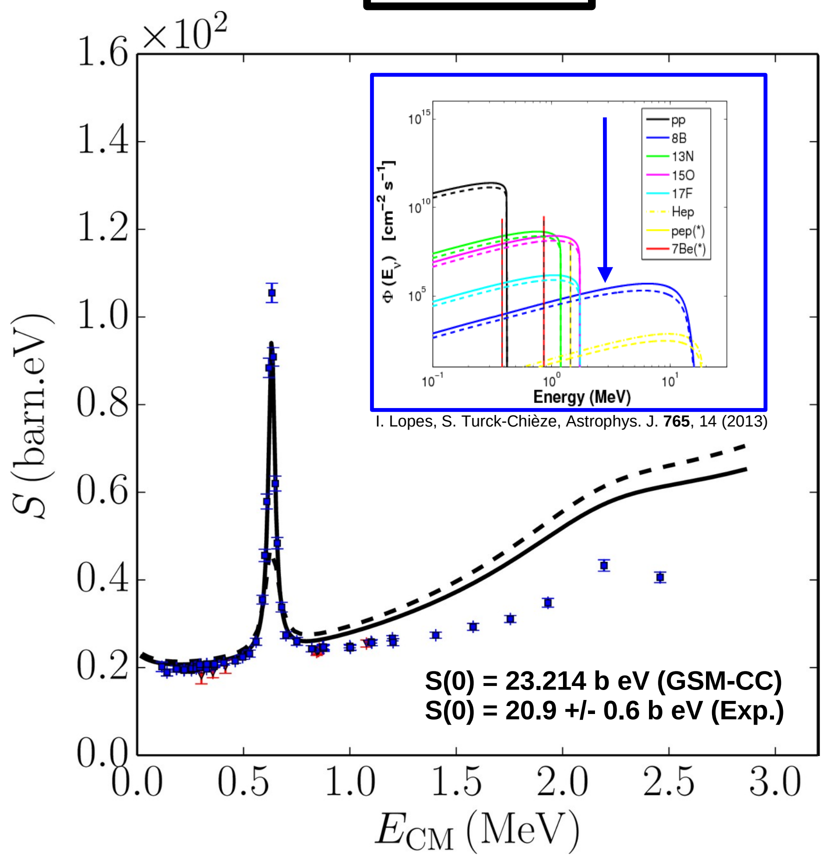
- ^{14}O and ^{15}F spectra fitted to experimental data
- ^{15}F unbound: continuum effects prominent in this reaction
  - Presence of a narrow 1/2- state above the barrier: effect seen on calculated cross section
- Good agreement with experimental data

Valence protons above a ^{12}C core
psd Berggren space
FHT (Furutani-Horiuchi-Tamagaki) interaction

F. de Grancey et al., Phys. Lett. B 758, **26** (2016)

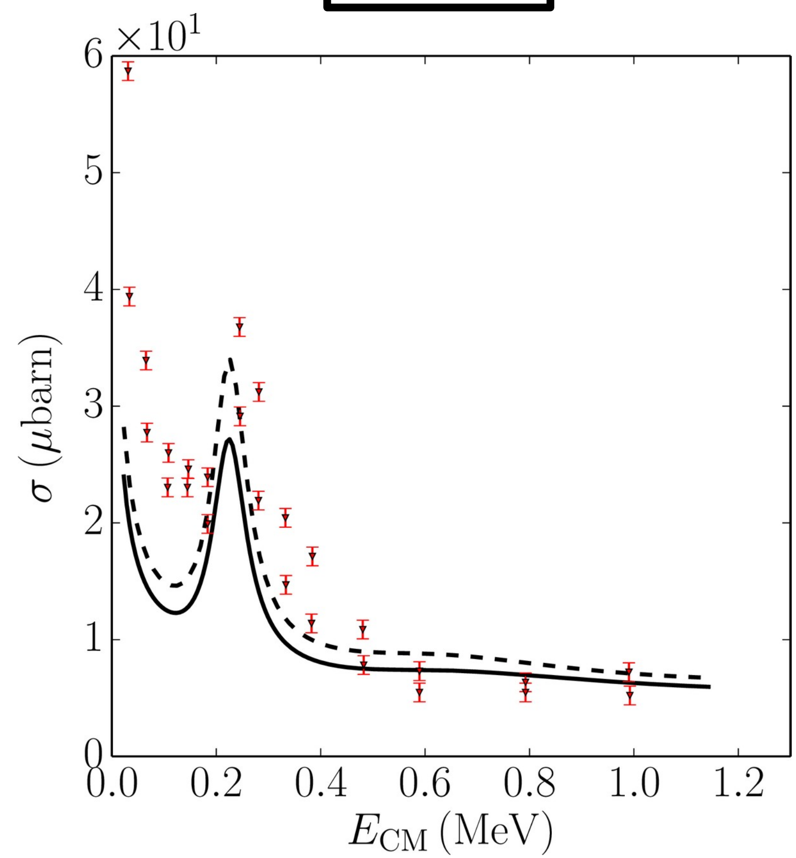
$^7Be(p, \gamma)$, $^7Li(n, \gamma)$ radiative captures

