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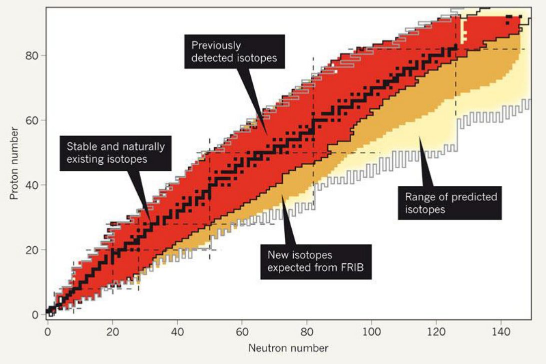
# Analysis of one-neutron knockout observables : sensitivity to the projectile's structure and dynamical effects

Chloë Hebborn and Pierre Capel

May, 6 2021

# Why study unstable nuclei ?

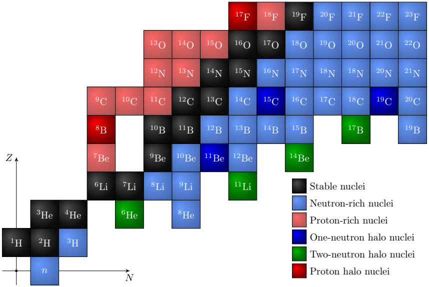
**Unstable nuclei** crucial to understand the formation of matter in stars




[Nature 477, 15, 2011]

**FRIB will access unexplored regions of the nuclear chart**

In the light neutron-rich sector :




**Halo nuclei** exhibit a very large matter radius

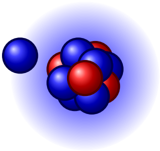
Compact core + one or two loosely-bound nucleons

Ex : $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + n$

$S_n = 501$ keV

$^{15}\mathrm{C} \equiv {}^{14}\mathrm{C} + n$

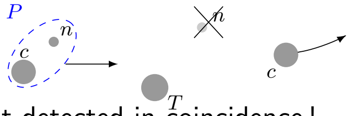
$S_n = 1218$ keV



Short-lived ($\tau_{^{11}\mathrm{Be}} \sim 13$ s) : studied through **reaction processes**

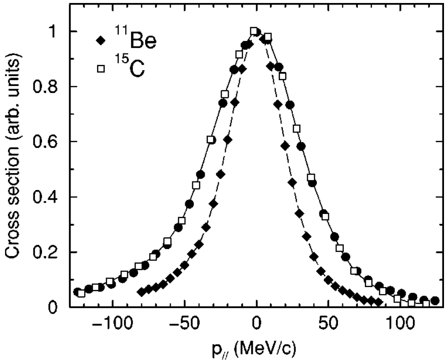
**One-neutron knockout :**

$P(\equiv c+n)+T\rightarrow c+X$




$\Rightarrow$ high statistics since the neutron is not detected in coincidence!

**Knockout carries information about the nucleus size**




[J. A. Tostevin *et al.*, PRC **66**, 024607 (2002)]

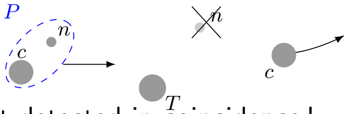
KO Reactions at $60A$ to $100A$ MeV

Sudden approximation :
distribution of $c$ inside the nucleus

$+$ Uncertainty principle : $\Delta r \Delta p > \hbar/2$

$\Rightarrow$ width linked to the nucleus size

# Knockout reactions a useful probe

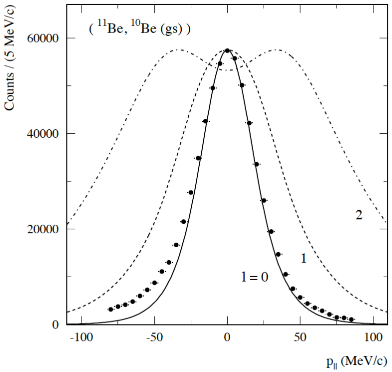



**One-neutron knockout :**

$P(\equiv c+n)+T\rightarrow c+X$

$\Rightarrow$ high statistics since the neutron is not detected in coincidence !

## Knockout used as a spectroscopic tool




Ex : $^{11}\text{Be}+{}^{9}\text{Be}\rightarrow{}^{10}\text{Be}$ @ 60$A$ MeV

Shell model predicts $1/2^-$ g.s.

| | | | |
|---|---|---|---|
| | $5/2^+$ | 1.274 | $d5/2$ |
| $\epsilon=0$ | | $^{10}$Be+$n$ | |
| | $1/2^-$ | -0.184 | $0p1/2$ |
| | $1/2^+$ | -0.501 | $1s1/2$ |

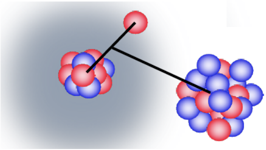
$^{11}$Be spectrum

Parity inversion of $1/2^+$ and $1/2^-$

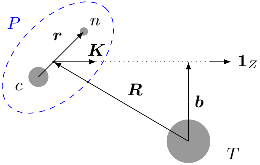
$\rightarrow$ visible in KO observables !

# Reaction model and eikonal approximation

**Three-body model of reaction**



$\Rightarrow$




- effective $c$-$n$ Hamiltonian $h_{cn}$ adjusted on low-energy spectrum
- $P-T$ interactions : optical potentials $V_{cT}$ and $V_{nT}$

**Three-body Schrödinger equation :**

$$[T_R + h_{cn} + V_{cT} + V_{nT}]\,\Psi(\boldsymbol{R},\boldsymbol{r}) = E\,\Psi(\boldsymbol{R},\boldsymbol{r})$$

Eikonal approximation : $\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\,\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})$ and $\Delta_{\mathbf{R}}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}) \ll K\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})$

$$\Rightarrow i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}) = [h_{cn} - \epsilon_0 + V_{cT} + V_{nT}]\,\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}),$$

**Dynamical Eikonal Approximation** (DEA) [Baye, Capel, and Goldstein, PRL **95**, 082502 (2005)]

# Eikonal model

$$\text{DEA}: i\hbar v \frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}) = [h_{cn} - \epsilon_0 + V_{cT} + V_{nT}]\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}),$$

Adiabatic approximation : $h_{cn} \approx \epsilon_0$

$$\Rightarrow i\hbar v \frac{\partial}{\partial Z}\widehat{\Psi}^{\text{eik}}(\boldsymbol{b},Z,\boldsymbol{r}) = [V_{cT} + V_{nT}]\widehat{\Psi}^{\text{eik}}(\boldsymbol{b},Z,\boldsymbol{r}),$$

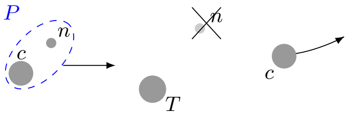
$$\boxed{\Psi^{\text{eik}}(\boldsymbol{b},Z,\boldsymbol{r}) \underset{Z\to+\infty}{\longrightarrow} e^{iKZ} e^{-\frac{i}{\hbar v}\int_{-\infty}^{+\infty}[V_{cT}(\boldsymbol{b_{cT}},Z)+V_{nT}(\boldsymbol{b_{nT}},Z)]dZ}\Phi_0(\boldsymbol{r}),}$$

**Usual eikonal model** [Glauber, *High energy collision theory*, (1959)]

**KO cross sections :** $\boxed{\sigma_{th} = \sum_i SF_i \times \sigma_{ko}^{sp,i}}$

→ occupancy of a s.p. orbital $i$ $SF_i$

→ s.p. KO cross section $\sigma_{ko}^{sp,i} = \underbrace{\sigma_{bu}^{sp,i}}_{\boxed{1}} + \underbrace{\sigma_{str}^{sp,i}}_{\boxed{2}}$



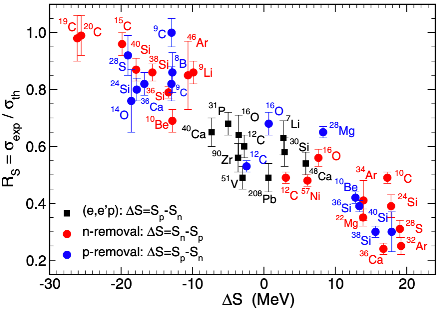

**This work :**

1 Diffractive breakup $\sigma_{bu}^{sp,i}$ : DEA or eikonal model

2 Stripping $\sigma_{str}^{sp,i}$ : eikonal model combined with Hussein-McVoy formalism

[Hussein and McVoy, NPA **445**, 124 (1985)]

# Knockout also used to study more bound nuclei




[Tostevin and Gade. PRC **90**, 057602 (2014)]