$^7Be(p,\gamma)$



K. Fossez, N. Michel, M. Płoszajczak, Y. Jaganathen, R. M. Id Betan, Phys. Rev. C **91**, 034609 (2015)

$^7Li(n,\gamma)$

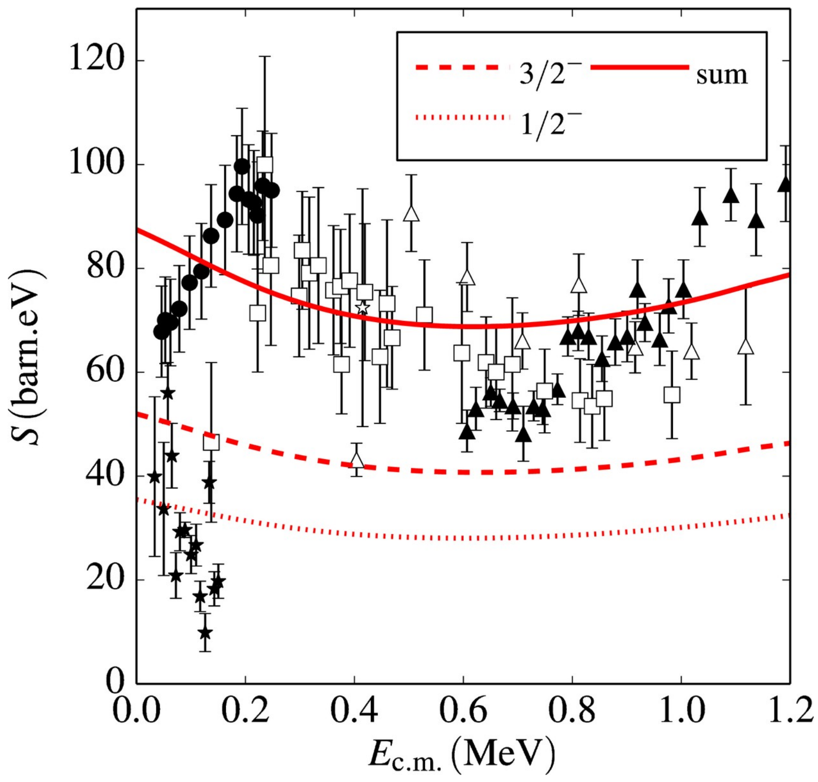


$^{7\text{-}8}$Be and $^{7\text{-}8}$Li spectra fitted to experimental data
Solar neutrino flux from ^{7}Be and ^{8}B
Exact fine tuning of ^{8}B separation energy necessary (proton halo)
Good agreement with experimental data