$\Delta S$ : $p$-$n$ asymmetry of the nucleus

$R_S = \sigma_{exp}/\sigma_{th}$

with $\sigma_{th} = \sum_i SF_i \times \sigma_{ko}^{sp,i}$

shell-model $SF_i$ and eikonal $\sigma_{ko}^{sp,i}$

$\Rightarrow$ $R_S$ interpreted as the deviations from shell-model calculations

→**Asymmetry-dependence in KO not seen in other reactions**

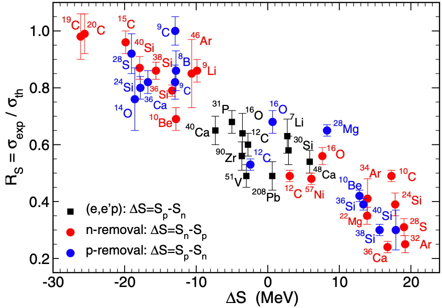
**ex : transfer, quasi-free scattering** [Aumann *et al.* Prog. Part. Nucl. Phys **118**, 103847 (2021)]

① Which part of the w.f. is probed for halo nuclei (top left) ?

② How does this evolve with the binding energy (going down right) ?

③ Improvement of the reaction model : extension of the DEA to stripping ?

# ① Which part of the w.f. is probed for halo nuclei (top left) ?




[Tostevin and Gade. PRC **90**, 057602 (2014)]

# Halo-EFT model of the projectile

**Test case :** $^{11}\mathrm{Be} + {}^{9}\mathrm{Be} \rightarrow {}^{10}\mathrm{Be} + X$ @ 60$A$ MeV

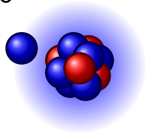
| | | |
|---|---|---|
| $5/2^+$ | 1.274 | $d5/2$ |
| $\epsilon = 0$ | $^{10}\mathrm{Be}+n$ | |
| $1/2^-$ | -0.184 | $0p1/2$ |
| $1/2^+$ | -0.501 | $1s1/2$ |

$^{11}$Be spectrum

$^{11}$Be :
g.s. $\epsilon_{1/2^+} = -0.501$ MeV
e.s. $\epsilon_{1/2^-} = -0.184$ MeV

**Halo-EFT model of $^{11}$Be :** uses the **separation of scale** to expand low-energy behaviour with $R_{\mathrm{core}}/R_{\mathrm{halo}} \sim 0.4$

[H.-W. Hammer *et al.* JPG **44**, 103002 (2017)]



$\Rightarrow$ $^{10}$Be-$n$ **effective potential**

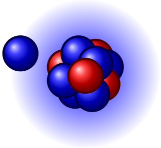
At NLO : $V_{lJ}(r) = V_{lJ}^{(0)} e^{-\frac{r^2}{2r_0^2}} + V_{lJ}^{(2)} r^2 e^{-\frac{r^2}{2r_0^2}}$ with $r_0$ cutoff

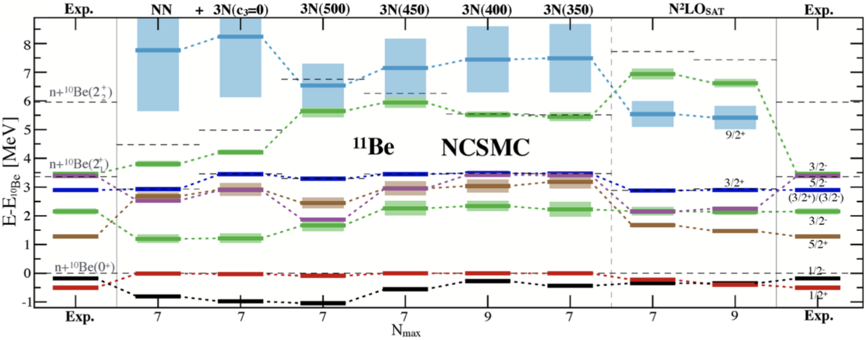
We constrain $V^{(0)}$ and $V^{(2)}$ in $s1/2$ and $p1/2$

1. Experimental binding energies of $1/2^+$ and $1/2^-$
2. Asympt. Norm. Constant (ANC) from *ab initio* calculations

No $p3/2$ interaction : negligible phase shifts at low $\epsilon$ [Calci *et al.* PRL **117**, 242501 (2016)]

**NCSMC description of $^{11}$Be**
**reproduces the energy levels and the parity inversion !**

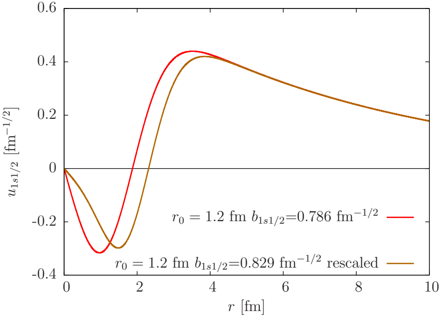




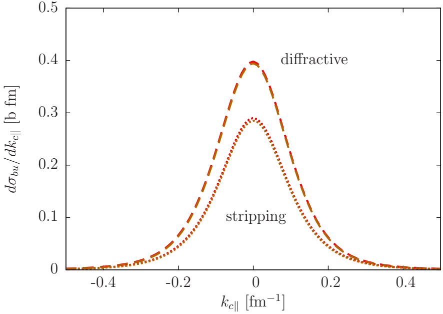

[Calci *et al.* PRL **117**, 242501 (2016)]

# Sensitivity of KO observables of halo nuclei

$^{10}$Be-$n$ g.s. wavefunction




$^{11}\text{Be} + {}^{9}\text{Be} \rightarrow {}^{10}\text{Be} + X$ @ 60$A$ MeV




[Hebborn and Capel, PRC **100**, 054607 (2019)]

Reference calculation : ANC=0.786 fm$^{-1/2}$ [Calci *et al.* PRL **117**, 242501 (2016)]

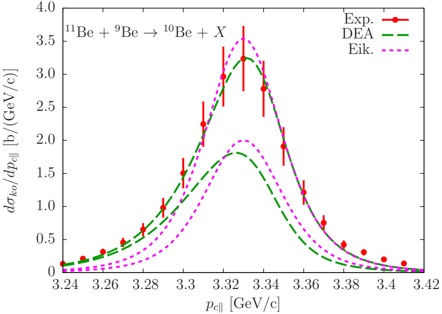
$\sigma_{bu} > \sigma_{str}$

Same ANC but SF=0.9 : same cross sections !

**KO of halo nuclei sensitive only to the asymptotics !**

⇒ Possibility to extract an ANC

# How does it compare to experimental data ?



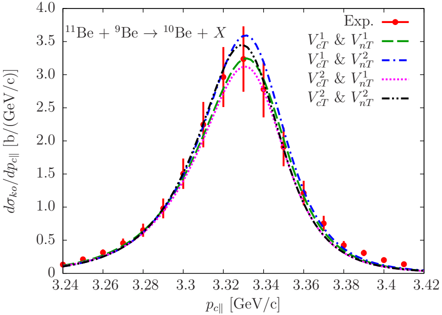

[Exp. : Aumann *et al.* PRL **84**, 35 (2000)]

Halo-EFT model of $^{11}$Be using ANCs of NCSMC

Eikonal **lacks asymmetry** due to the adiabatic approximation

$\sigma_{bu}$ computed with the DEA → Asymmetry well reproduced

# How does it compare to experimental data ?




[Exp. : Aumann *et al.* PRL **84**, 35 (2000)]

Halo-EFT model of $^{11}$Be using ANCs of NCSMC

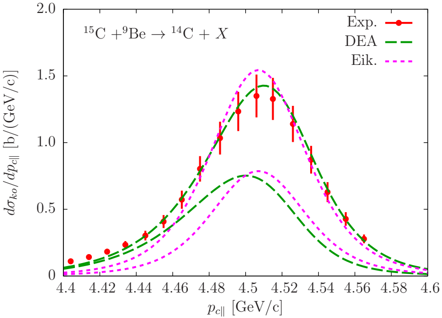
Eikonal **lacks asymmetry** due to the adiabatic approximation

$\sigma_{bu}$ computed with the DEA → Asymmetry well reproduced

Sensitivity to optical potentials : **ANC**$^2 = 0.62 \pm 0.06 \pm 0.09$ **fm**$^{-1}$

⇒ **Excellent agreement with ab initio value ANC$^2$=0.618 fm$^{-1}$**

# Similar analysis for $^{15}$C

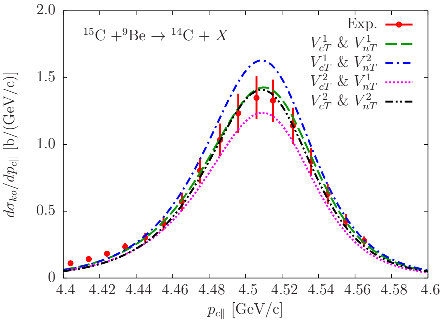



[Exp. : Tostevin *et al.* PRC **66**, 024607 (2002)]