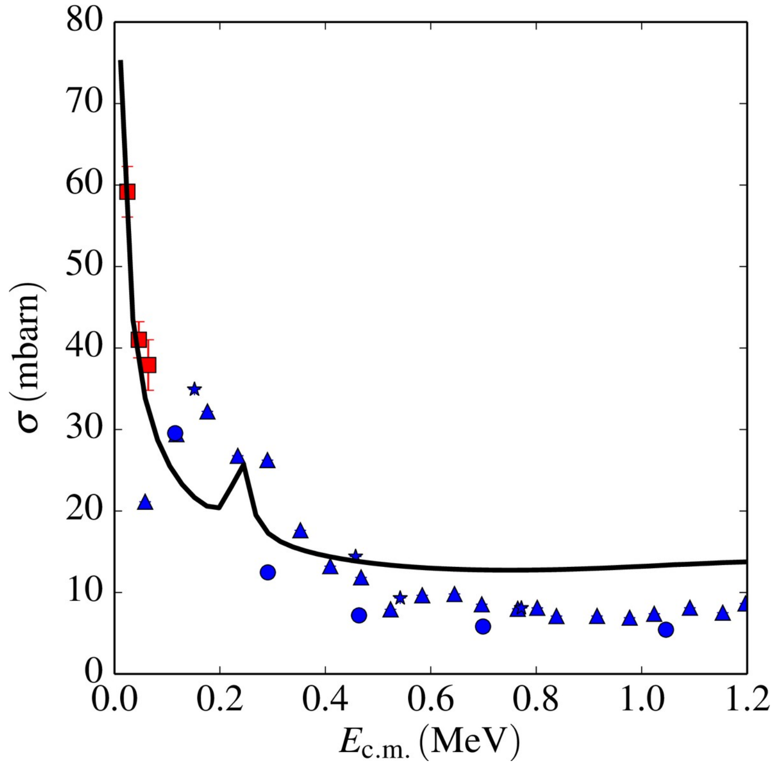
Valence nucleons above a ^{4}He core
psd Berggren space
FHT interaction

^{6}Li(p,γ), ^{6}Li(n,γ) radiative captures

^{6}Li(p,γ)



^{6}Li(n,γ)

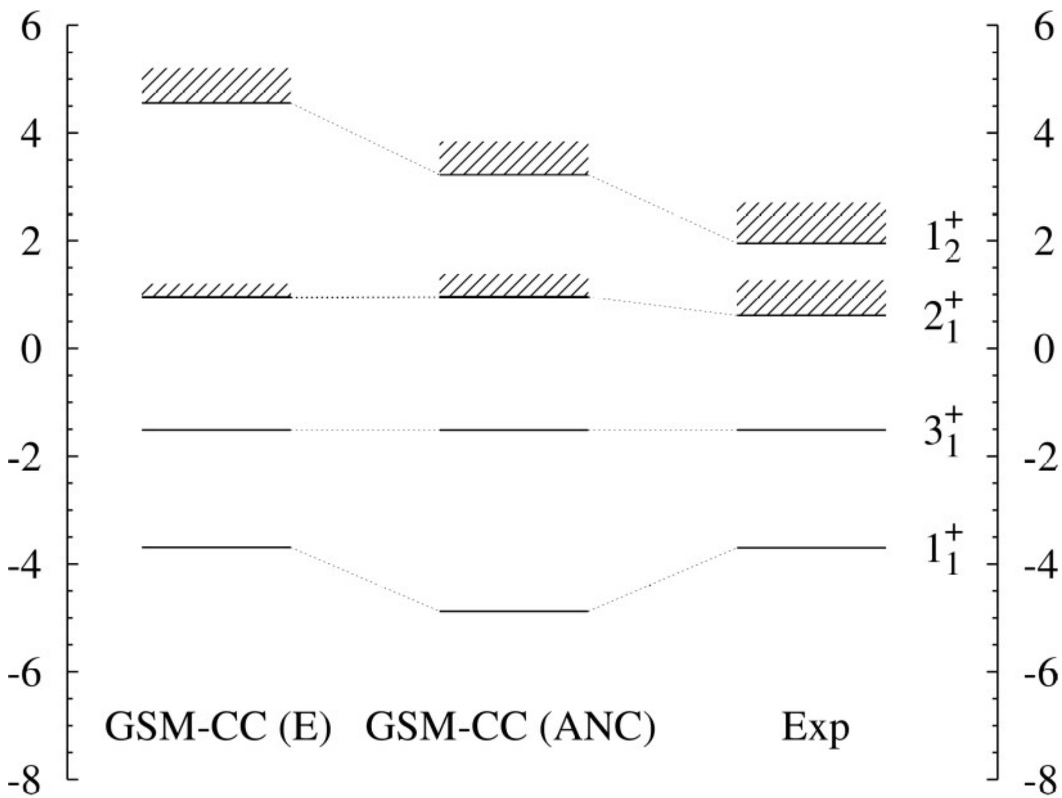


G.X. Dong, N. Michel, K. Fossez, M. Płoszajczak, Y. Jaganathen, R.M. Id Betan,
J. Phys. G: Nucl. Part. Phys. **44**, 045201 (2017)