Halo-EFT model of $^{15}$C using ANCs extracted from transfer (and NCSMC)

$\sigma_{bu}$ computed with the DEA → **Asymmetry well reproduced**

# Similar analysis for $^{15}$C




[Exp. : Tostevin *et al.* PRC **66**, 024607 (2002)]

Halo-EFT model of $^{15}$C using ANCs extracted from transfer (and NCSMC)

$\sigma_{bu}$ computed with the DEA → **Asymmetry well reproduced**

Strong sensitivity to optical potentials : **ANC**$^2 = 1.57 \pm 0.30 \pm 0.18$ **fm**$^{-1}$

⇒ **Excellent agreement with ab initio value ANC$^2$=1.644 fm$^{-1}$**

**ANCs of $^{11}$Be and $^{15}$C reproduce knockout data,...**

# ANCs of $^{11}$Be and $^{15}$C reproduce knockout data,...

## diffractive breakup data

PHYSICAL REVIEW C **98**, 034610 (2018)

**Dissecting reaction calculations using halo effective field theory and *ab initio* input**

P. Capel,[1,2,3,4,*] D. R. Phillips,[5,3,4,†] and H.-W. Hammer[3,4,‡]
[1]*Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany*
[2]*Physique Nucléaire et Physique Quantique (CP 229), Université libre de Bruxelles (ULB), B-1050 Brussels, Belgium*
[3]*Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*
[4]*ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*
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Reliable extraction of the $dB(\mathrm{E1})/dE$ for $^{11}$Be from its breakup at 520 MeV/nucleon

L. Moschini[a,*], P. Capel[b,a]
[a] Physique Nucléaire et Physique Quantique (C.P. 229), Université libre de Bruxelles (ULB), 50 avenue F.D. Roosevelt, B-1050 Brussels, Belgium
[b] Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, Johann-Joachim-Becher Weg 45, D-55099 Mainz, Germany

## transfer data,

PHYSICAL REVIEW C **98**, 054602 (2018)

**Systematic analysis of the peripherality of the $^{10}$Be$(d,p)^{11}$Be transfer reaction and extraction of the asymptotic normalization coefficient of $^{11}$Be bound states**

J. Yang[1,2,*] and P. Capel[1,3,†]
[1]*Physique Nucléaire et Physique Quantique (CP 229), Université libre de Bruxelles (ULB), B-1050 Brussels, Belgium*
[2]*Afdeling Kern-en Stralingsfysica, Celestijnenlaan 200d-bus 2418, B-3001 Leuven, Belgium*
[3]*Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany*

## and radiative capture data !

PHYSICAL REVIEW C **100**, 044615 (2019)

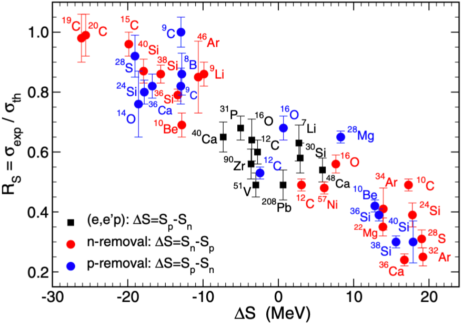
**$^{15}$C: From halo effective field theory structure to the study of transfer, breakup, and radiative-capture reactions**

Laura Moschini,[1,*] Jiecheng Yang,[1,2,†] and Pierre Capel[3,1,‡]
[1]*Physique Nucléaire et Physique Quantique (C.P. 229), Université libre de Bruxelles (ULB), 50 avenue F.D. Roosevelt, B-1050 Brussels, Belgium*
[2]*Afdeling Kern-en Stralingsfysica, Celestijnenlaan 200d-bus 2418, 3001 Leuven, Belgium*
[3]*Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, Johann-Joachim-Becher Weg 45, D-55099 Mainz, Germany*

# Summary for halo nuclei

① Halo nuclei : peripherality of knockout reactions

Halo-EFT bridges *ab initio* and reaction theory




[Tostevin and Gade. PRC **90**, 057602 (2014)]

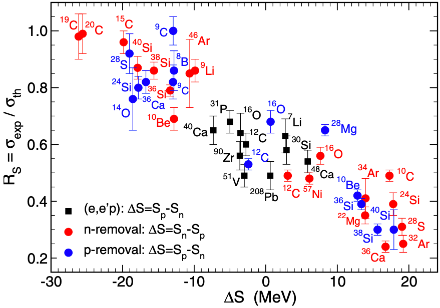
⇒ No sensitivity to the SF

⇒ Good agreement probably due to use of a realistic ANC

Sensitivity to the optical potentials → Need for a more systematic study

# ② What happens when the binding energy increases (going down right) ?

(suggested by D. Bazin and F. Nunes @ Reaction Seminar 2020)




[Tostevin and Gade. PRC **90**, 057602 (2014)]

# Deeply-bound projectile description

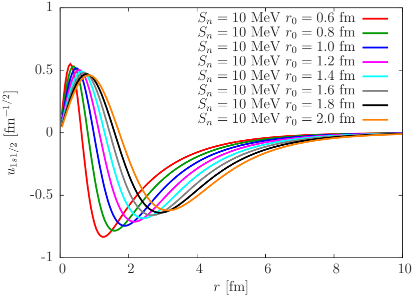
Irrealistic $^{11}$Be : $1/2^+$ g.s. $S_n = 10$ MeV

**Beyond Halo-EFT :** use a Gaussian potential $V_{s1/2}$

$$V_{s1/2}(r) = V^{(0)}_{s1/2} e^{-\frac{r^2}{2r_0^2}}$$

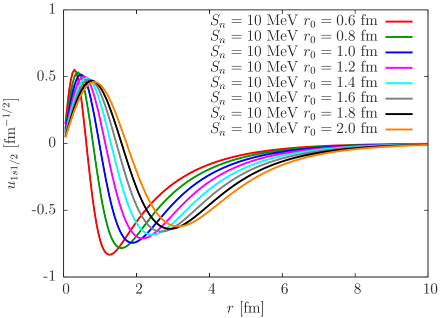
We constrain $V^{(0)}_{s1/2}$ with separation energy $S_n$

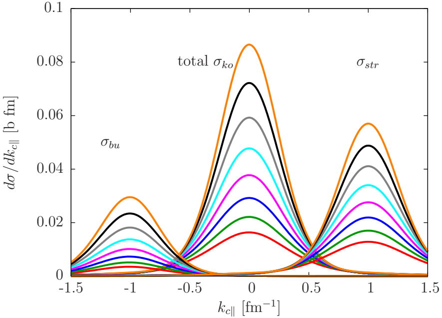
Generation of different g.s. wavefunctions with various $r_0$




Larger $r_0$ → larger ANC

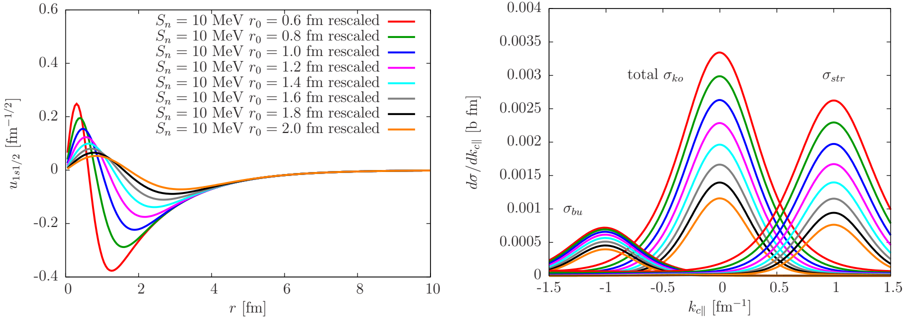
# Sensitivity for deeply-bound projectile




- Larger $r_0$ → larger ANC → larger $\sigma_{str}$ and $\sigma_{bu}$ (with $\sigma_{str} > \sigma_{bu}$)

# Sensitivity for deeply-bound projectile

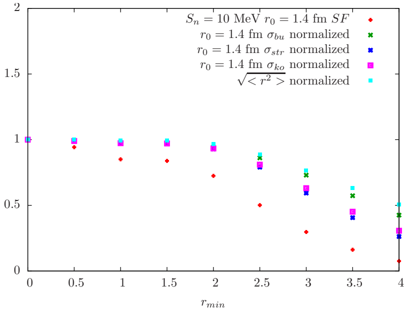



- Larger $r_0$ → larger ANC → larger $\sigma_{str}$ and $\sigma_{bu}$ (with $\sigma_{str} > \sigma_{bu}$)
- Rescale with the ANC → same asymptotics but SF=0.2–0.01
  $\sigma_{bu}$ : smaller spread → stays mainly peripheral
  $\sigma_{str}$ : no scaling (inverse order) & exhibit different shapes
  ⇒ $\sigma_{str}$ **is more sensitive to the inner part of the wavefunction**