Valence nucleons above a ^{4}He core
psd Berggren space
FHT interaction

Problem of puzzled abundances of $^{6\text{-}7}$Li
$^{6\text{-}7}$Li and ^{7}Be spectra fitted to experimental data
Good agreement with experimental data

^{4}He(d,d) scattering reaction (1/3)



Valence nucleons above an alpha core
Partial waves up to l=4
FHT interaction between valence nucleons

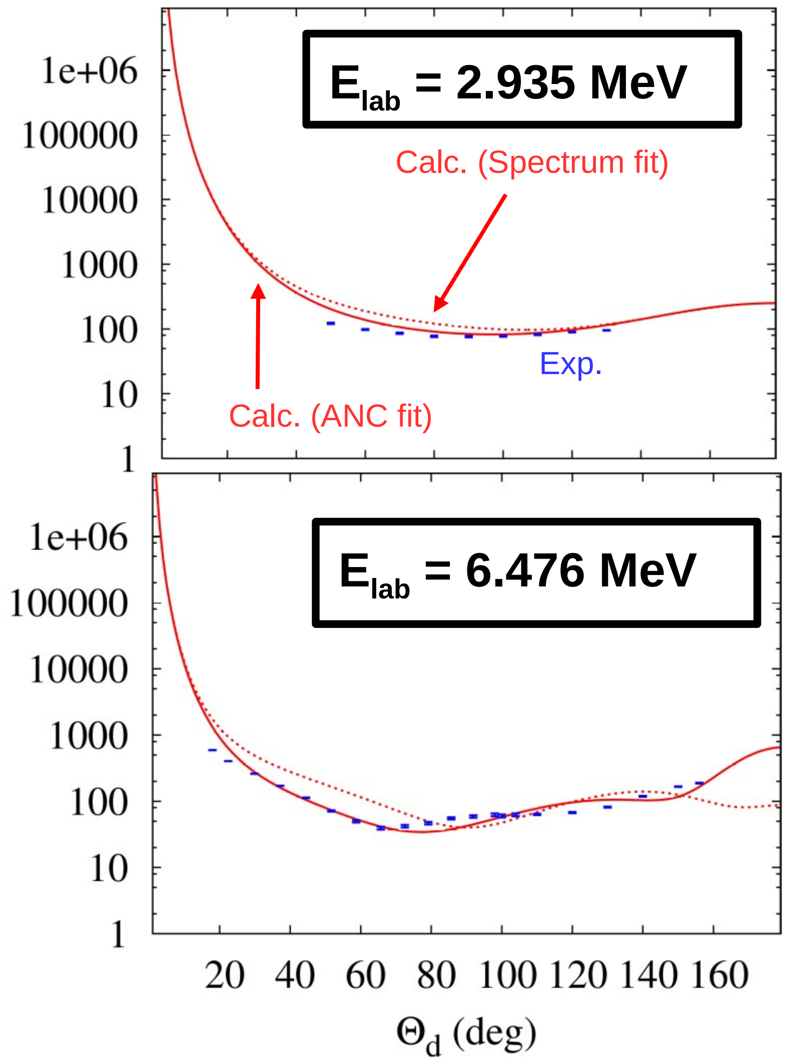
Structure and reactions features
^{6}Li: spectrum fit vs ANC fit

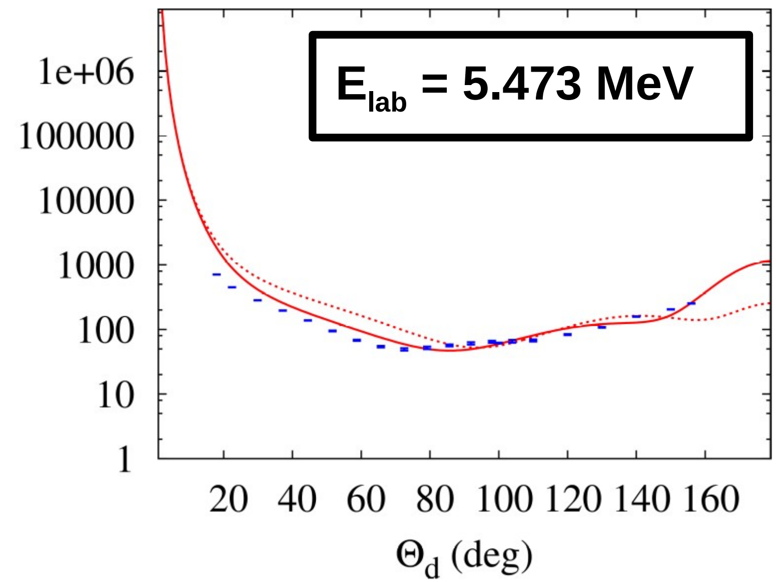
Spectrum fit: Neglects important correlations
Not sufficient for reactions

ANC fit: Slightly shifted ground state
Better reproduction of cross sections

A. Mercenne, N. Michel, M. Płoszajczak, Phys. Rev. C **99**, 044606 (2019)

^{4}He(d,d) scattering reaction (2/3)



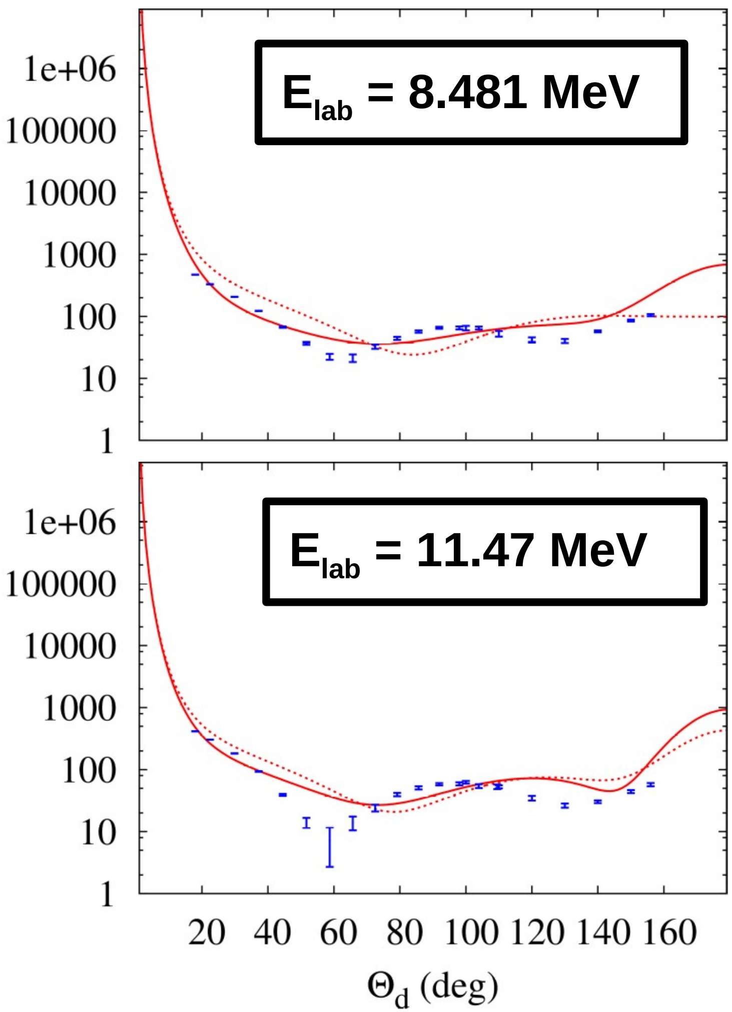


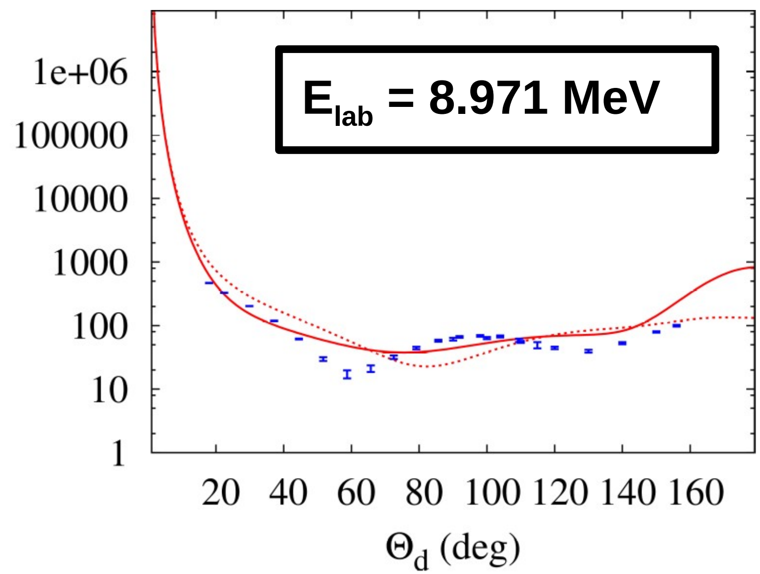
- Valence nucleons above an alpha core
- Partial waves up to l=4
- Deuteron structure from N³LO interaction
- Break-up included with scattering deuteron states
- FHT interaction between valence nucleons