  1. From which $r$ is $\sigma_{str}$ sensitive ? 2. How does it depend on SF ?

# Dependence of $\sigma_{ko}$ on SF

1. From which $r$ is $\sigma_{str}$ sensitive ? 2. How does it depend on SF ?




$$u^{r_{min}}_{1s1/2}(r) = \begin{cases} 0 & \text{if } r < r_{min} \\ u_{1s1/2}(r) & \text{if } r \geq r_{min} \end{cases} \qquad SF = \int_{r_{min}}^{+\infty} |u^{r_{min}}_{1s1/2}(r)|^2 \, dr$$

⇒ SF sensitive to all distances

⇒ $\sigma_{ko}$ insensitive to $r < 1.5$ fm (decrease by only 3%)

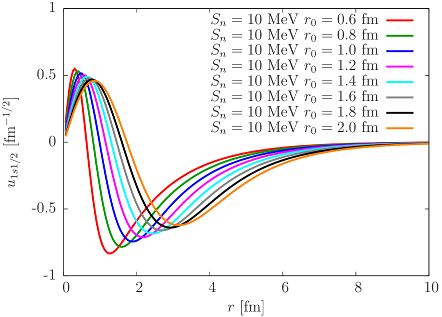
→ insensitivity to the internal node

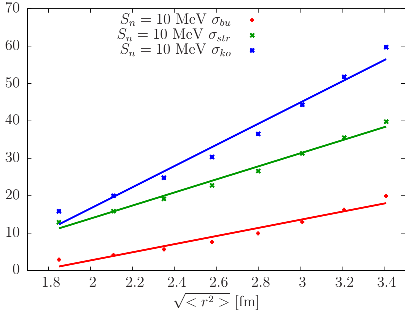
⇒ **non-linear dependence of $\sigma_{ko}$ on the normalization SF**

⇒ $\sigma_{ko}$ **behaves similarly with** $r_{min}$ **as** $\sqrt{\langle r^2 \rangle}$

# Dependence of $\sigma_{ko}$ on $\sqrt{\langle r^2 \rangle}$

Each $r_0$ generates wave function with various $\sqrt{\langle r^2 \rangle}$




**Approximate linear dependence of $\sigma_{ko}$ in $\sqrt{\langle r^2 \rangle}$**

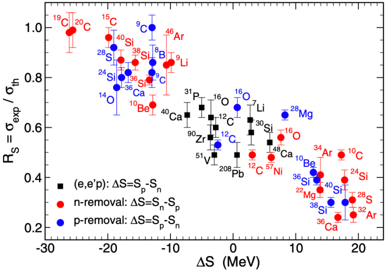
→ also observed in [Gade *et al.* PRC 044306 (2008)]

# Summary for deeply-bound nuclei

② Deeply-bound projectile $S_n = 10$ MeV :

- $\sigma_{ko}$ is sensitive to the inner part but only above a certain distance

→ $\sigma_{ko}$ **does not depend linearly on SF but approximatively on** $\sqrt{\langle r^2 \rangle}$




[Tostevin and Gade. PRC **90**, 057602 (2014)]

→ Still not clear why there is a strong reduction of exp.-th. ratio

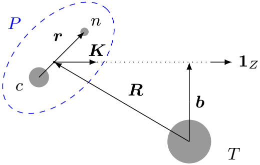
⇒ **Improvement of the few-body model of reaction are still needed**

ex : core excitation as in X-CDCC ? [Louchart, Obertelli, Boudard, Flavigny PRC **83** 011601(R) (2011)]

dynamical treatment of the stripping as in TC ? [Flavigny *et al.* PRL **108**, 252501 (2012)]

③ Improvement of the reaction model :
study of the extension of the DEA to stripping reactions

# Application of the ERT to the DEA




**Eikonal Reaction theory (ERT) :**

treats short-range interaction adiabatically

and long-range dynamically

[M. Yahiro *et al.* PTP **126**, 167 (2012)]

Application to Eikonal-CDCC : $S$-matrix $\widehat{S} = \widehat{S}^{\text{eik}}_{nT} \cdot \widehat{S}^{\text{E-CDCC}}_{cT}$