- Simple model for alpha cluster ($0s_{1/2}$ occupied only)
- Spectrum fit not sufficient already for low energies
- Good reproduction of experimental data with ANC fit

A. Mercenne, N. Michel, M. Płoszajczak, Phys. Rev. C **99**, 044606 (2019)

$^4He(d,d)$ scattering reaction (3/3)

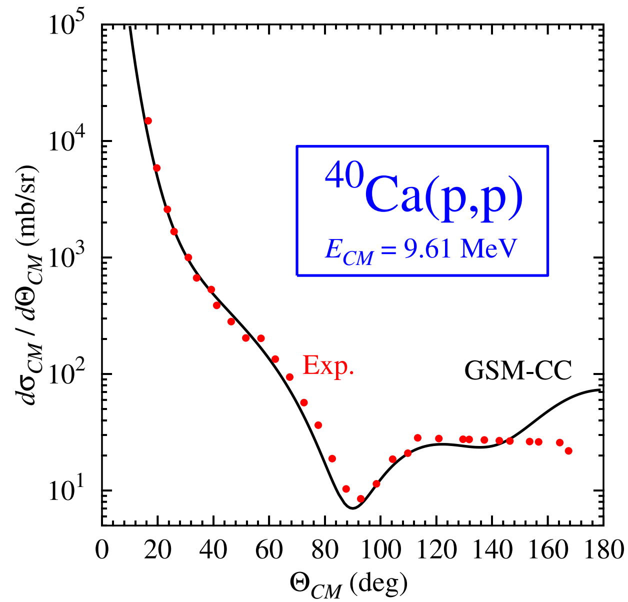


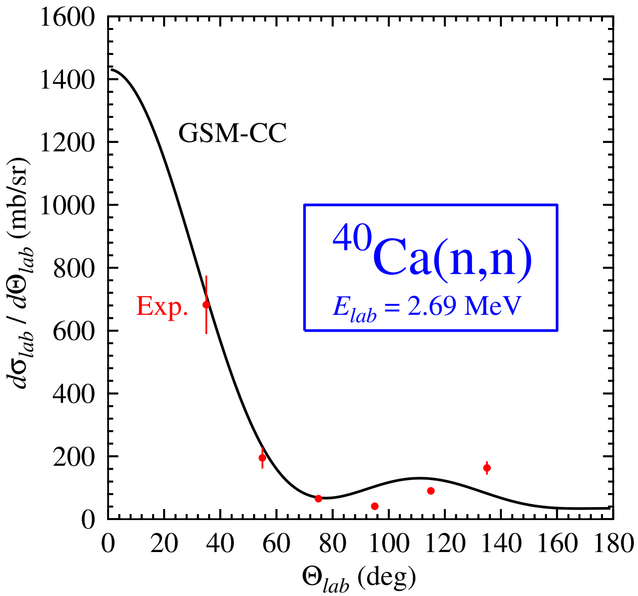


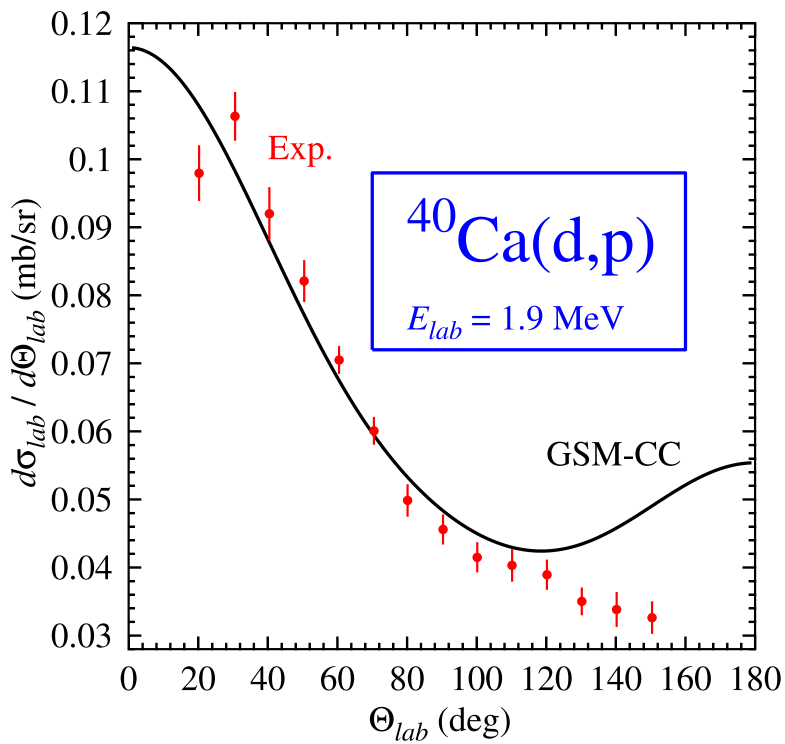
Cross sections qualitatively correct at higher energy
Spectrum fit provides with poor results
ANC fit gives a good reproduction of experimental data
Effect of missing channels and absence of 3-body force visible

A. Mercenne, N. Michel, M. Płoszajczak, Phys. Rev. C **99**, 044606 (2019)

^{40}Ca(d,p) transfer reaction







Valence nucleons above a ^{40}Ca core
Same framework used as for ^{4}He(d,d)

Scattering and transfer cross sections properly described
Simple model : no excitation from ^{40}Ca core,
Inter-nucleon correlations missing : larger model space needed

A. Mercenne, N. Michel, M. Płoszajczak, in preparation

Book on Gamow shell model

The Gamow Shell Model: the unified theory of nuclear structure and reactions

Authors : N. Michel and M. Płoszajczak

Publisher : Lectures Notes in Physics (Springer)

Background

Functional analysis, linear algebra, differential equations, standard quantum mechanics

Main topics