⇒ 5% error on $\sigma_{bu}$ of halo nuclei on light and heavy targets

⇒ ERT factorisation of $S$-matrix allows to use Hussein-McVoy formalism

**Study of the ERT applied to the DEA :**

Adiabatic treatment of [1] Nucl. $c$-$T$ interaction $\text{ERT}^{(c)}$ $\widehat{S} = \widehat{S}^{\text{eik}}_{\text{Nucl.}\,cT} \cdot \widehat{S}^{\text{DEA}}_{nT+\text{Coul.}\,cT}$

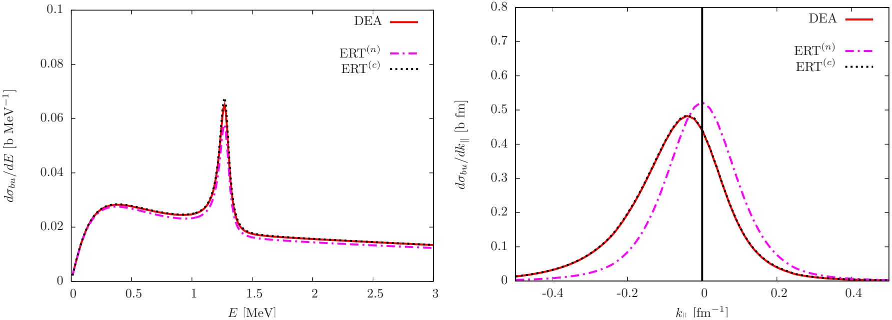
[2] $n$-$T$ interaction $\text{ERT}^{(n)}$ $\widehat{S} = \widehat{S}^{\text{eik}}_{nT} \cdot \widehat{S}^{\text{DEA}}_{cT}$

→ 5% error on $\sigma_{bu}$ for light and heavy targets [Hebborn and Capel, arXiv :2104.04712]

→ **Is the ERT accurate for energy and momentum distributions ?**

# Analysis of the ERT for light targets

$^{11}$Be + $^{12}$C → $^{10}$Be + $n$ + $^{12}$C @ 67$A$ MeV




[Hebborn and Capel, arXiv :2104.04712]

DEA accurate for these reactions [Goldstein, Baye and Capel, PRC **73**, 024602 (2006)]

→ Asymmetric and shifted center caused by projectile's **dynamics**

ERT$^{(c)}$ accurate for both $E$ and $k_\parallel$ distributions

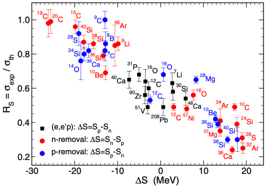
ERT$^{(n)}$ accurate for $E$ distribution but lacks asymmetry in $k_\parallel$

⇒ $n$-$T$ has to be treated dynamically

⇒ **No simple extension of the DEA to stripping observables**

**Knockout reactions** : used to probe the s.-p. structure of exotic nuclei

Asymmetry-dependence of the ratio exp-th.
not understood




[Tostevin and Gade, PRC **90**, 057602 (2014)]

① **Which part of the w.f. is probed for halo nuclei (top left) ?**

- peripherality of knockout reactions
  - ⇒ No sensitivity to the SF
  - ⇒ $\sigma_{exp}/\sigma_{th} \sim 1$ probably due to use of realistic ANCs
- Halo-EFT bridges *ab initio* and reaction theory

  ⇒ **One unique Halo-EFT description of $^{11}$Be and $^{15}$C reproduces knockout, transfer and diffractive breakup data**
- Sensitivity to **optical potentials** ⇒ Need for a more systematic study

# Conclusions and prospects

② **How does the sensitivity evolve with the binding energy ?**

Deeply-bound nucleus $S_n = 10$ MeV : $\sigma_{str}$ dominant

- $\sigma_{ko}$ is sensitive to the inner part but only above a certain distance
- $\sigma_{ko}$ **does not depend linearly on SF but approximately on** $\sqrt{\langle r^2 \rangle}$

→ Still not clear why there is a strong reduction of exp.-th. ratio

⇒ **Improvements of the few-body model of reaction are still needed**

③ **Study of the extension of the DEA to stripping using ERT**

- Adiabatic treatment of nucl. $c$-$T$ accurate for $\sigma_{bu}$, energy and momentum distributions
- Adiabatic treatment of $n$-$T$ accurate for $\sigma_{bu}$ and energy distributions
  → fails to reproduce the asymmetry of momentum distributions

⇒ **No simple generalization of the Hussein-McVoy approach & extension of the DEA to stripping still needed**

**Thank you for your attention**