Introduction with one-body and two-body systems
Many-body theory of complex-energy physics
Halos and resonances in molecules and nuclei
Examples of applications in nuclear structure and reactions

Exercises and codes

Theoretical details about used methods
Computational applications using codes available from internet

Conclusion

Current status

GSM: structure model including the continuum
GSM-CC: reaction model including structure

Elastic and inelastic scattering cross sections
Radiative capture cross sections
Deuteron projectile scattering and transfer cross sections

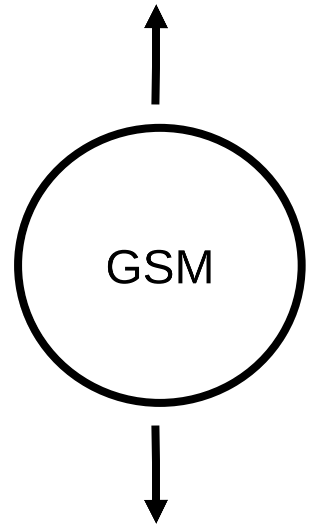
Energetics of the lightest nuclei
Nice agreement with experimental data
Structure effects on reaction observables explicit
Fine tuning of Hamiltonian necessary

Book on GSM and GSM-CC published
Theory, exercises, GSM codes publicly available

Perspectives

GSM: Isotopic chains with effective and realistic interactions
GSM-CC: non-resonant channels, few-nucleon clusters
Calculation of theoretical uncertainties with Bayesian statistics

Effective interaction at drip-lines for light nuclei
Observables dependence on spectrum ($^{6-8}$Li,^{7}Be,^{8}B)
Two-neutron and two-proton halos (^{31}F, ^{17}Ne)



Reactions with GSM-CC
^{6}Li(p,γ), ^{6}Li(n,γ),
^{7}Be(p,γ), ^{7}Li(n,γ),
^{14}O(p,p) ,^{18}Ne(p,p),
^{4}He(d,d),
^{40}Ca(p,p), ^{40}Ca(n,n), ^{40}Ca(d,p)